Math 2701 - Topology 2
Syllabus and Topics List

Course Overview: This is a course in algebraic topology, a subject which has as its primary (or at least initial) goal the development of algebraic invariants to distinguish certain topological spaces up to homeomorphism or homotopy equivalence. The spaces studied in algebraic topology usually lack distinguishing features from the standpoint of point-set topology.

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Course website: `http://www.pitt.edu/~jdeblois/S18_Top2.html`
Office Hours: TBD. Check the course website.

Textbook: *Algebraic Topology*, by Allen Hatcher. It is freely available on the internet at `https://www.math.cornell.edu/~hatcher/AT/ATpage.html`

Prerequisites: Basic knowledge of point-set topology (even if only from metric spaces) and abstract algebra. Here is a list of topics from each subject with which you should be familiar:

Point-set: Open and closed sets, bases, product and subspace topologies, connectedness, compactness, continuous functions, the Hausdorff property.

Algebra: Groups, subgroups, quotient groups and homomorphisms, presentations, direct sums, direct products, and finitely generated abelian groups.

Grades: Your course grade will be determined as follows:

- Homework, 25%. (Assigned weekly on the course website.)
- Midterms 1 and 2, 25% each.
- Final project, 25%: either a 3-5 page paper or a talk in front of the class.

Final Project Caveat: Students taking the spring analysis prelim may present their favorite prelim topology problem in class as a final project.

Homework Policy: Selected solutions are to be presented *verbally*, the week after a homework set is assigned, in a 20-minute office hours appointment. You may select one problem to present, and I will select one problem for you to present. You have the right to veto my first selection.

Academic Integrity: No resource is off-limits, and in particular you are welcome to work on problems with your peers. But *all content that you produce for this class* (homework and exam solutions, final projects, etc.) *must reflect* ***your*** *thoughts and* ***your*** *comprehension.* I will give a zero score to any assignment which I find to contain material copied from another source.

Topics: a brief list of the main points I hope to cover.

Basic Topology.

- Quotient spaces
- CW-complexes: construction and recognition (Hatcher, Prop. A.2).
- Retractions, deformation retractions, homotopy equivalence, and homeomorphism.
- The basic examples: $\mathbb{R}^n$, Σ_g ($g \geq 1$), S^n, $\mathbb{R}P^n$, $\mathbb{C}P^n$ ($n \leq \infty$). CW structures.

Fundamental group.

- Definition and basic properties: functoriality, basepoint-independence, homotopy invariance, $\pi_1(X \times Y) \cong \pi_1(X) \times \pi_1(Y)$.
- Free groups, free products, and presentations (in particular, Tietze transformations).
- Van Kampen's theorem & applications: $\pi_1(\text{graph})$ (Hatcher Prop. 1A.2), $\pi_1(\text{CW-cx})$.
- Fundamental groups of the basic examples.
- Classical theorems: Brouwer fixed point & Borsuk–Ulam, fund. theorem of algebra.

Covering spaces.

- Definition and basic properties: π_1-injectivity and # sheets.
- Path lifting, homotopy lifting, and map lifting.
- The classification of covering spaces, existence of the universal cover, Galois correspondence.
- Deck transformations, normal covers, and covering space actions.

Higher homotopy groups. (This will be covered to some extent.)

- Definition(s) and basic properties: abelian, action of π_1.
- Cellular approximation; $\pi_k S^n$, $k < n$.
- Whitehead's theorem; weak homotopy equivalence; CW approximation.
- Fiber bundles, homotopy lifting, LES of a fiber bundle.
  - Example: Hopf bundle $S^{2n+1} \to \mathbb{C}P^n$
- Freudenthal suspension theorem, $\pi_n S^n$, $\pi_3 S^2$.
- Degree of $f\colon S^n \to S^n$; basic computations (reflections & antipodal map).

Homology.

- Chain complexes and homology groups.
- CW-homology.
  - Euler characteristic χ of a finite CW-complex, the Euler–Poincaré formula.
  - CW-homology of the basic examples.
- Homology with arbitrary (abelian group) coefficients.
- Δ-complexes, simplicial complexes, and simplicial homology
  - Equivalence with singular/cellular homology.
  - simplicial approximation; the Lefschetz fixed point theorem.
- Singular homology
  - Definition and basic properties: functoriality and homotopy invariance.
  - Basic computations: $H_0(X)$, $H_n(\{\text{pt}\})$, $H_n(\bigsqcup X_\alpha) \cong \bigoplus H_n(X_\alpha)$.
  - Basic homological algebra.
    - Reduced homology, relative homology.
    - SES of chain cxes $\rightsquigarrow$ LES in homology (definition of ∂ in particular).
    - The LES of (X, A) and of (X, A, B).
    - The five lemma.
  - Excision and the LES of X/A, for a good pair (X, A).
  - Local homology, invariance of dimension.
  - Equivalence of singular & cellular homology; homotopy classification of basic examples.
  - The Mayer-Vietoris sequence.
- Axioms and the categorical perspective.
- Degree via homology; its computation using local degree.