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The Large-Scale Structure of Inductive Inference: A Précis

John D. Norton
Department of History and
Philosophy of Science
University of Pittsburgh

1. The Material Theory of Induction

My earlier volume, *The Material Theory of Induction*, asserts that inductive inferences are warranted materially by facts and not by conformity with universally applicable schemas. A few examples illustrate the assertion.

Marie Curie inferred that all samples of radium chloride will be crystallographically like the one sample she had prepared. The inference was warranted, not by the rule of enumerative induction, but by factual discoveries in the 19th century on the properties of crystalline substances.

Galileo inferred to the heights of mountains on the moon through an analogy with mountain shadows formed on the earth. The inference was not warranted by a similarity in the reasoning in the two cases conforming with some general rule, but by the warranting fact that the same processes of linear light propagation formed the patterns of light and dark in both cases.

Probabilistic inductive inferences are not warranted by the tendentious supposition that all uncertainties can be represented probabilistically. They are warranted on a case-by-case basis by facts specific to the case at hand. That we can infer probabilistically from samples to the population as a whole depends on the fact that the samples were taken randomly, that is, with each individual having an equal probability of selection.

If no such warranting facts prevail, we are at serious risk of spurious inferences whose results are an artifact of misapplied logic.

2. Inductive Inference in the Large

The Large-Scale Structure of Inductive Inference addresses a problem arising from the earlier volume. Each of our inductive inferences is warranted by a proposition that can only serve to warrant if it is a truth. If we are to be secure in these inductions, we must be assured of the truth of these warranting propositions. We gain that assurance through further inductive inferences; and they in turn employ further warranting propositions. What results is a cascade of inductive inferences. The present volume seeks to understand its overall structure.

2.1 A Regress?

As we trace back through a sequence of warranting facts, does each new inference require a warranting fact more general than the one preceding it? If so, what would result is an unsustainable infinite regress to ever more general warranting facts. This threatened regress arises only in abstract speculation. It tacitly assumes that warranting facts can be arranged in a hierarchy of increasing generality such that warranting facts at one level can only warrant inferences to facts of lesser generality in the hierarchy. If we examine inductive inferences in a mature science, we find no such hierarchy. Instead, in *many* examples presented in the volume, we find inductive inferences that cross over one another in ways that defy such a hierarchy. The inductive inferences that use traditional historical methods to date archaeological artifacts are warranted by the efficacy of radiocarbon dating. Conversely, inferences to the efficacy and even the calibration of radiocarbon dating are warranted by the reliability of historical dating methods.

A helpful metaphor is of an arch, in which stones higher in the arch represent propositions of greater generality. Stones in any position in the arch are supported by stones both lower down and higher up in the arch. Stones on one side of the arch support those on the other side; and conversely.

2.2 Getting Started

A second question is how any inductive enterprise can be initiated. Our experience of the world supplies us particular facts, but not the generalities sought in science. How can a collection of particular facts provide inductive support for these generalities? If we know no warranting fact of general scope, how can we infer inductively from particulars to generalities?

The analogy is to a tower, in which courses of stones lower in the tower correspond to propositions of lesser generality. In the analogy, when a tower is constructed, each course is only supported by stones lower in the tower. Correspondingly, when inductive investigations begin, propositions can only call on warranting facts of lesser generality, which means that inductive inferences to propositions of greater generality cannot be supplied with warranting facts.

The problem is solved when we realize that the tower is a poor analogy and that the arch is a better one. We construct arches by temporarily supporting stones with scaffolding, called "centring." Once all the stones are in place, we remove the centring and find that each stone in the arch is well-supported.

In inductive inference, hypotheses are the analogs of the stones supported temporarily by centring. To begin our inductive explorations, we warrant inductive inferences to propositions of greater generality by hypothesizing a warranting proposition that would suffice. In doing so, we take on an inductive debt that must be discharged eventually by providing independent support for the warranting proposition hypothesized. If this independent support cannot be found, we must judge the original inductive inference to be unwarranted.

Once again, the volume contains *many* examples. For millennia, distances to the sun, moon and planets could not be determined by direct observational methods. Inferences to them required hypotheses. In the Ptolemaic, geocentric system, the warranting hypothesis was of the closest packing of the hypothetical Ptolemaic spheres. Ptolemaic inferences to planetary distances failed when this hypothesis did not secure independent support. Where the Ptolemaic hypothesis failed, Copernicus' heliocentric hypothesis succeeded. It supported inferences to solar system distances and secured extensive independent support, most notably through the success of Newton's mechanics. (In another instance of the crossing over of inductive support, Copernican astronomy played a central role in the inductive support for Newton's mechanics.)

2.3 Self-Supporting Inductive Structures

The material theory of induction asserts the possibility of self-supporting inductive structures. These consist of a large set of propositions whose relations of mutual support are so extensive that each proposition is well-supported inductively by the others. Instances of such self-supporting structures are provided by mature sciences. In them, if we select any proposition of importance, we can display how it is supported inductively by other propositions in the science and, perhaps in neighboring sciences. This close-knit arrangement of mutual support is why mature sciences are so stable. One cannot negate one of its propositions without the negation propagating through the entire structure and destabilizing it.

Such destabilization is rare. Since all inductive inferences are fallible, it is always possible if, for example, new, troublesome evidence is found. When it happens, a scientific revolution results.

2.4 Circularities?

The examples above of mutually supporting pairs of propositions are just the simplest form of mutual support. In a mature science, the relations of mutual support form a massively entangled thicket of great complexity. The analogy to a complicated, vaulted ceiling improves only slightly on that of an arch, since the relations of inductive support are much more complicated than the support relations in such a ceiling.

Since the structure is non-hierarchical, it is inevitable that we can trace out circles in the relations of support. It is tempting to let that fact alone damn the structure. That is hasty. Circularities can be benign, such as is, I argue, the case in relations of support in a mature science. We should judge circularities to be harmful only if we can specify the harm they cause. That harm happens in two ways; and both are readily remedied within the material theory if they arise.

The first way is a vicious circularity, in which a contradiction arises. This can happen in relations of inductive support; and when it does it is generally a standard exercise to revise the affected proposition in such a way as to eliminate the contradiction. If no such revision is possible, the particular propositions should be abandoned. It is not a failure of the material theory of induction, but the discovery of an internal contradiction in the specifics of a particular application.

The second way is when a circularity leaves a system of propositions underdetermined. That can happen in inductive investigations when there is a lack of empirical evidence. The natural and common remedy is to seek further evidence that would remove the underdetermination. If that turns out to be impossible—and sometimes demonstrably so—the common diagnosis is that the underdetermination is not factual but an arbitrariness in our descriptions.

Benign circularities are common in science. Maxwell's celebrated equations for electric and magnetic fields are an example. The configuration of the magnetic field is conditioned by the time variations of the electric field; and the configuration of the electric field is conditioned by the time variations of the magnetic field. The circularity yields time varying electromagnetic fields corresponding to propagating light waves.

2.5 Inductive Uniqueness

The idea of self-supporting inductive structures raises an intriguing possibility. Might one body of empirical evidence sustain multiple, competing, self-supporting structures? If that were to happen, it would not just be that a multiplicity of contradicting propositions are supported. Since those propositions are also the warrants for inductive inference, what would result are multiple, competing inductive logics.

These multiple competing systems do not arise in science. That is, once a science has achieved maturity, it remains stable, without viable competitors, possibly in perpetuity, unless destabilizing evidence emerges. The material theory of induction has within it a mechanism that suppresses and resolves competition between systems of propositions: these competitions are unstable. Consider two systems in a viable competition. If new evidence emerges that favors and strengthens one of the systems, that system and its inferential capacities are enhanced, since it can draw on more warranting facts. Since the two systems disagree factually, the other system and its inferential capacities will be correspondingly weakened. The resulting instability is self-reinforcing and, if inductive investigations continue, will lead to the collapse of one of the two systems.

3. The Problem of Induction

3.1 The Problem Eluded

Since its revival at the start of the twentieth century by Russell, Reichenbach and Popper, Hume's problem of induction has been the epitome of an intractable philosophical problem. In the version of Hume, Russell and Reichenbach, no rule or principle of inductive inference can be justified on pain of a harmful circularity. We cannot justify a rule to generalize regularities by noting that it has always worked in the past, since that justification employs to the very rule in need of justification. In Popper's regress version, each rule of inductive inference is justified by a different rule. A fanciful infinite regress of rules follows.

The material theory of induction escapes this venerable problem. The theory has no universal rules of inductive inference. It follows that the problem of induction in either circularity or regress versions cannot be stated within the theory. The escape employs no innovation of great philosophical brilliance. Every problem of philosophy requires a framework in which it is formulated. The material theory of induction does not use the framework needed for formulating the problem of induction. The problem cannot be stated within the material theory.

3.2 Failure of Attempts to Recreate It

The problem of induction has otherwise resisted attempts at solutions. The common experience has been that promising solutions fail since a careful examination of them shows that the original circularity or regress reappears within the solution but perhaps in a disguised form. Thus, it is reasonable to expect that a similar recreation of the problem afflicts the material theory's escape. Several ways for the problem to reappear have been suggested. None, however, succeed in recreating it.

First, we might ask if the cascade of warrants for inductive inferences recreates an infinite regress akin to that of the original problem of induction. We have already seen that such an infinite regress requires an assumption of the hierarchical structure of relations of support. Relations of support in the material theory do not respect such a hierarchy. Moreover, the regress of rules in the problem of induction is fanciful. We might imagine the rule of rule of enumerative induction to be justified by the rule of inference to the best explanation; and that rule to be justified by the rule of severe testing. We are three steps into the regress and it is already fanciful speculation. The corresponding regress in the material theory is just one of prosaic facts. As we trace back the justification of warranting facts, we merely trace out, again and again, how one mundane fact of science finds its warrant in another.

Second, we might ask if the circularities in relations of support are akin to the circularities in the problem of induction. These latter circularities are harmful. They leave the affected rule of induction underdetermined. As we have seen, the circularities in the material theory are benign. They may temporarily lead to contradictions or underdeterminations. Both are remedied routinely by adjustments to the theories or by seeking new evidence.

4. How to do History and Philosophy of Science

This volume presents a model of how I believe we can integrate history of science and philosophy of science productively. A facile object to the integration is that attempts to read philosophy of science from historical case studies cannot escape the temptation to cherry-pick the historical case studies. The integrative methodology of this volume is different.

To begin, the philosophical theses of the volume are established by arguments that are independent of the history of science. The danger with such arguments, however, is that we have dreamt up a lofty-sounding account that has little connection to real science. Such cases arise too often. An example is the still popular support for a strong form of the underdetermination thesis. It asserts that, as a matter of general principle, no body of empirical evidence is sufficient to determine a theory. Its support depends on a naïve hypothetico-deductive account of evidential support. In another example, too much of present Bayesian analysis in philosophy of science is merely elementary exercises in probability theory, aggrandized by tendentious labeling of its terms.

As a discipline to avoid such separation from the actuality of science, the volume goes to some pains to display real cases in the history of science that instantiate each of its claims.

The history of science also has a role heuristically. Time and again, when the philosophical analysis was stalled, I found that examining real cases in the history of science revealed novel ideas that allowed the philosophical analysis to proceed. An example came with the concern that a single body of empirical evidence might support two distinct bodies of theory and thus two distinct logics of induction. In mature sciences, this problem does not appear. An examination of the background to these sciences revealed the instability of inductive competition sketched above that leads to the uniqueness of a well-developed science.