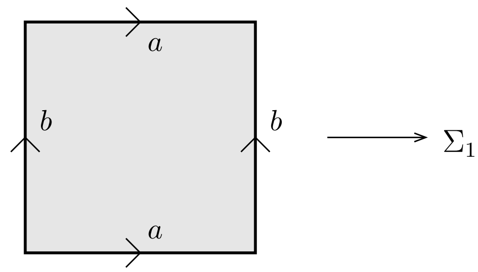
MIDTERM EXAM – TOPOLOGY 2

(1) A topological space X has the *fixed point property* if every continuous map $f\colon X \to X$ has a fixed point, $x \in X$ such that $f(x) = x$. For example, $\mathbb{D}^2$ has the fixed point property by the Brouwer fixed point theorem.

(a) (3 pts) Prove or give a counterexample: if a space X with the fixed point property retracts to a subspace $A \subset X$, then A also has the fixed point property.

(b) (3 pts) Prove or give a counterexample: $S^1 \wedge S^1$ has the fixed point property.



(2) (3 pts) Prove or give a counterexample: for Σ_1 (the torus) as described in class (see above) with its natural CW-complex structure, there is no retraction from Σ_1 to its one-skeleton.

(3) (a) (3 pts) Describe a CW-complex X with $\pi_1(X) \cong \mathbb{Z}_3$.

(b) (4 pts) Describe a CW structure on the universal cover $\widetilde{X}$ of the complex X from part (a).

(4) (4 pts) Show that the group $\langle a, b \,|\, aba^{-1} = b^2, bab^{-1} = a^2 \rangle$ is trivial.

Bonus (2 pts) Show that $\langle a, b, c \,|\, aba^{-1} = b^2, bcb^{-1} = c^2, cac^{-1} = a^2 \rangle$ is trivial.