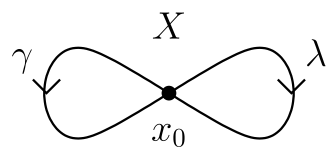
TOPOLOGY 2 - HOMEWORK 3

(1) Prove that $\mathbb{R}^n/\mathbb{Z}^n$ is homeomorphic to $S^1 \times S^1 \times \ldots \times S^1$, where the Cartesian product is taken n times and "$\mathbb{R}^n/\mathbb{Z}^n$" refers to the quotient by the following equivalence relation:

$$(x_1, \ldots, x_n) \sim (x_1 + a_1, \ldots, x_n + a_n) \quad \text{for all } (x_1, \ldots, x_n) \in \mathbb{R}^n \text{ and } (a_1, \ldots, a_n) \in \mathbb{Z}^n$$

(Hint: use the lemma from problem 1 of assignment 1. Note that the second part still holds for non-compact X such that X^* is compact.)

(2) For X and $a = [\gamma], b = [\lambda] \in \pi_1(X, x_0)$ as below, prove that $a^m b^n \neq 1$ for all $(m, n) \in \mathbb{Z}^2 - \{0\}$.



(Hint: you might use some of the many maps $X \to S^1$.)

(3) Show that the restriction to $[0, 1)$ of $p\colon t \mapsto (\cos(2\pi t), \sin(2\pi t))$ is not a covering map to S^1.

(4) Hatcher, Section 1.3, Exercise 1

(5) Hatcher, Section 1.3, Exercise 3

(6) Hatcher, Section 1.1, Exercise 18 (3 notes:
- Proposition 1.14 is the proof that $\pi_1(S^n) = 0$.
- The *wedge sum* $X \vee Y$ of X with Y is constructed as follows: fix $x \in X$ and $y \in Y$, and take $X \vee Y = X \sqcup Y / x \sim y$. You can feel free to choose x and y as you see fit — assuming X and Y are path-connected, different choices yield homotopy-equivalent spaces.
- Don't worry about the case of infinitely many cells.)