

May 2016, Problem 4

Let $D = \{(x, y) \mid x^2 + y^2 < 1\}$ be the unit disk in \mathbb{R}^2 . Let $f, g \in C^2(D)$ be such that g is bounded on D , $f(x, y) \rightarrow +\infty$ as $x^2 + y^2 \rightarrow 1$, and moreover $\Delta f = e^f$ and $\Delta g \geq e^g$ at all points of D . Here $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian. Show that $f(x, y) \geq g(x, y)$ for any $(x, y) \in D$.