

August 2012, Problem 4.

Let Ω be an open set in \mathbb{R}^n . Let $F: \Omega \rightarrow \mathbb{R}^n$ and $G: \mathbb{R}^n \rightarrow \mathbb{R}$ be two continuously differentiable functions such that $G \circ F = 0$ on Ω . Suppose that

$$\sum_{j=1}^n \left(\frac{\partial G(x)}{\partial x_j} \right)^2 > 0$$

for every $x \in F(\Omega)$. Prove that $\det(DF) = 0$ on Ω .