TOPOLOGY 2 - HOMEWORK 10

(1) For a short exact sequence $0 \to A_* \xrightarrow{i} B_* \xrightarrow{j} C_* \to 0$ of chain complexes, prove that $\mathrm{Im}(j_*) = \mathrm{Ker}(\partial)$ in the associated sequence

$$\cdots \to H_n(A) \xrightarrow{i_*} H_n(B) \xrightarrow{j_*} H_n(C) \xrightarrow{\partial} H_{n-1}(A) \to \cdots$$

(2) If $A_* \subset B_*$ is a chain subcomplex; ie, $A_n \subset B_n$ and $\partial(A_{n+1}) \subset A_n$ for all n, prove that $(B/A)_* \doteq \{B_n/A_n\}_{n\in\mathbb{N}}$ inherits the structure of a chain complex from B_* and that

$$0 \to A_* \to B_* \to (B/A)_* \to 0$$

is a short exact sequence of chain complexes.

(3) Hatcher, Section 2.1, Exercise 14.

(4) Hatcher, Section 2.1, Exercise 16.

(5) Hatcher, Section 2.1, Exercise 26.
Hint: You may use the following fact: for a topological space X and $x_0 \in X$, $H_1(X) \cong \pi_1(X, x_0)^{ab}$.