Math 3670 - Topics in Topology
Guidelines and Syllabus

Overview: This course is structured around the classification problem for closed manifolds, and it will touch on some fundamental examples and topological and geometric tools that inform this problem. The last two thirds (or so) will focus on dimension three, where *effective* classification is provably possible but rather involved. The solution intrinsically involves geometry as exemplified by the "geometrization theorem", conjectured by W. Thurston in the early 1980's and proved by G. Perelman in the early 2000's. We will discuss the topological results referenced in the statement of the theorem before pivoting to the classification and some structure theory of geometric manifolds.

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Course website: `http://www.pitt.edu/∼jdeblois/TopixInTop.html`

Grades: will be based on a project that expands on a theme related to the course material.

Course Outline

I. The classification problem in arbitrary dimensions.

(1) Manifolds: topological versus differentiable versus PL (piecewise-linear).
(2) The classifications of one- and two-manifolds.
(3) The non-classifiability of n-manifolds, $n \geq 4$.
(4) Classification of simply connected 4-manifolds.

II. Topology of three-manifolds.

(1) Examples: surface bundles, Seifert fibered spaces, Heegaard splittings, Dehn surgery, etc.
(2) Dehn's lemma and the loop and sphere theorems.
(3) The prime decomposition (along spheres).
(4) The JSJ decomposition (along tori).

III. Geometry of two- and three-manifolds.

(1) The Uniformization Theorem and the *geometric* classification of two-manifolds.
(2) The classification of three-dimensional locally homogeneous geometries.
(3) The geometrization theorem, and the classification of 3-manifolds.
(4) Examples of geometric manifolds.
(5) The structure of hyperbolic manifolds.
  (a) Mostow rigidty.
  (b) Margulis' Lemma and the thick-thin decomposition.
  (c) The hyperbolic Dehn surgery theorem.

General References

- *Differential Topology*, V. Guillemin and A. Pollack.
- Two free (and good!) books by Allen Hatcher:
  Algebraic Topology and *Notes on Basic 3-Manifold Topology*
  Both are downloadable at `http://pi.math.cornell.edu/~hatcher/`
- Two free references on hyperbolic geometry (and more):
  An Introduction to Geometric Topology, B. Martelli. `https://arxiv.org/abs/1610.02592`
  Geometry and topology of three-manifolds, mimeographed lecture notes of W. Thurston.
  `http://library.msri.org/books/gt3m/`

Specific References (for course outline topics)

I. The classification problem in arbitrary dimensions.

(1) Triangulations of Manifolds, C. Manolescu. *Notices of the ICCM*, 2014.
On manifolds homeomorphic to the 7-sphere, J. Milnor. *Ann. of Math. (2)* **64**, 1956.
The Kirby torus trick for surfaces, Allen Hatcher. `https://arXiv.org/abs/1312.3518`

(2) Both in *The American Mathematical Monthly*:
The Classification of 1-Manifolds, a Take-Home Exam, D. Gale. (**94**, No. 2, 1987.)
Conway's ZIP proof, D. Francis & J. Weeks. (**106**, No. 5, 1999.)

(3)

(4) *The Wild World of 4-manifolds*, A. Scorpan.

II. Topology of three-manifolds – see Hatcher's general reference above.

III. Geometrization.

(1)

(2) The geometries of 3-manifolds, P. Scott. *Bull. LMS* **15** (5), 1983, 401–487.

(3) Recent progress on the Poincaré conjecture and the classification of 3-manifolds,
J.W. Morgan. *Bull. AMS*, **42** (1), 57–78.
The homeomorphism problem for closed 3-manifolds, P. Scott & H. Short. *A.G.T.* **14**, 2014.

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