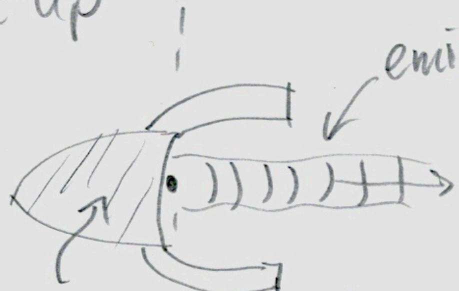
Differential form of thought experiment in Einstein (1906) "Principle of conservation of motion..."

Set up



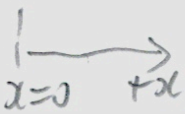
emits unidirectional radiation at rate s. ← constant

Energy emitted $S(t) = st$

Process starts at $t=0$

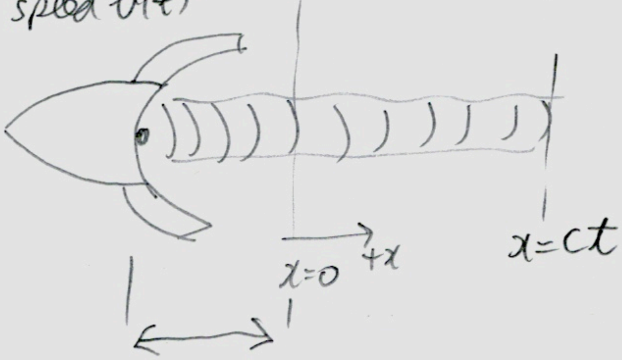
mass m, at rest at $t=0$

center of gravity located at $x=0$



After time t

mass $m(t)$
speed $v(t)$ ← negative



$X(t) = \int_0^t v(t)\,dt$

↑ negative

Require

① momentum is conserved.

Radiation has momentum $\frac{S}{c} = \frac{st}{c}$

$\therefore\ m(t)v(t) + \frac{st}{c} = 0$ ← momentum at $t=0$

② Center of gravity remains fixed.

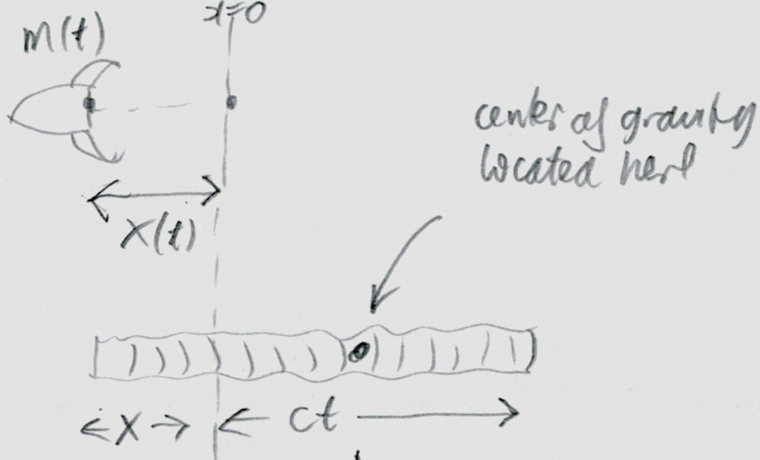
This <u>requires</u> that emitted radiation has mass, else center of mass moves with $m(t)$

Assign mass $I(S)$ to radiant energy S

corresponds to

Rate of addition mass to radiation emitted → $i(s)$ for s

center of gravity at time t



$$\boxed{\frac{ct-x}{2}+x = \frac{ct+x}{2}}$$

... NB $x<0$!!

{ $ct-x$ = length of radiation system.
Then shift center by x.

$$\text{center of gravity} = \left[\frac{m(t)x(t) + \underbrace{i(s)t}_{\text{mass of radiation}}\left[\frac{ct+x}{2}\right]}{\text{total mass}}\right]$$

$$0 = \frac{d}{dt}\left[\text{center of gravity}\right] \Rightarrow 0 = \frac{d}{dt}\left[m(t)x(t) + i(s)\frac{ct^2}{2} + i(s)\frac{xt}{2}\right]$$

since total mass = constant

$$\therefore\ 0 = \frac{dm(t)}{dt}x(t) + \underbrace{m(t)\underbrace{\frac{dx(t)}{dt}}_{v(t)}}_{m(t)v(t) = -\frac{st}{c}} + i(s)ct + i(s)\frac{x}{2}$$

$m(t)v(t) = -\frac{st}{c}$ from momentum conservation combining:

$$0 = \frac{dm(t)}{dt}x(t) - \frac{st}{c} + i(s)ct + i(s)\frac{x}{2}$$

solve for $i(s)$:

$$i(s) = \underbrace{\frac{-1}{ct}\frac{dm(t)}{dt}\cdot x(t)}_{0} + \frac{s}{c^2} - \underbrace{\frac{i(s)x}{2ct}}_{0}$$

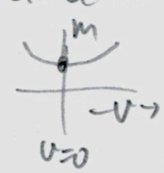
Evaluate when $t=0$

$\downarrow$

$$i(s) = \frac{s}{c^2} \Rightarrow \boxed{I = s/c^2}$$

Vanishing of terms

$$\frac{dm}{dt} = \frac{dm}{dv}\cdot\frac{dv}{dt} = 0$$ at $t=0$ since m is an <u>even</u> function of v

$\therefore$ 

$\frac{dm}{dv} = 0$ at $v=0$

$\frac{X(t)}{t} = 0$ at $t=0$ by L'Hopital's rule since

$X(t) \to 0$ as $t \to 0$ $\therefore$ $$\lim_{t\to 0}\frac{x(t)}{t} = \lim_{t\to 0}\frac{dx/dt}{dt/dt} = \lim_{t\to 0}\frac{v(t)}{1} = v(0) = 0$$