TOPOLOGY 2, HOMEWORK 9
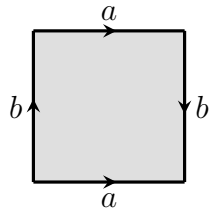
(1) The *Klein bottle* K is the quotient space of the square by the edge identifications pictured below.

    (a) Give a presentation for $\pi_1 K$.

    (b) Show that K is homeomorphic to the orbit space $\mathbb{R}^2/G$, where G is generated by $(x, y) \mapsto (x, y+1)$ and the *glide reflection* $(x, y) \mapsto \left(x + \frac{1}{2}, -y\right)$. *Hint*: embed the square in $\mathbb{R}^2$ as $\left\{(x, y) \,|\, 0 \le x \le \frac{1}{2}, -\frac{1}{2} \le y \le \frac{1}{2}\right\}$.

    (c) Describe a degree-two covering $p\colon T \to K$, where T is the (two-dimensional) torus, and identify $p_*(\pi_1 T)$ as a subgroup of $\pi_1 K$.

(2) Hatcher, Section 1.3 #12

(3) Hatcher, Section 1.3 #20

(4) Hatcher, Section 1.3 #23

(5) Hatcher, Section 1.3 #25