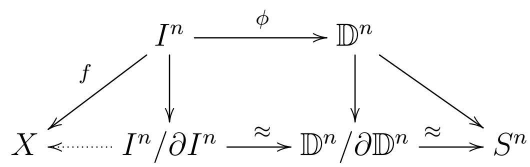
TOPOLOGY 2, HOMEWORK 10

(1) For a fixed space X and $x_0 \in X$, we showed in class that a map $f\colon (I^n, \partial I^n) \to (X, x_0)$ induces a map $\bar{f}\colon (S^n, \mathbf{s}_0) \to (X, x_0)$, where $\mathbf{s}_0 = (0, \ldots, 0, 1)$, obtained by going right-to-left on the bottom line of the diagram below.



For two such maps f and g, show that the map $\overline{f+g}$ induced in this way is the same as $(\bar{f} \vee \bar{g}) \circ c$, where $c\colon S^n \to S^n/(\{0\} \times S^{n-1})$ is the quotient map and $S^n/(\{0\} \times S^{n-1})$ is homeomorphically identified with $S^n \vee S^n$.

(2) Show that $\mathbb{C}P^n = (\mathbb{C}^{n+1} - \{\mathbf{0}\})/\mathbf{z} \sim \lambda\mathbf{z}\ \forall\lambda \in \mathbb{C}^*,\ \forall\mathbf{z}$ is Hausdorff.

(3) Hatcher, Section 4.1 #2

(4) Hatcher, Section 4.1 #10

(5) Hatcher, Section 4.1 #14
Hint: for any CW complex X, $X \times I$ inherits a natural CW complex structure from that of X and one on I (say with two 0-cells and a single 1-cell). See Hatcher's Theorem A.6.