TOPOLOGY 2, HOMEWORK 3

(1) (2 pts) The *mapping cylinder* of a continuous map $f\colon X \to Y$ is the identification space obtained by attaching $X \times I$ to Y along $X \times \{1\}$ using the map $(x, 1) \mapsto f(x)$. Show that the mapping cylinder of any continuous map $f\colon S^1 \to S^1$ admits the structure of a CW complex.

(2) (2 pts) Hatcher, Section 1.1 #1

(3) (2 pts) Hatcher, Section 1.1 #4

(4) (4 pts) Hatcher, Section 1.1 #5

(5) (2 pts) A *topological group* is a group equipped with a topology such that the inversion and multiplication maps are continuous. (The *inversion map* $G \to G$ is given by $g \mapsto g^{-1}$, and the *multiplication map* $G \times G \to G$ is given by $(a, b) \mapsto a \cdot b$, the product of a and b.) For a topological group G with identity element e, show that $\pi_1(G, e)$ is abelian.
(*Hint*: For loops γ and λ based at e, consider the map $(s, t) \mapsto \gamma(s) \cdot \lambda(t)$.)