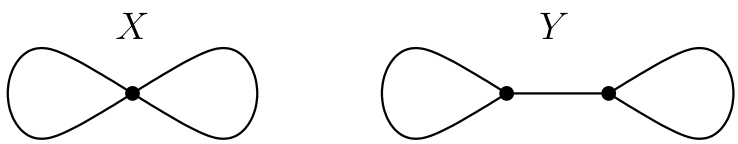
TOPOLOGY 2 - HOMEWORK 2

(1) For $n \geq 0$, let $\mathbb{R}P^n = S^n/\mathbf{x} \sim -\mathbf{x}$ with the quotient topology. Describe a CW-structure on $\mathbb{R}P^n$ with a single k-cell for each $k \in \{0, 1, \ldots, n\}$ and no others.

(2) Prove that the identification space Σ_g described in class has the structure of a CW-complex with a single 0-cell, $2g$ 1-cells, and a single 2-cell.

(3) Prove that the graphs X and Y below are homotopy-equivalent but not homeomorphic.



(4) Prove that a homotopy equivalence $f\colon X \to Y$ induces a bijection from the set of components of X to the set of components of Y.

(The set $\mathcal{C}(X)$ of components of X can be described as $X/\sim$, taking $x \sim y$ if a connected subset of X contains both x and y. A continuous map $f\colon X \to Y$ has a well-defined induced map $f_*\colon \mathcal{C}(X) \to \mathcal{C}(Y)$ defined by $f_*([x]) = [f(x)]$, to which I refer above.)

(5) Hatcher, Section 1.1, Exercise 5.