Pugh Ed. 2, Ch. 4 #30.

Give an example of a continuous map of a compact, nonempty, path-connected metric space into itself that has no fixed point.

August 2014, Question 8.

Let $\mathcal{F} \subset C^\infty[0,1]$ be a uniformly bounded and equicontinuous family of smooth functions on $[0,1]$ such that $f' \in \mathcal{F}$ whenever $f \in \mathcal{F}$. Suppose that

$$\sup_{x\in[0,1]} |f'(x) - g'(x)| \le \frac{1}{2} \sup_{x\in[0,1]} |f(x) - g(x)|$$

for all $f, g \in \mathcal{F}$. Show that there exists a sequence f_n of functions in $\mathcal{F}$ that tends uniformly to Ce^x, for some real constant C.