TOPOLOGY 2, HOMEWORK 11

(1) For a fiber bundle $p\colon E \to B$, $x_0 \in B$ and $\tilde{x}_0 \in p^{-1}(x_0)$, show that the sequence below is exact at $\pi_n(B, x_0)$ for $n \geq 1$.

$$\pi_n(E, \tilde{x}_0) \xrightarrow{p_*} \pi_n(B, x_0) \xrightarrow{\partial} \pi_{n-1}(p^{-1}(x_0), \tilde{x}_0)$$

Here ∂ is the *connecting homomorphism* defined by $\partial[f] = [\tilde{f}_1]$ for $[f] \in \pi_n(B, x_0)$, where $f\colon (I^n, \partial I^n) \to (B, x_0)$ is regarded as a homotopy on $I^{n-1} \times I$ and lifted to $\tilde{f}\colon I^n \to E$ with $\tilde{f}\left((I^{n-1} \times \{0\}) \cup (\partial I^{n-1} \times I)\right) = \{\tilde{x}_0\}$.

(2) Hatcher, Section 4.2 #30

(3) Hatcher, Section 4.2 #32

(4) (a) Show that every self-map of S^n with degree 0 has a fixed point. (*Hint*: Show that a map with no fixed points is homotopic to the antipodal map.)

(b) Prove the Brouwer fixed point theorem for maps $f\colon \mathbb{D}^n \to \mathbb{D}^n$ by applying part (a) to the map $S^n \to S^n$ that sends the northern and southern hemispheres to the southern hemisphere using f.

(5) Construct a surjective map $S^n \to S^n$ of degree 0 for each $n \geq 1$.