

April 2013, Problem 3. Prove that $\sum_{n=1}^{\infty} \frac{x^\alpha}{\sqrt{n}(n^2 + x^3)}$ is uniformly convergent on $[0, \infty)$ if $\alpha = 2$, but not uniformly convergent on $[0, \infty)$ if $\alpha = 3$.