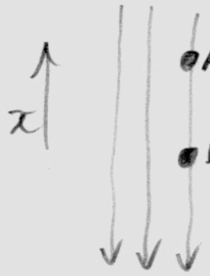
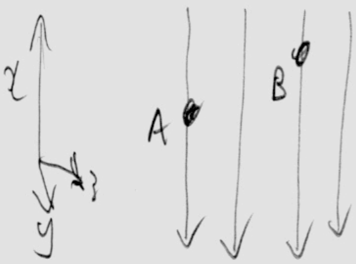
General Relativity, Part 1 Answer

Easier case



$$\left.\begin{aligned} \frac{d^2 x_A}{dt^2} &= -\frac{\partial \phi}{\partial x_A} = -g \\ \frac{d^2 x_B}{dt^2} &= -\frac{\partial \phi}{\partial x_B} = -g \end{aligned}\right\} \therefore \frac{d^2}{dt^2}\underbrace{(x_A - x_B)}_{\text{distance between A ad B}} = -g-(-g) = 0$$

Harder case



$$\frac{d^2 \underset{\sim}{x}_A}{dt^2} = -\underset{\sim}{\nabla}_A \phi = -g\,\hat{\underset{\sim}{i}}$$ ← unit vector in x direction

$$\frac{d^2 \underset{\sim}{x}_B}{dt^2} = -\underset{\sim}{\nabla}_B \phi = -g\,\hat{\underset{\sim}{i}}$$

$$\therefore \frac{d^2}{dt^2}\underbrace{(\underset{\sim}{x}_A - \underset{\sim}{x}_B)}_{\substack{\text{vector}\\ \text{separating}\\ A,B}} = -g\hat{\underset{\sim}{i}} - (-g\hat{\underset{\sim}{i}}) = 0$$

<u>BUT</u> if $d_{AB} = |\underset{\sim}{x}_A - \underset{\sim}{x}_B| = \sqrt{(\underset{\sim}{x}_A - \underset{\sim}{x}_B)\cdot(\underset{\sim}{x}_A - \underset{\sim}{x}_B)}$

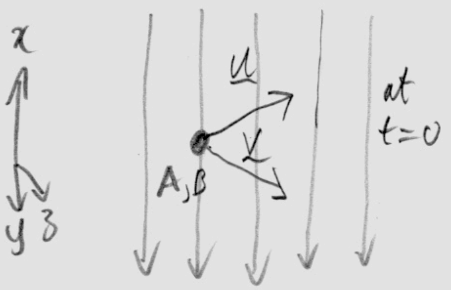
we do <u>not</u> in general have $\frac{d^2}{dt^2} d_{AB} = 0$

When do we? PTO

$$\boxed{\text{Sufficient condition for } \frac{d^2}{dt^2} d_{AB} = 0}$$

conjecture: Also necessary

$$\boxed{\text{Bodies A and B have same origin point at } t=0}$$



Then we have

$$\underset{\sim}{x}_A = \underset{\sim}{u}t - \tfrac{1}{2}gt^2\,\hat{\underset{\sim}{i}}$$

$$\underset{\sim}{x}_B = \underset{\sim}{v}t - \tfrac{1}{2}gt^2\,\hat{\underset{\sim}{i}}$$

write $\Delta\underset{\sim}{x} = \underset{\sim}{x}_A - \underset{\sim}{x}_B = (\underset{\sim}{u} - \underset{\sim}{v})t$

$$d_{AB} = \sqrt{\Delta\underset{\sim}{x} \cdot \Delta\underset{\sim}{x}} = \sqrt{(\underset{\sim}{u}-\underset{\sim}{v})t \cdot (\underset{\sim}{u}-\underset{\sim}{v})t} = \underbrace{|\underset{\sim}{u}-\underset{\sim}{v}|}_{\text{constant}}\, t$$

$$\therefore \frac{d}{dt} d_{AB} = |\underset{\sim}{u} - \underset{\sim}{v}|$$

$$\boxed{\frac{d^2}{dt^2} d_{AB} = 0}$$