DIFFERENTIAL GEOMETRY 2, HOMEWORK 2 ADDENDUM

(1) (a) For the affine charts $\phi_1 = (x^1, \dots, x^n)$ and $\phi_2 = (y^1, \dots, y^n)$ on $\mathbb{R}P^n$ express the $\frac{\partial}{\partial x^j}|_p$ in terms of the $\frac{\partial}{\partial y^i}|_p$ at $p = [1, \dots, 1]$. (Recall that the affine chart $\phi_i \colon \{[u^1, \dots, u^{n+1}] \in \mathbb{R}P^n \mid u^i \neq 0\} \to \mathbb{R}^n$ is given by

$$\phi_i \left[u^1, \dots, u^{n+1}\right] = \left(\frac{u^1}{u^i}, \dots, \frac{u^{i-1}}{u^i}, \frac{u^{i+1}}{u^i}, \dots, \frac{u^{n+1}}{u^i}\right)$$

(b) For the map $P[x, y, z] = [x^2 - yz, y^2 - xz, z^2]$ on $\mathbb{R}P^2$, describe $DP|_{[1,1,1]}$ in local coordinates. (Why is P well-defined?)

(2) (a) Show that the quotient map $\mathbb{R}^2 \to T^2$ is a local diffeomorphism, where T^2 is the abstract torus $\mathbb{R}^2/\mathbb{Z}^2$.

(b) Show that the map $F(x, y) = (\cos(2\pi x), \sin(2\pi x), \cos(2\pi y), \sin(2\pi y))$ induces an immersion from T^2 to $\mathbb{R}^4$. (The *induced map* takes the equivalence class $[(x, y)]$ of (x, y) to $F(x, y)$; you must check this is well-defined.)

(3) (Gullemin & Pollack, §2.4 #11)

(a) The $n \times n$ matrices with determinant 1 form a group denoted $SL(n)$. Prove that $SL(n)$ is a submanifold of $M(n)$ (the set of $n \times n$ matrices, identified with $\mathbb{R}^{n^2}$). (*Hint*: The only critical value of the determinant $\det\colon M(n) \to \mathbb{R}$ is 0; indeed, if $\det(A) \neq 0$ then the restriction of det to $\{tA \mid t \neq 0\}$ is already a submersion.)

(b) Check that the tangent space to $SL(n)$ at the identity matrix consists of all matrices with trace equal to 0.