TOPOLOGY 2 - HOMEWORK 11

(1) Prove the **Lebesgue Covering Lemma**: If X is a compact metric space then for any open cover $\{U_j\}$ of X there exists $\epsilon > 0$ with the following property: every subset C of X such that $\mathrm{diam}(C) < \epsilon$ is entirely contained in U_j for some j. (Here $\mathrm{diam}(C) = \sup\{d(x,y) \mid x, y \in C\}$.)

(2) Hatcher, Section 2.1, Exercise 17.

(3) Explicitly describe a homeomorphism of pairs $(\Delta^n, \partial\Delta^n) \to (\mathbb{D}^n, \partial\mathbb{D}^n)$

(4) Let $\Lambda^n = \partial\Delta^n - int(\{0\} \times \Delta^{n-1})$. Explicitly describe a deformation retraction $\Delta^n \to \Lambda^n$. (**Hint**: First describe a retraction $r\colon \Delta^n \to \Lambda^n$, then use the straight-line homotopy $x \mapsto r(x)$.)