Math 2701 - Topology 2
Syllabus and Topics List

Course Overview: This is a course in algebraic topology, a subject which has as its primary (or at least initial) goal the development of algebraic invariants to distinguish certain topological spaces up to homeomorphism or homotopy equivalence. The spaces studied in algebraic topology usually lack distinguishing features from the standpoint of point-set topology.

Instructor: Jason DeBlois. My office is Thackeray 407, and my email is `jdeblois@pitt.edu`.
Course website: `http://www.pitt.edu/~jdeblois/S16_top2.html`
Office Hours: TBD. Check the course website.

Textbook: *Algebraic Topology*, by Allen Hatcher. It is freely available on the internet at `https://www.math.cornell.edu/~hatcher/AT/ATpage.html`

Prerequisites: Basic knowledge of point-set topology and abstract algebra. Here is a list of topics from each subject with which you should be familiar:

Point-set: Open and closed sets, bases, product and subspace topologies, connectedness, compactness, continuous functions, the Hausdorff property.

Algebra: Groups, subgroups, quotient groups and homomorphisms, presentations, direct sums, direct products, and finitely generated abelian groups.

Grades: Your course grade will be determined as follows:

- Homework, 50%. (Assigned weekly on the course website.)
- Midterm, 25%
- Final project, 25%.

The final project will consist of either a 3-5 page paper or a talk in front of the class (I will decide which one later.)

Topics

Here is a brief list of the main points I hope to cover.

Basic Topology.

- Quotient spaces
- CW-complexes
- Retractions, deformation retractions, homotopy equivalence, and homeomorphism.
- The basic examples: $\mathbb{R}^n$, Σ_g $(g \geq 1)$, S^n, $\mathbb{R}P^n$, $\mathbb{C}P^n$ $(n \leq \infty)$. CW structures.

Fundamental group.

- Basic properties: functoriality, homotopy invariance, $\pi_1(X \times Y) \cong \pi_1(X) \times \pi_1(Y)$.
- Free groups, free products, and presentations.
- Van Kampen's theorem; application: π_1(CW-cx).
- Fundamental groups of the basic examples.
- Classical theorems: Brouwer fixed point & Borsuk–Ulam, fund. theorem of algebra.

Covering spaces.

- Definition and basic properties: π_1-injectivity and # sheets.
- Map lifting and homotopy lifting.
- The classification of covering spaces, existence of the universal cover.
- Deck transformations and properly discontinuous group actions.

Higher homotopy groups.

- Definition and basic properties (eg. abelian).
- Fibrations, LES of a fibration.
  - Example: Hopf fibration $S^{2n+1} \to \mathbb{C}P^n$
- Cellular approximation; $\pi_k S^n$, $k \leq n$.
- Degree of $f\colon S^n \to S^n$; basic computations.
- Whitehead's theorem; weak homotopy equivalence; CW approximation.

Homology.

- Chain complexes and homology groups.
- CW-homology.
  - Euler characteristic; in particular, χ(ft CW-cx).
  - CW-homology of the basic examples.
- Singular homology
  - Definition and basic properties: functoriality and homotopy invariance.
  - Reduced homology, relative homology, and local homology.
  - Basic homological algebra.
    - SES of chain cxes $\rightsquigarrow$ LES in homology (definition of ∂ in particular).
    - The five lemma.
  - The LES of (X, A) and of (X, A, B).
  - Excision and the LES of X/A, for a good pair (X, A).
  - Equivalence of singular and cellular homology; homotopy classification of basic examples.
  - The Mayer-Vietoris sequence.
- Δ-complexes, simplicial complexes, and simplicial homology
  - equivalence with singular homology.
  - simplicial approximation; the Lefschetz fixed point theorem.
- Homology with arbitrary (abelian group) coefficients.
- Axioms and the categorical perspective
- Degree via homology; its computation using local degree.

Cohomology. (time permitting, which it probably won't)

- Definition and basic properties; analogies with homology.
- Universal coefficient theorem and Ext.
- Cup product; $H^*(\mathbb{R}P^n)$ and $H^*(\mathbb{C}P^n)$.
- Cohomology of manifolds; Poincaré duality.