April 2002, Problem 4

(a) Suppose the functions $\{f_n(x) : n = 1, 2, 3, \ldots\}$ are integrable and uniformly bounded on $[a, b] \subset \mathbb{R}$. For each n, let $F_n(x) = \int_a^x f_n(t)\,dt$, for $x \in [a, b]$. Show that there exists a subsequence F_{n_k} of F_n which converges uniformly on $[a, b]$.

(b) Evaluate

$$\sum_{n=0}^{\infty} \frac{n+1}{2^n}.$$