

**August 2016, Problem 2**

Prove that there is an increasing sequence of integers  $a_1 < a_2 < a_3 < \dots$  such that for every  $k \in \mathbb{N}$ , the sequence  $\{\sin(ka_n)\}_{n=1}^{\infty}$  converges.

**April 2010, Problem 1**

Let  $I$  be the interval  $[0, \infty)$ . For  $n \in \mathbb{N}$  and  $t \in I$ , let

$$f_n(t) = \sin(\sqrt{t + 4n^2\pi^2}).$$

- (i) Show that the sequence  $\{f_n\}$  is equicontinuous on  $I$ ;
- (ii) Show that  $\{f_n\}$  does not contain a subsequence which is uniformly convergent on  $I$ .