Pugh Ed. 2, Ch. 2, #72. Let H be the hyperbola $\{(x, y) \in \mathbb{R}^2 \,|\, xy = 1 \text{ and } x, y > 0\}$ and let X be the x-axis.

(a) Is the set $S = X \cup H$ connected?
(b) What if we replace H with the graph G of any continuous positive function $f\colon \mathbb{R} \to (0, \infty)$; is $X \cup G$ connected?
(c) What if f is everywhere positive but discontinuous at just one point?

Pugh Ed. 2, Ch. 2, #83. The open cylinder is $(0, 1) \times S^1$. The punctured plane is $\mathbb{R}^2 \backslash \{0\}$.

(a) Prove that the open cylinder is homeomorphic to the punctured plane.
(b) Prove that the open cylinder, the double cone, and the plane are not homeomorphic.

April 2008, #6. Let f be a continuous function on the unit square $[0, 1] \times [0, 1]$, and for $s \in [0, 1]$ let $g(s) = \max\{f(s, t) \,|\, t \in [0, 1]\}$. Show that g is a continuous function on $[0, 1]$.