

# A Simple Minkowskian Time-Travel Spacetime

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## 1. Introduction

Einstein's general theory of relativity admits spacetimes in which time travel is possible, in the sense that they harbor closed timelike curves. That this is so has been known since at least Goedel's (1949) time travel solution of Einstein's  $\lambda$ -augmented gravitational field equations. Since then, many other time-travel spacetimes have been found within Einstein's theory, such as are afforded by Kerr black holes. They have attracted considerable attention in both physics and philosophy, as reported in Earman et al. (2022) and Smeenk et al. (2023). Some proposals require exotic physics, such as is needed by Morris et al. (1988), to open a wormhole that connects different parts of spacetime. Others escape this complication in wormhole creation by simply stipulating a topology altering connection between two parts of the spacetime.

The simplest proposal takes a Minkowski spacetime and merely identifies two spacelike hypersurfaces, so that the present evolves back to itself. This "cylinder" universe has the global topology of  $S \times R^3$ . While such stipulations produce spacetimes that are, in my view, admissible within Einstein's theory, one could be forgiven for the sense that time travel has been introduced artificially into the theory by our meddlesome stipulation as opposed to a sound reason of physics. Earman et al. (2022) have reviewed the difficult and still open question of whether we could do something that might bring about a time machine, that is, bring about closed timelike curves. At least some time-travel universes are so structured that we can point to a cause of temporal anomaly. In Tipler's (1974) proposal, frame dragging effects are produced by a rapidly rotating cylinder of matter and they are sufficient to lead to closed timelike curves.

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<sup>1</sup> My thanks to John Earman, John Manchak and David Malament for helpful comments on an earlier draft; and to Bob Wald for helpful discussion on characterizing conical singularities.

This paper presents one of the simplest time-travel universes admitted by Einstein's theory in the sense that it is topologically  $R^4$ , matter free and everywhere geometrically flat, except for a singular, two-dimensional surface around which timelike geodesics are deflected back into their past. The example is offered as a pedagogically useful addition to our repertoire of time-travel universes. We might, if we are so inclined, attribute the possibility of time travel to the disturbing influence of the singularity.<sup>2</sup> The spacetime is otherwise unremarkable, in being everywhere locally flat, like the spacetime of special relativity.

As a preview, Figure 1 is a caricature of the time travel spacetime. The spacetime to the left of the figure is roughly Minkowskian and is more accurately so as we go farther left. The usual time direction there is left-right. The spaceships shown are moving inertially along timelike geodesics. No rocket motors fire. As they approach the singularity on the right of the figure, their geodesics are deflected so that their motion in time is reversed when they return to the spacetime on the left of the figure. They are there able to communicate with their past selves, by, for example, sending them a light signal as shown. The spacetime is not time orientable. That is, no consistent division of timelike motions into future and past is possible. Any such division will be contradicted by the return of the motion after it passes the singularity. Time travelers encounter their past selves aging in the opposite local time sense.

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<sup>2</sup> My view is that such attributions are purely of heuristic value. It is argued in Norton (2003, forthcoming) that seeking a notion of causal influence over and above the relations already provided by the prevailing physical theory is yet another of the many attempts at a priori physics, all of which have met with little success over millennia.

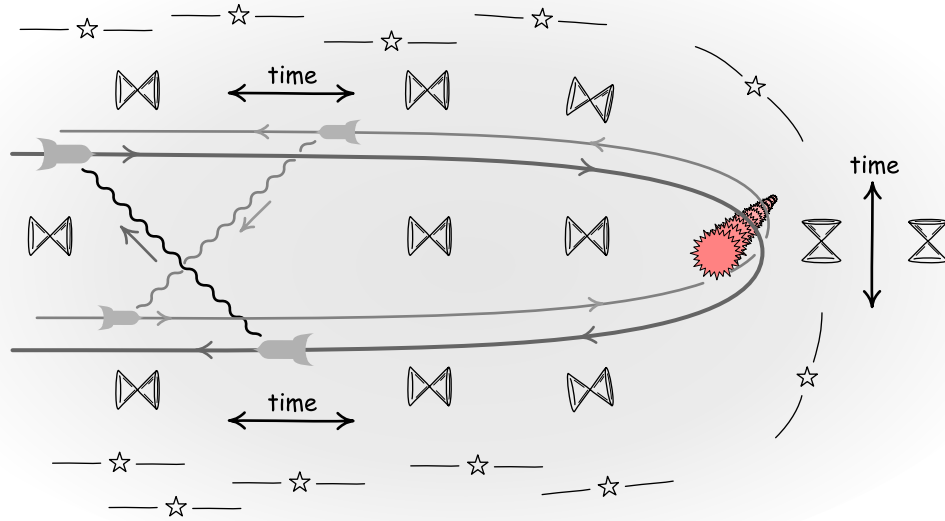


Figure 1. Time travel in the time travel spacetime

## 2. The Spacetime

The geometrical structure of the spacetime is given by the line element for the interval  $s$ :

$$ds^2 = (r^2/4) \cos \theta d\theta^2 - \cos \theta dr^2 + r \sin \theta d\theta dr - dy^2 - dz^2 \quad (1)$$

where polar coordinates  $r, \theta$  have values  $r > 0$  and  $0 \leq \theta < 2\pi$ . Cartesian coordinates  $y, z$  have values  $-\infty < y, z < \infty$ . The metrical coefficients in a coordinate basis  $x^i = (\theta, r, y, z)$  are:

$$g_{ik} = \begin{bmatrix} \frac{r^2}{4} \cos \theta & \left(\frac{1}{2}\right) r \sin \theta & 0 & 0 \\ \left(\frac{1}{2}\right) r \sin \theta & -\cos \theta & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad (2)$$

This coordinate system, shown in Figure 2, “goes bad” at the origin  $r = 0$ , since an event there would be assigned all values of the angle coordinate  $\theta$ . It will turn out, as shown in Section 7 below, that there is a singularity at  $r = 0$  in the sense that taking a continuous limit from the surrounding spacetime provides no unique extension of the metrical structure to  $r = 0$ . If events are allowed at or added to the manifold at  $r = 0$ , then the spacetime manifold is globally  $\mathbb{R}^4$ . For all events other than at  $r = 0$ , it will become apparent in Section 3 that the spacetime is metrically flat with the usual Lorentz signature of a Minkowski spacetime. That is, any open,  $\mathbb{R}^4$  neighborhood of the spacetime excluding  $r = 0$  is isometric with an open  $\mathbb{R}^4$  neighborhood of a

Minkowski spacetime. Thus, it satisfies Einstein's unaugmented, source-free field equations everywhere, except at  $r = 0$ . However, as Figure 1 already suggests, the spacetime is not time orientable.

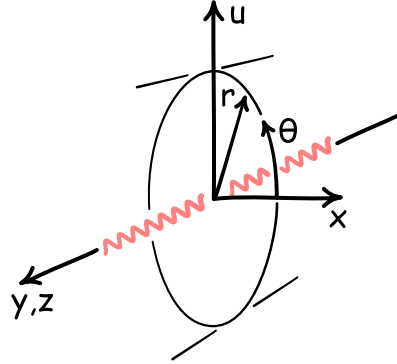


Figure 2. Cylindrical coordinates of the spacetime and its singularity

In Figure 2, the singularity at  $r = 0$  is drawn as a line within a three-dimensional space. The figure suppresses one dimension of the four-dimensional spacetime by collapsing the  $y$ - and  $z$ -axes. Thus, the singularity is really a two-dimensional surface enclosed within a four-dimensional space.

It follows immediately from (1) and (2) that the metrical structure is translation invariant in the  $y$  and  $z$  directions. That is,  $y \rightarrow (y + \text{constant})$  and  $z \rightarrow (z + \text{constant})$  are both isometries. The interesting, time-travel related physics happens in surfaces of constant  $y$  and  $z$ ; that is in surfaces spanned by the coordinates  $r$  and  $\theta$ .

We shall see below that two families of intersecting lightlike curves in these surfaces are given by:

$$r = \frac{k}{\sqrt{1-\sin\theta}}, r = \frac{k}{\sqrt{1+\sin\theta}} \quad (3)$$

for  $k > 0$ . The lightcones adapted to these curves are illustrated in Figure 3 for a surface spanned by the coordinates  $r$  and  $\theta$ .

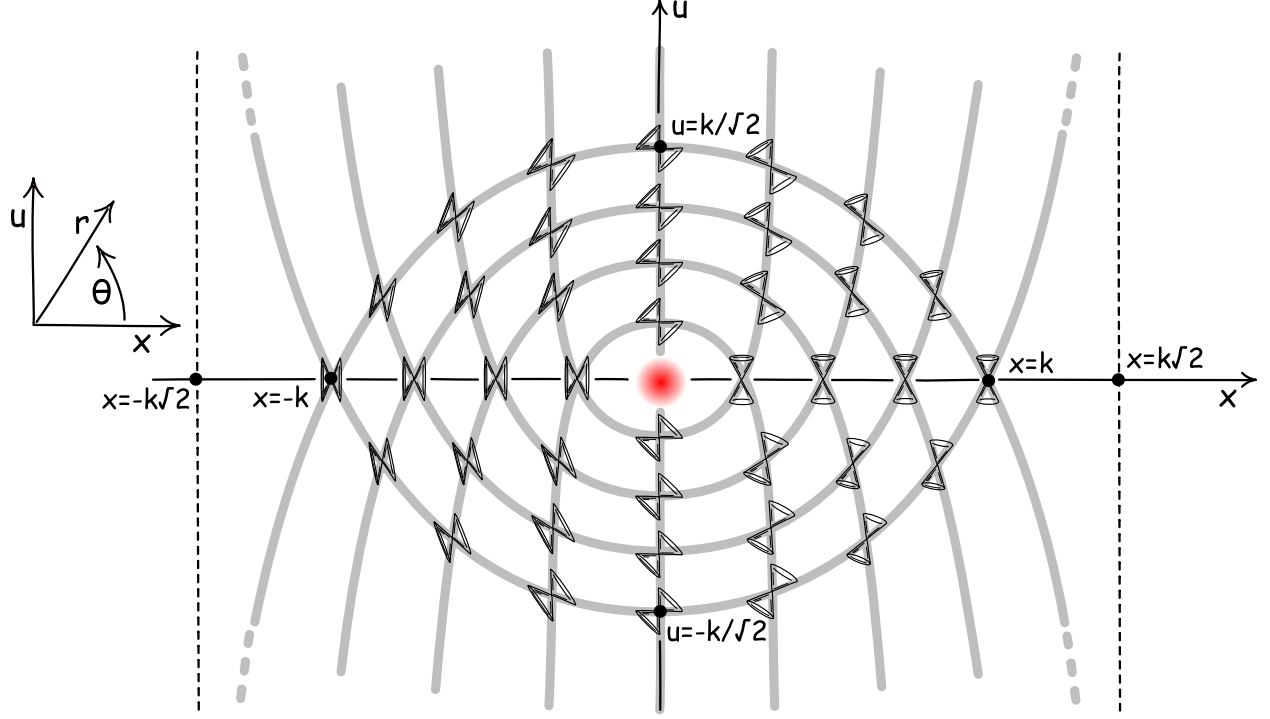


Figure 3. Light cone structure in a surface of constant  $y$  and  $z$

The Cartesian coordinates shown in Figures 2 and 3 are defined by

$$u = r \sin \theta \quad x = r \cos \theta \quad y = Y \quad z = Z \quad (4)$$

Briefly, at  $\theta = 0$ , the light cones indicate a timelike direction orthogonal to the radial direction. As we proceed in both the  $+\theta$  and  $-\theta$  directions, the light cones tip towards the singularity at  $r = 0$ . At  $\theta = \pm\pi$ , they meet such that timelike curves can pass directly into the singularity. If the lightlike curves intersect the  $x$  axis at  $x = \pm k$ , then they intersect the  $u$  axis at  $u = \pm k / \sqrt{2}$  and are asymptotic to  $x = \pm k \sqrt{2}$ .

The region of spacetime for very negative  $x$  coordinates will appear much like an ordinary Minkowski spacetime. That is, for regions of very large, negative  $x$  in the vicinity of the  $x$ -axis,  $\theta \approx \pi$ ,  $\cos \theta \approx -1$  and  $\sin \theta \approx 0$ , the line element (1) then approximates the Minkowskian  $ds^2 = dr^2 - dv^2 - dy^2 - dz^2$  if we introduce the new coordinate  $v$  such that  $dv = r d\theta$ . It is only for events near the singularity at  $x = r = 0$  that the light cones tip toward the  $-x$ -axis and such that lightlike curves are eventually deflected around the singularity.

Timelike geodesics, similarly, behave much like in an ordinary Minkowski spacetime for the region with very negative  $x$  coordinates. As we enter regions close to the singularity at  $x = u = r = 0$ , the timelike geodesics are deflected around the singularity and reversed in their

direction. More precisely, we shall see below that a family of timelike geodesics, for arbitrary fixed values of  $y$  and  $z$ , is given by

$$r = k/\cos(\theta/2) \tag{5}$$

where  $k > 0$  is an arbitrary constant. The disposition of these geodesics is shown in Figure 4.<sup>3</sup>

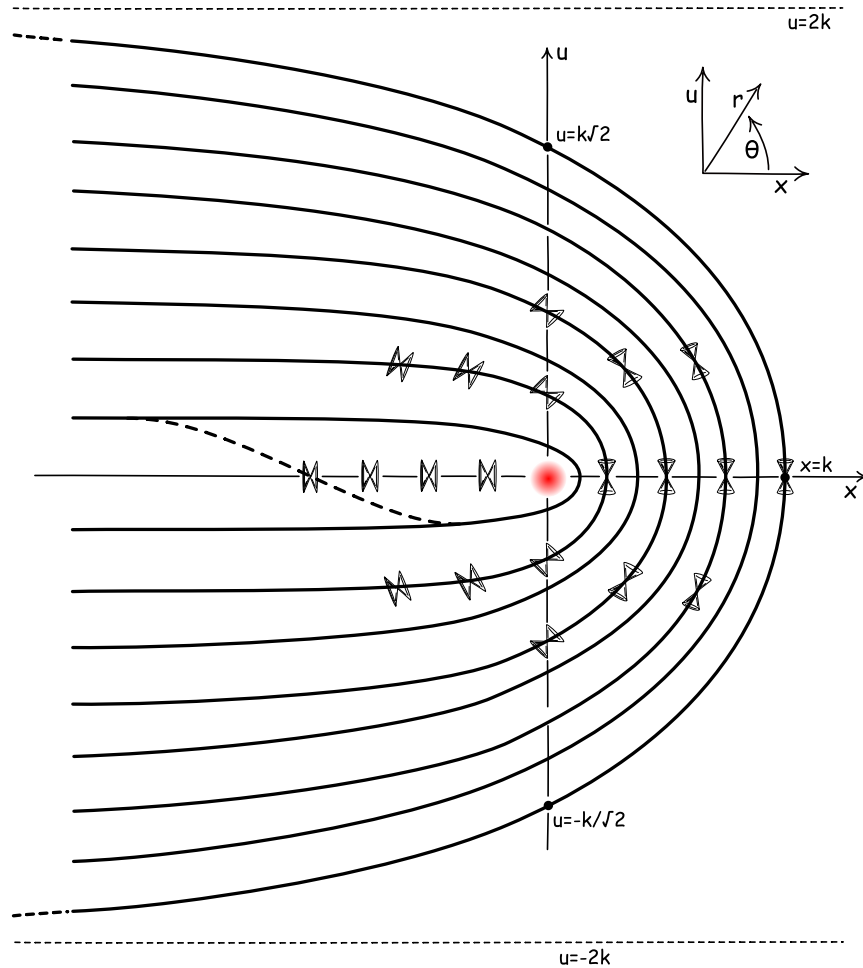


Figure 4. A family of timelike geodesics

The geodesics intersect the  $x$  axis at  $x = k$ , the  $u$  axis at  $u = k/\sqrt{2}$  and, as they extend in the  $-x$  direction, are asymptotic to  $u = \pm 2k$ . While these geodesics do not form closed curves, if we adopt a position far enough in the  $-x$  direction, the two parts of each geodesic can be connected by another timelike curve, shown as a dashed curve in Figure 4. If we connect parts of these

<sup>3</sup> Caution is advised in reading these diagrams. The polar coordinates  $r$  and  $\theta$  and Cartesian coordinates  $u$  and  $x$  do not have their usual metrical significance. Metrical judgments using them should be mediated by the line element (1) and metrical coefficients (2).

curves in obvious ways, we get closure in the sense of a single timelike curve that intersects its past self.

An observer whose worldline coincides with one of these timelike geodesics, needs no acceleration to travel back in time. After sufficient proper time has passed, that observer will encounter the observer's past self, but aging in the opposite time sense.

More generally, this spacetime is not time orientable in the usual sense of the existence of an everywhere non-vanishing, continuous, timelike vector field. That is, if we stipulate that some timelike vector, at some event, points in the future direction, parallel transporting that vector along one of these timelike geodesics in both directions will eventually return it to a neighborhood where it has a contradicting time sense.

### 3 Constructing the Spacetime

The narrative so far has given no clue to the mode of construction of the spacetime. That has been done with the hope that the resulting spacetime will be assessed on its merits and not discounted because of the simplicity of the construction method. We arrive at the spacetime of (1) by an identification on a familiar Minkowski spacetime in the following way. This "source" Minkowski spacetime has the line element

$$ds^2 = dT^2 - dX^2 - dY^2 - dZ^2 \tag{6}$$

where as usual  $-\infty < T, X, Y, Z < \infty$ . The "target" spacetime of (1) is recovered by introducing the coordinate systems of (1) and (4) in such a way as to cover the half of the source Minkowski spacetime specified by  $X \geq 0$ , as illustrated in Figure 5.

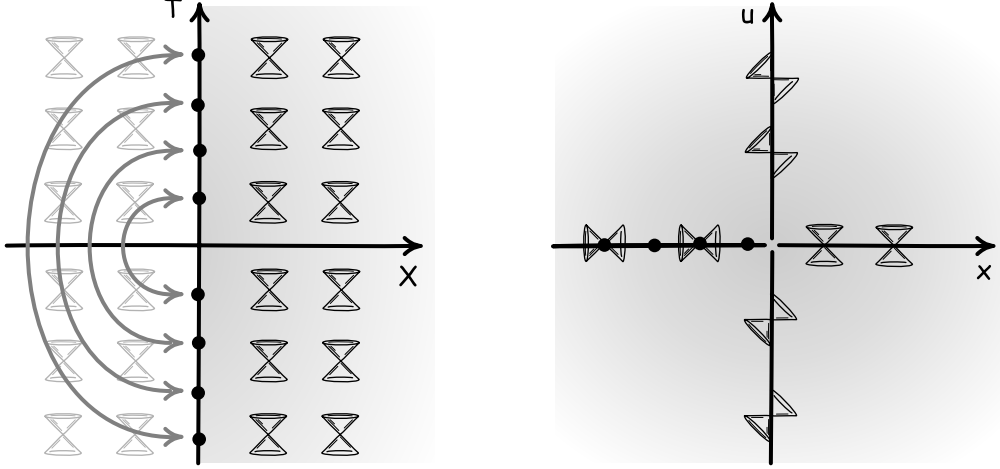


Figure 5. Construction of the time-travel spacetime

The construction requires identification of each event of (6)  $(T, X=0, Y, Z)$  with  $(-T, X=0, Y, Z)$ . To give further details, it is convenient to replace the  $T, X$  coordinates of (6) with polar coordinates  $r, \phi$ , defined by

$$T = r \sin \phi \quad X = r \cos \phi \quad (7)$$

where  $r > 0$  and  $0 \leq \phi < 2\pi$ . The mapping from the half-plane of the source (6) to the full plane of the target (1) is carried out by taking the metrical structure of the half Minkowski spacetime of (6) at the event  $(\phi, r, Y, Z)$  and mapping it to the event  $(\theta = 2\phi, r, y, z)$  in the target spacetime. Loosely speaking, the new metrical structure (1) is recovered by a doubling expansion of the angle variable about  $\phi = 0$  of the source Minkowski spacetime. It can be written compactly as

$$\theta = 2\phi \quad (8)$$

#### 4. Recovering the Metrical Structure and its Flatness

The line element (1) can be recovered from the line element (6) of the source Minkowski spacetime by a two-step transformation. First, cylindrical coordinates are introduced into (6) by the transformation (7). From (7) we have

$$dT = r \cos \phi d\phi + \sin \phi dr$$

$$dX = -r \sin \phi d\phi + \cos \phi dr$$

After substitution and some manipulation, the Minkowski line element (6) becomes

$$ds^2 = r^2 \cos(2\phi) d\phi^2 - \cos(2\phi) dr^2 + 2r \sin(2\phi) d\phi dr - dY^2 - dZ^2 \quad (9)$$



The second step maps half the source Minkowski spacetime to the target spacetime by the substitutions:

$$\theta = 2\phi \quad r = r \quad y = Y \quad z = Z$$

The expression for the line element (9) becomes the corresponding expression (1):

$$ds^2 = (r^2/4) \cos \theta d\theta^2 - \cos \theta dr^2 + r \sin \theta d\theta dr - dy^2 - dz^2$$

Since the source Minkowski metric is everywhere flat and the target spacetime is produced by a coordinate transformation, it follows that the new spacetime is also everywhere flat, excluding the singularity at  $r = 0$ .

## 5. Recovering the Light Cone Structure

This mapping (8) shows how deflections of the light cone structure of Figure 2 arise. We simply need to track how the light cones of the source half Minkowski spacetime are relocated and reoriented in the target spacetime under the mapping (8), as shown in Figure 6:

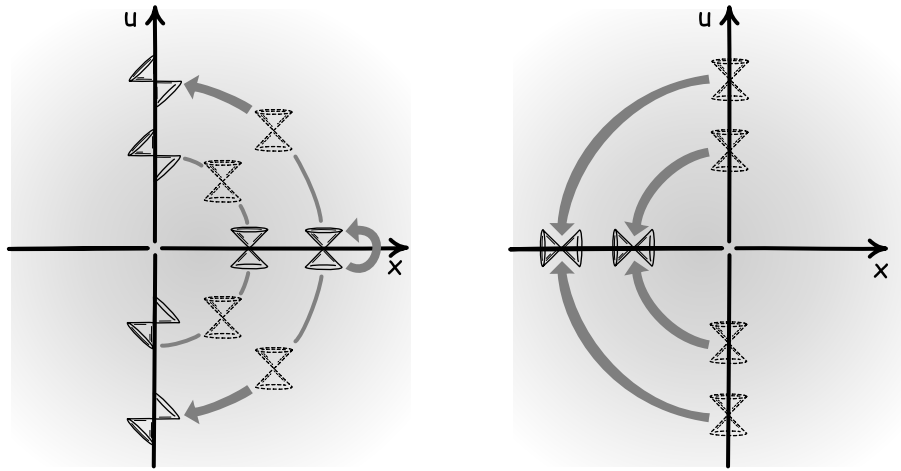


Figure 6 Mapping of light cone structure

An important property of the mapping concerns light cones mapped under  $\phi = \pi/2 \rightarrow \theta = \pi$  and those mapped under  $\phi = -\pi/2 \rightarrow \theta = -\pi$ . Since  $\theta = \pi$  and  $\theta = -\pi$  are coordinates of the same event (if other coordinates equal), it is essential that the two different mappings yield the same light cone. The two mappings deliver light cones such that one is the temporal inverse of the other. But since the light cones are time inversion invariant, the two mappings yield the same result.

Hence the ensuing metrical structure is fully regular at all events  $\theta = \pm\pi$  (where  $r > 0$ ). It is this inversion, however, that precludes the new spacetime being time orientable.

The analytic expressions (3) for the lightlike curves of Figure 2 are recovered from this mapping. First consider “future” ( $= +T$ ) directed lightlike curves in the source Minkowski spacetime:

$$T = X + k$$

for constant  $-\infty < k < \infty$ . In polar coordinates introduced by (7), the curves are

$$r \sin \phi = r \sin \phi + k$$

or

$$r = \frac{k}{\sin \phi - \cos \phi}$$

where the range of values of  $\phi$  to which this formula applies must be restricted to ensure that  $r$  remains positive. (The case of  $k = 0$  is excepted and addressed below.) Under the mapping (8) and similar angle restrictions, the expression becomes the first formula of (3)<sup>4</sup>

$$r = \frac{k}{\sin(\frac{\theta}{2}) - \cos(\frac{\theta}{2})} = \frac{|k|}{\sqrt{1 - \sin \theta}}$$

where the restriction to the absolute value of  $k$  is all that is needed in the final  $\sin \theta$  formula to ensure positive values for  $r$ . Applying this formula is complicated by the fact that *two* lightlike curves of the source Minkowski spacetime are mapped to form a *single* lightlike curve in the target spacetime. That is, the lightlike curves  $T = X + k$  and  $T = X - k$  for  $k > 0$  are mapped under (8) to give a single curve of (3). These curves are illustrated in Figure 7.

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<sup>4</sup> The second equality requires the trigonometric half angle identity,  $\sin \theta = 2 \sin(\theta/2) \cos(\theta/2)$ .

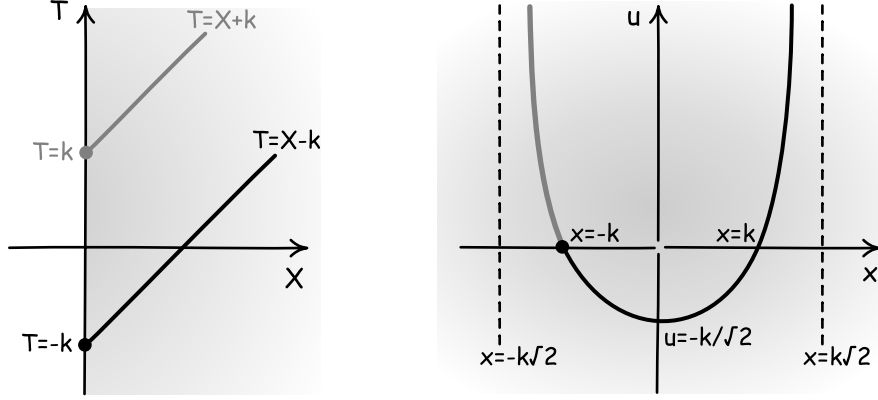


Figure 7. Mapping of lightlike curves

The mapping of the two lightlike curves from the source Minkowski spacetime joins at  $u = 0, x = -k$  in the target spacetime to yield a single curve. The resulting lightlike curve in the target spacetime diverges in the  $+u$  direction. That it is asymptotic to  $x = \pm k\sqrt{2}$  cannot be recovered directly from (3). It can be affirmed by re-expressing (3) in the Cartesian coordinates  $u, x$  of (4) and considering the limit as  $u \rightarrow \infty$ .

An analogous computation gives similar results for the “past” ( $= -T$ ) directed timelike curves  $T = -X + k$  for constant  $-\infty < k < \infty$ . We recover the second expression of (3):

$$r = \frac{k}{\sin\left(\frac{\theta}{2}\right) + \cos\left(\frac{\theta}{2}\right)} = \frac{|k|}{\sqrt{1 + \sin\theta}}$$

Once again, two lightlike curves of the source Minkowski spacetime,  $T = -X + k$  and  $T = -X - k$  for  $k > 0$  are mapped under (8) to give a single curve of (3). These curves are illustrated in Figure 8.

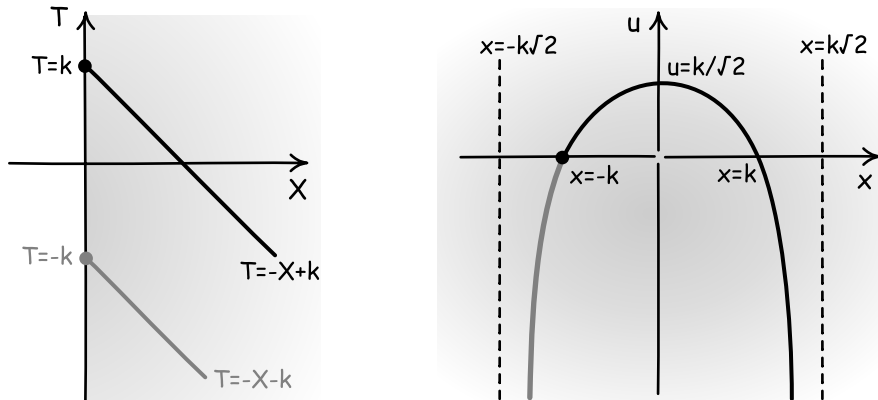


Figure 8. Second mapping of lightlike curves

The resulting curve in the target spacetime diverges in the  $-u$  direction and is also asymptotic to  $x = \pm k\sqrt{2}$ .

The special case of  $k = 0$  corresponds to the two lightlike curves  $T = X$  and  $T = -X$ . The formulae (3) are degenerate for them. It is easy to see however that these two curves map under (8) to the lightlike curves  $\theta = \pi/2$  and  $\theta = -\pi/2$ , that is, curves that lie on the  $u$  axis.

A check on the consistency of these results for lightlike curves employs the line element (1). If we set  $ds^2 = 0$ , the line element provides a differential equation that characterizes lightlike curves in the  $r, \theta$  plane:

$$\left(\frac{dr}{d\theta}\right)^2 - r \tan \theta \frac{dr}{d\theta} - \frac{r^2}{4} = 0$$

Some manipulations affirms that the expressions (3) each solve this equation.

## 6. Recovering Timelike Geodesics

This same mapping makes recovery of timelike geodesics straightforward. In the source Minkowski spacetime of (6), a family of geodesics is defined by

$$X = k \quad r \cos \phi = k$$

for fixed values of  $Y$  and  $Z$  and for  $k > 0$ . These correspond under (8) to

$$r \cos (\theta/2) = k$$

in the target spacetime. Figure 9 shows the geodesics in the source Minkowski spacetime on the left and in the target spacetime on the right.

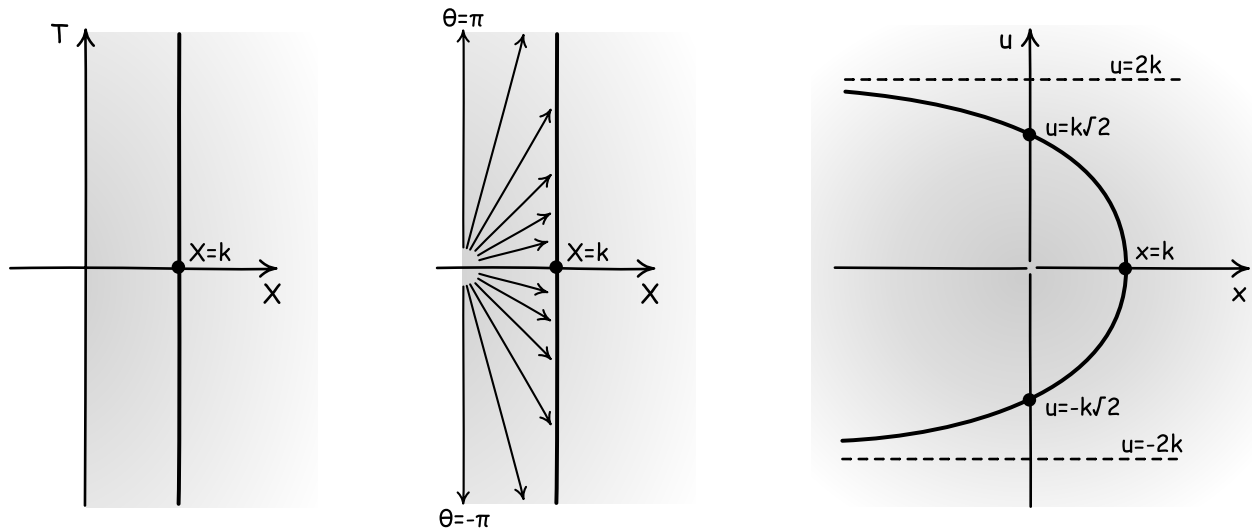


Figure 9. Timelike geodesics

The construction in the center shows how one can recover qualitative features of the transformed geodesic by inspection of the figure. It shows radial lines of constant  $\theta$  from the target spacetime

superimposed on the source Minkowski spacetime. The two radial lines  $\theta = \pi$  and  $\theta = -\pi$  are parallel to the geodesic. Since these two radial line coincide with the  $-x$ -axis in the target spacetime, we can conclude that the geodesic mapped to the target spacetime will approach lines parallel to the  $-x$ -axis for very negative  $x$ . Similar interpretations can be applied to Figures 7 and 8.

## 7. The Singularity

The status of the target spacetime at  $r = 0$  is, so far, unclear. That the polar coordinates “go bad” at  $r = 0$  may merely be an artifact of that coordinate system and may not represent a pathology of spacetime. Such a benign result arises when polar coordinates are used in a Euclidean space. That is not the case here. There is a singularity in the metrical structure of (1) at  $r = 0$ . It is the type of singularity that is found in the intrinsic geometry of a cone.

The simplest cone singularity is produced, figuratively, by taking a flat sheet of paper, excising a pie shaped segment and connecting the exposed edges to form a cone, as shown in Figure 10. The intrinsic geometry of the surface of the cone remains everywhere flat. However, something goes awry at the apex. The familiar way to illustrate it, is to note that the circumference of a circle, centered on the apex, no longer obeys the Euclidean result of (circumference) =  $2\pi$ (radius).

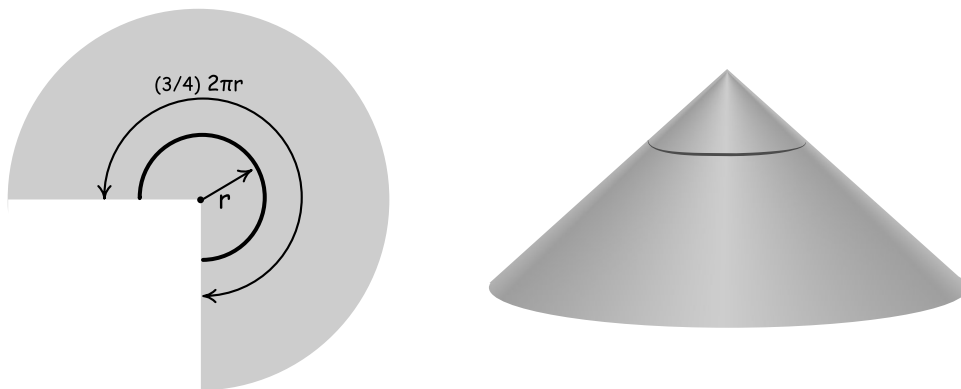


Figure 10. The simplest cone singularity

A fuller analysis shows that the singular character of the geometry *intrinsic to the cone's surface* resides in a failure of uniform convergence. That is, if we seek to assign a metric to the apex by taking the limit of the metrical structure on radial lines leading to the apex, we find different metrics according to the radial line chosen.

Since the construction of the time-travel spacetime is similar to that of the cone, the same sort of cone singularity arises at  $r = 0$  in (1). Spacetime singularities of this type have been investigated by Ellis and Schmidt (1977, §3). That something is amiss at  $r = 0$  follows if we seek to assign light cones to events at  $r = 0$ . At all regular events in the spacetime where  $r > 0$ , timelike curves through the event form the familiar double cone, no matter how close that event is to  $r = 0$ . If we collect the timelike curves converging towards an event at  $r = 0$ , they form a single cone. Whatever metrical structure we might assign to events at  $r = 0$ , that structure will be unlike the metrical structure at all neighboring events since it must produce a single-lobed lightcone.

The construction of this single-lobed cone is shown in Figure 11. The left of the figure shows timelike curves in the source Minkowski spacetime that will transform to this single cone in the target spacetime. The right of the figure shows these curves after they are transformed under (8) in the target spacetime.

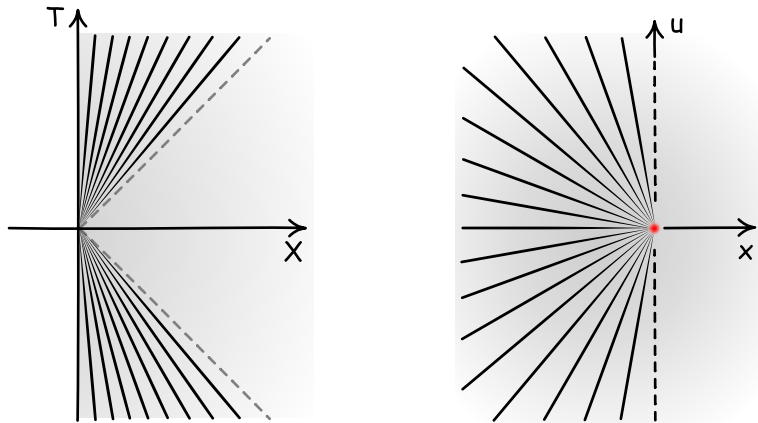


Figure 11. Degenerate light cone structure

Proceeding more fully, we cannot use the cylindrical coordinate system of (1) to show the singular character of (1) at  $r = 0$ , since that coordinate system is badly behaved at  $r = 0$ . Instead, we *stipulate* that there are manifold points at  $r = 0$ . We seek to investigate the metrical structure there using the Cartesian coordinate system  $u, r, y, z$ , defined in (4), since that coordinate system is regular at  $r = 0$ .

We transform the line element (1) to this new coordinate system using differentials derived from (4):

$$d\theta = (x/r^2) du + (u/r^2) dx \quad dr = (u/r) du + (x/r) dx$$

After considerable manipulation, the line element (1) transforms to<sup>5</sup>

$$ds^2 = \frac{x}{r} \left[ \frac{x^2/4}{u^2 + x^2} \right] du^2 - \frac{x}{r} \left[ \frac{3u^2/4 + x^2}{u^2 + x^2} \right] dx^2 - \frac{u}{r} \left[ \frac{u^2 + 3x^2/2}{u^2 + x^2} \right] dudx - dy^2 - dz^2$$

where, as before,  $r = (u^2 + r^2)^{1/2}$ . Using (4) and with some manipulation, this form of the line element can be rewritten in terms of  $\theta$  as

$$ds^2 = \cos\theta [(\cos^2\theta)/4] du^2 - \cos\theta [(3/4) \sin^2\theta + \cos^2\theta] dx^2 - \sin\theta [\sin^2\theta + (3/2) \cos^2\theta] dudx - dy^2 - dz^2$$

Its metrical coefficients in a coordinate basis  $x^i = (u, x, y, z)$  are

$$g_{ik} = \begin{bmatrix} \cos\theta(\cos^2\theta)/4 & -(\sin\theta)/2 \left[ \sin^2\theta + \left(\frac{3}{2}\right)\cos^2\theta \right] & 0 & 0 \\ -(\sin\theta)/2 \left[ \sin^2\theta + \left(\frac{3}{2}\right)\cos^2\theta \right] & -\cos\theta \left[ \left(\frac{3}{4}\right)\sin^2\theta + \cos^2\theta \right] & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The distinctive property of this form of the line element is not so much the specific values that these metrical coefficients take. Rather it is just that these coefficients are functions of  $\theta$  only and they are different for different values of  $\theta$ .

This fact reveals the character of the singularity at  $r=0$ . We may try to define a metrical structure at  $r = 0$  in the time-travel spacetime by the requirement of continuity with the metrical structure at neighboring events. That is, we seek to assign a metric to the event  $r=0$  as the limit of the metric taken along a constant  $\theta$ , radial line terminating in  $r=0$ . It now follows that this requirement of continuity produces a different metrical structure according to the radial line of constant  $\theta$  along which we approach  $r = 0$ .<sup>6</sup> The singular character of the metrical structure at  $r = 0$  resides in its necessary discontinuity with the metrical structure of neighboring events.

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<sup>5</sup> This formula explains why the lightcones in Figure 2 appear distorted. If we solve for  $ds^2 = 0$ , we find for light cones on the  $x$ -axis, where  $u=0$ , that  $du/dx = du/dy = du/dz = \pm 2$ . The distortion is a coordinate artifact.

<sup>6</sup> These metrics, with different values of  $\theta$ , are isometric, since all are flat. If realize all of them at the origin, however, they will assign different norms to the same vector. The vector  $(1, 0, 0, 0)$  will be assigned the norm  $\cos\theta [(\cos^2\theta)/4]$ , for example. This is an invariant failure to agree.

## 8. Conclusion

What do we learn from this example? In my view, we reaffirm a familiar result: that the spacetimes of general relativity admit time travel. Whether one finds this example more or less illuminating than others is, in the end, decided by what each of us finds more or less natural or intuitive. In this regard, I find it appealing since the spacetime is locally everywhere flat excepting for the singular surface whose presence makes the difference between an everywhere flat Minkowski spacetime without time travel and one with time travel.

One possible reaction—heard anecdotally—is that this spacetime is somehow lesser since it is “unphysical” or, in a different but related concern, “artificial.” This notion of being “physical” is an important part of the pragmatics of practical physics. It is invoked to dismiss some particular result from consideration in the particular context at hand.

In another application, Norton (2008, §3.2) found four different, precise senses for the notion. None apply here in so far as the goal is merely to explore the range of spacetimes admitted by general relativity. There is no proposal that this form of time travel is realized in our universe; and no assertion that it is not. The situation is analogous to the intrinsic geometry of the cone of Figure 10. It may or may not be the geometry of our space. Nonetheless, its analysis lies within the scope of geometry and that is underscored by the fact that we can build something close to it in a paper model. This example is connected to what is possible in our world by a slender thread: it is a model of our current best theory of space and time, general relativity. In that, however, it keeps company with many other much stranger models.

## References

- Earman, John; Wüthrich, Christian; and Manchak, JB (2022) “Time Machines,” *The Stanford Encyclopedia of Philosophy* (Spring 2022 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2022/entries/time-machine/>.
- Ellis, George F. R. and Schmidt, B. G. (1977) “Singular Spacetimes,” *General Relativity and Gravitation*, **8**, pp. 915-953.
- Goedel, Kurt (1949) “An Example of a New Type of Cosmological Solutions of Einstein’s Field Equations of Gravitation,” *Reviews of Modern Physics*, **21**, pp. 447-50.
- Morris, Michael S.; Thorne, Kip S.; and Yurtsever, Ulvi (1988) “Wormholes, Time Machines, and the Weak Energy Condition,” *Physical Review Letters*, **61**, pp. 1446-49.



- Norton, John D. (2003) "Causation as Folk Science," *Philosophers' Imprint* Vol. 3, No. 4.  
Reprinted in pp. 11-44, H. Price and R. Corry, *Causation, Physics and the Constitution of Reality*. Oxford: Oxford University Press.
- Norton, John D. (2008) "The Dome: An Unexpectedly Simple Failure of Determinism," *Philosophy of Science*, **75**, pp. 786-98.
- Norton, John D. (forthcoming) "The Metaphysics of Causation: An Empiricist Critique," in Yafeng Shan, ed., *Alternative Approaches to Causation*. Oxford University Press.
- Smeenk, Christopher; Arntzenius, Frank; and Maudlin, Tim (2023) "Time Travel and Modern Physics," *The Stanford Encyclopedia of Philosophy* (Spring 2023 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = <https://plato.stanford.edu/archives/spr2023/entries/time-travel-phys/>.
- Tipler, Frank J. (1974) "Rotating cylinders and the possibility of global causality violation," *Physical Review D*, **9**, pp. 2203-2206.