Math 2701 - Topology 2
Possible Final Project Topics List

The classification of one-manifolds. There do not seem to be many proofs of this classical result in print, though there are more on the internet. Guillemin–Pollack's *Differential Topology* gives one in the smooth case, and David Gale's *American Mathematical Monthly* article "The Classification of 1-Manifolds: A Take-Home Exam" gives a topological proof (in exercises; google it).

The classification of surfaces. A cornerstone result in many different areas of topology. There's a classical proof using triangulations which is presented, for instance, in Massey's *A Basic Course in Algebraic Topology*, Chapter 1 §7. "Conway's ZIP Proof" (*American Mathematical Monthly* article by Francis–Weeks; google it) presents a "lighter, fat-free *nouvelle cuisine* approach that retains all the classical flavor of elementary topology", due to John H. Conway.

The Jordan Curve theorem, the Schoenfliess Theorem, and the Alexander horned sphere. The first asserts that every embedded circle in $\mathbb{R}^2$ divides it into two components, each of which it bounds; the second, that one of these is homeomorphic to a disk. These are discussed in most basic topology books, eg. Armstrong's *Basic Topology* and Munkres' *Topology.* The Jordan curve theorem extends to higher dimensions whereas the Schoenfliess theorem does not (in the topological category, at least), and the Alexander horned sphere is a counterexample: a two-sphere embedded in $\mathbb{R}^3$ that does not bound a ball. See Hatcher's *Algebraic Topology*, Example 2B.2.

First homology vs. the abelianization of π_1. Section 1.A of Hatcher's *Algebraic Topology* gives a proof of the fundamental computational result that for a path connected space X, $H_1(X)$ is naturally isomorphic to the abelianization $\pi_1(X)^{\text{ab}}$ of $\pi_1(X)$.

$K(G,1)$ **spaces and graphs of groups.** These important objects in low-dimensional topology and geometric group theory are nicely introduced in Hatcher's *Algebraic Topology*, Section 1.B.

The Wirtinger Presentation for π_1(knot complement). A *knot* is a subspace K of $\mathbb{R}^3$ (or $\mathbb{S}^3$) homeomorphic to a circle, and its *complement* is $\mathbb{S}^3 - K$. There is a "Wirtinger presentation" for $\pi_1(\mathbb{S}^3 - K)$ associated to a "nice" projection of K to a plane. It is nicely explained in Section 10.2 of Armstrong's *Basic Topology*, complete with a discussion between a "fussy algebraist" and an "optimistic geometer". See also Hatcher's Section 1.2, exercise 22.

Orbifolds In studying group actions on topological spaces one often encounters actions with fixed points which are nonetheless "like" covering space actions. The quotient space by such an action (on, say, $\mathbb{R}^n$) inherits the structure of what topologists call an *orbifold.* The two-dimensional case of this is presented in Section 1 of Peter Scott's lovely article "The Geometries of 3-Manifolds" in the *Bulletin of the London Mathematical Society.*