

## TOPOLOGY 2 - HOMEWORK 4

- (1) Compute  $\pi_1(\mathbb{R}P^2)$ .
- (2) Hatcher, Section 1.2, Exercise 1.
- (3) Hatcher, Section 1.2, Exercise 7.
- (4) Hatcher, Section 1.2, Exercise 14. (In this problem you may assume the result of Exercise 6.)
- (5) Let  $H$  be a regular hexagon with sides  $e_0, \dots, e_5$ , ordered and oriented counterclockwise. Compute  $\pi_1(X)$ , where  $X$  is the quotient space of  $H$  in which each point of  $e_i$  is identified with the corresponding point on the opposite side  $\bar{e}_{i+3}$  (taking  $i+3$  modulo 6) with opposite orientation.

More precisely:  $t \cdot x_i + (1-t) \cdot x_{i+1} \sim (1-t) \cdot x_{i+3} + t \cdot x_{i+4}$  for each  $t \in I$  and  $i \in \{0, \dots, 5\}$ , where  $x_i = e_i \cap e_{i-1}$  and all indices are taken modulo 6.

**Overall hint:** First construct a CW structure, then apply the final result from Friday's class.