August 2008, Problem 4 Let f be a differentiable function on $\mathbb{R}$. Assume that there is no x such that $f(x) = 0 = f'(x)$. Show that the set

$$S = \{x \in [0,1] : f(x) = 0\}$$

is finite.