1 Enumerative Induction.

Enumerative induction is the reluctant offspring of traditional syllogistic logic. It exists partly because something like it is pervasive and partly because the framework of syllogistic logic allows it to be defined.

The Pervasiveness of Enumerative Induction

An enumerative induction or, to use its more formal name, an induction by simple enumeration has the form

Some As are B

Therefore, All As are B.

It is the simplest form of inductive inference, even the most ancient ancestor of all inductive inference. But it is not a venerated ancestor. As we shall see, it is routinely approached with hesitation and mistrust. That seems to me to be undeserved.[2] Whatever may be its weaknesses, its use is quite pervasive in ordinary life and in science. Our investigations in all areas would be greatly impoverished if it was denied us. The argument form is typically introduced with somewhat frivolous examples that mask its importance: some squirrels have bushy tails; therefore all squirrels have bushy tails. These examples make it easy to display its form, but mask its importance.

We can find examples in all areas of the sciences. Euclid based his geometry on the postulate that we can always draw a straight line between any two points. Euclid's geometry and his postulate hold true of the space of our common experience and that of much of science, at

[2] It is hard to find anyone in the literature willing to defend enumerative induction. The exception may be Nicod (1930, pp. 203-205)

least to a level of accuracy of measurement that transcends our instruments.[3] How do we know the truth of the postulate, at least at the level of virtually all observational error? We know by enumerative induction. We have massive experience that tells us that we can always connect two points with a straight line. Perhaps we use a ruler of a stretched string to connect the points. Or, recalling that light moves in straight lines, we affirm that a pair of points is connected by a straight line when we sight from one to the other. The light reaching our eye traverses the straight line. We know that some—many—pairs of points can be connected by a straight line. We conclude that all pairs of points can be connected by a straight line. I know of no more fundamental justification for Euclid's postulate.[4]

In fundamental particle physics, many properties of particles can be recovered from the theory. But others are simple known to us a brute facts. We have no account of why the particles have those properties. We just have always found that particular instances do. The simplest example is the electron. It has a mass of 9.1 x 10^{-28}g and a charge of –1.6 x 10^{19} coulombs. We know this because every time we measure reliably we get these results. Similarly we know there are no magnetic monopoles and no paraparticles (particles that superpose integer and non-inter spin) simply because every individual we check has the property of not being a monopole or paraparticle. This is not to say that theories may not emerge that will explain why the electron has its particular charge; and so on. But when and if they do, we will already have had a long history of belief in the generality on the basis of induction by simple enumeration.

We can proceed through the sciences in this way, listing generalizations whose sole support was originally enumerative induction and may still be its sole support today. Thus we

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[3] Einstein's general theory of relativity finally informs us that Euclid's geometry holds only approximately in our space. The deviations are only discernible by the most refined of observations. A ray of starlight grazes the edge of the sun and is momentarily visible in photographic plates taken at the time of a solar eclipse. The photos reveal that the ray is slightly deflected as the light falls into the sun. Only half the deflection is due to that fall. The remainder is due to the slight deviation of the geometry of space from Euclidean near the sun.

[4] Typically induction by simple enumeration is the only way we can establish our most fundamental laws. So Newton was assured that all masses attract one another gravitationally by an inverse square law simply because every pair he could check attracted in this way.

knew through enumerative induction that cats cannot mate with dogs long before we had any account of genes and DNA. We knew that green plants need light to grow before we had accounts of photosynthesis and chlorophyll. We knew that drying or salting preserves food long before we knew of the micro-organisms these processes destroyed. We knew that intentional maggot infestation is an effective way of cleaning a wound and promoting its healing well before we knew that they acted in part by inhibiting bacterial infection. We knew that gold does not tarnish and that all pieces of wood burn before we had an elaborated theory of chemistry. We knew that all samples of diamond are the hardest of minerals before we knew of diamond’s unique molecular structure. After every set of numbers we tried conformed to Fermat's last theorem, we knew pretty surely that is was true, long we found a proof. The examples multiply without apparent limit. Science without enumerative induction could not have become what it is today and could not continue.

The Connection to Traditional Syllogistic Logic

When we consult the literature, the description given of enumerative induction (or what amounts to the same thing) may appear diffuse. The Port Royal Logic merely asserts "When from the examination of many particular instances we conclude to a general statement, we have made an induction." (Arnaud, 1662, p. 264) This, or something close to it, is the standard formula. The induction consists in an inference from particulars to a generality.[5] Other accounts

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[5] This standard formula is usually given by first applying it to perfect or complete induction (described below) and then apologetically extending it to the incomplete case. So Keynes (1921, p.274), as part of an historical survey of the use of the term induction, attributes to Aristotle a notion of induction "in which we generalize after the complete enumeration and assertion of *all* particulars which the generalisation embraces." Induction by simple enumeration "approximates" this sense if the enumeration is incomplete "as the number of instances is increased." Eaton (1931, pp.486-87) in related discussion also speaks of induction in terms of the passage from particulars to a generalization. Joseph (1916, p.378) in initiating his survey of the historical use of the term induction allows that: "…and induction meant primarily to Aristotle, proving a proposition to be true universally, by showing empirically that it was true in each particular case or kind of case…" This becomes the familiar (incomplete) enumerative induction as long as not

2 Complaints about Enumerative Induction.

Enumerative induction has been unfairly judged inadequate because it may fail; and it has been fairly judged so, because the characterization spawned by syllogistic logic is too weak. There is no consensus on how to repair it.

The Vilification of Enumerative Induction

Enumerative induction has become the flatulence of philosophy of science. Everyone has it; everyone does it; and everyone apologizes for it. It will not go away and cannot go away without compromising the vital function of science. The most celebrated jibe is Francis Bacon's (1620, First Book, §105)

> The induction which proceeds by simple enumeration is puerile, leads to uncertain conclusions, and is exposed to danger from one contradictory instance, deciding generally from too small a number of facts, and those only the most obvious.

Reading earlier in the work, we see that Bacon's reservations about enumerative induction are coupled with a denunciation of its practitioners: they are slothful and intellectually dishonest. So, earlier (§19), Bacon described the then current practice of enumerative induction as one that "…hurries on rapidly from the senses and particulars to the most general axioms…" He then lamented (§20) that this haste is natural to the understanding "when left to itself…[;] for the mind is fond of starting off to generalities, that it may avoid labor, and after dwelling a little on the subject is fatigued by experiment." The result is poorly grounded science, defended by sophistry (§25):

> The axioms now in use are derived from a scanty handful, as it were, of experience, and a few particulars of frequent occurrence, whence they are of much the same dimensions or extent as their origin. And if any neglected or unknown instance occurs, the axiom is saved by some frivolous distinction, when it would be more consistent with truth to amend it.

Is it just enumerative induction that has aroused his fury. Or is it enumerative induction as he believes it is practiced? Or does Bacon hold enumerative induction to be so flawed that no responsible practice is possible? The last would seem to be the case. For Bacon's remedy is not merely to call for more care and honesty; it is to replace enumerative induction by his new method.

In any case, the idea of enumerative induction as defective has maintained a persistent place in the literature. The Port Royal Logic (Arnaud, 1662, p. 264) includes a discussion of enumerative induction in a chapter "Sophisms: the Different Ways of Reasoning Incorrectly." Jevon's (1874, p. 149), having warned us of enumerative induction that "[W]e reap where we have never sown," proceeds with an elaborate denunciation:

> …I venture to assert that it never makes any real addition to our knowledge, in the meaning of the expression sometimes accepted. As in other cases of inference, it merely unfolds the information contained in past observations; it merely renders explicit what was implicit in previous experience. It transmutes but does not create knowledge.

A more measured Johnson (1924, Ch. II) captures it best in my view, in labeling enumerative induction "problematic induction." That represents at once that enumerative induction does present a problem but that it is a problem worthy of solution.

Counterexamples to Enumerative Induction…

Why is this most common form of inductive inference so vilified? The most common complaint against enumerative induction is that it admits counterexamples. This concern is ancient; it seems to be the essence of Sextus Empiricus' complaint against imperfect induction quoted above. ("…the induction will be infirm, it being possible that some of the particulars omitted in the induction should be contrary to the universal;…") In almost every era the counterexamples have been duly paraded. Indeed the goal of the parade is often to display the most striking exception drawn from the latest science. The message implicit in the examples--whether intended or otherwise--is not just that enumerative induction may fail. It is that the failure would obscure whatever the latest breakthrough is in science; or it would have, had we been so foolish as to rely solely on enumerative induction. I have collected a sampling of these counterexamples in Box 3: All Swans Are Not While.

Box 2: All Swans Are Not White

A fixture in the literature on enumerative induction is a display of counterexamples. They are typically chosen as illustrations of remarkable obtuseness of proponents of enumerative induction or of their overlooking major advances in science. Whether intended on not, they suggest that proponents of enumerative induction are foolish or benighted.

Sextus Empiricus, in the passage quoted above, already enunciated in antiquity[21] the simple objection that enumerative induction is infirm since some unchecked instance may contradict the conclusion. Elsewhere in the same work (Book II, 195), he contributed to the literature of surprising counterexamples. Horses, hogs, humans and hordes of other animals that spring naturally to mind all move their lower jaw. The enumerative induction to all animals is instinctive, but…:[22]

> Now this proposition--Everything human is an animal--is confirmed inductively from the particulars; for from the fact that Socrates, being human, is also an animal, and similarly with Plato and Dio and each of the particulars, it is thought possible to affirm that everything human is an animal. For even if one of the particulars were to appear contrary to the others, the universal proposition is not sound--e.g. since most animals move their lower jaw but the crocodile alone moves its upper jaw, the proposition 'Every animal moves its lower jaw' is not true.

Francis Bacon's assault on enumerative induction is rich in confident, polemical denunciation, but rather lean in concrete example. There is one quite memorable counterexample

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[21] See Franklin (2001, pp. 201-201) for Stoic examples that draw heavily on reports of freaks "…the man in Alexandria half a cubit high with a colossal head that could be beaten with a hammer, who used to be exhibited by the embalmers…" (Reported by the Eupicurean, Philodemus, *On Signs*.)

[22] Annas and Barnes (1994, p. 120)

that illustrates not so much the failure of enumerative induction but of the frailty of the human understanding that he elsewhere described as supporting overly hasty enumerative induction. It is always ready to add more support to propositions once laid down and slow to seek counterexamples.

> It was well answered by him [Footnote: Diagoras] who was shown in a temple the votive tablets suspended by such as had escaped the peril of shipwreck, and was pressed as to whether he would then recognize the power of the gods, by an inquiry. But where are those who have perished in spite of their vows? (Bacon, 1620, Book 1, §46)

Like Bacon, the *Port Royal Logic* complains forcefully that enumerative induction fails if conducted rashly and founded on too few instances. Arnaud (1662, pp. 284-85) writes:

> There are diseases which escape the detection of the most skilled physicians, and prescribed cures are not always successful. Rash minds, therefore, conclude that medicine is completely useless and is a craft of charlatans. There are women of easy virtue; this is warrant enough for the jealous to entertain unjust suspicions against the most upright women and for irresponsible writers to condemn all women. Great vices are often concealed beneath a façade of piety; hence, infers the freethinker, all devotion is but hypocrisy. Some things are difficult and obscure, and often we are grossly deceived. Therefore, say the ancient and modern Pyrrhonists, all things are obscure and uncertain--we cannot know the truth of anything with certainty.

The litany continues through improper generalization of some human failure or of a weakness of reason to the *finale*:

> From a few repeated actions we conclude to a custom; from three of four faults, a habit. What happens once a month or once every year is said to occur every day, every hour, even every moment. Men take so little pains to keep within the limits of truth and justice!

Where Bacon was not so clear, the *Port Royal Logic* makes clear that it has little sympathy for an enumerative induction even when conducted thoroughly and based on many instances, for it often leads to error. Arnaud (1662, pp. 264-65) continues:[23]

> …We cite but one noteworthy example.
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> Until recently all philosophers held two things indubitable truths: (a) A tight-fitting piston cannot be removed without bursting the pump; (b) suction pumps can raise water to any height. The philosophers came to hold fast to these generalizations in virtue of inductions based on a great many experiments. But new experiments have disclosed that both generalizations are false; the piston of a suction pump, however tight fitting, may be withdrawn if we use a force equal to the weight of a column of water whose cross section is the diameter of the pump and whose height is more that thirty feet; but a suction pump cannot raise water to a height of more than thirty-two or thirty-three feet.

Arnaud is referring here to what modern readers would call the discovery that a vacuum is possible after all; removing a piston without collapsing the cylinder forms a vacuum. Arnaud artfully avoids reporting the new results that way. With his Cartesian inclinations, presumably he does not want to challenge Descartes' deduction that space and matter are identical so that vacua--space without matter--really are impossible.

Jevons liked also to draw on recent science for illustrations of the failure of enumerative induction. One example came from astronomy. (Jevons, 1870, p.215)

> …there was a widespread and unbroken induction tending to show that all the Satellites in the planetary system went in one uniform direction round their planets. Nevertheless the Satellites of Uranus when discovered were found to move in a

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[23] Arnaud (1662, p.319) later gives another example drawn from the then recent breakthroughs in fluid dynamics:

> Not long ago we were quite certain that were water poured into a dish one end of which was much narrower than the other, the water would be at the same depth everywhere in the dish. Our certainty was derived from observation. Still, quite recently it was discovered that if one end of an irregularly shaped dish is very narrow, the water stands higher there than in the wider end.

> *retrograde* direction, or in an opposite direction to all Satellites previously known, and the same peculiarity attaches to the Satellites of Neptune more lately discovered.

Other examples came from chemistry. (Jevons, 1874, p. 238)

> Lavoisier, when laying the foundations of chemistry, met with so many instances tending to show the existence of oxygen in all acids, that he adopted a general conclusion to that effect, and devised the name oxygen accordingly. He entertained no appreciable doubt that the acid existing in sea salt also contained oxygen;[24] yet subsequent experience falsified his expectations.

This unreliability of natural patterns in chemistry was reinforced by the observation that many of the then newly discovered elements had simply been mistaken for others, until their distinct identity was discovered. (Jevons, 1870, p. 224; 1874, p. 237) So the recently discovered caesium and rubidium were mistaken for the chemically similar potassium. Other chemically similar elements were confused: tantalum and niobium, selenium and sulfur, and so on. His goal apparently was to shake the faith we may have in continuing any discovered regularity. Substances we presume of the same kind may later turn out not to be.

Yet more examples came from mathematics. Fermat believed, Jevons (1870, p. 222) reports, that $2^{2^x}+1$ is always prime, for any natural number x, presumably from the weight of positive instances. The regularity fails when x=4294967297, with the resulting number divisible by 641. Similarly (Jevons, 1870, p. 221; see also 1874, p. 230) "at one time it was believed"[25] that the formula x^2+x+41 yields primes, since it certainly does so for x=1, 2, 3 and many more values. "This was believed solely on the ground of trial and experience." It fails finally for x=40, for which the formula gives 40x40+40 +41 = 41x40 + 41 = 41x41 and that is not a prime number!

Jevons (1874, pp. 229-30) also invented simpler mathematical examples that a reader could easily see through. 5, 15, 35, 45, 65, 95 all end in the digit 5 and are divisible by 5. Does that allow us to infer that all numbers ending in the digit 5 are divisible by 5? The result is true

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[24] Jevon's footnote: "Lavoisier's *Chemistry*, translated by Kerr. 3rd ed., pp. 114, 121, 123."

[25] The failure at x=40 is so obvious that it is hard to see how this formula, attributed to Euler, could ever have caused serious confusion.

but cannot be founded on the enumerative induction. Otherwise, we might note that 7, 17, 37, 47, 67, 97 all end in the digit 7 and are primes. We would then infer to the falsity that all numbers ending in the digit 7 are prime.

Mill (1872, p.205) reports the venerable counterexample: all swans are not white. Black swans were found in Western Australia.[26] This counterexample has become such a familiar cliché that one rarely needs to complete the sentence beginning with the words "black swans…" for one's point to be all too apparent. Presumably its novelty had not yet dissipated when Mill invoked it, for he could readily classify it with other possible failures of enumerative induction that would be quite shocking were they now affirmed:

> But let us now turn to an instance apparently not very dissimilar to this [case of the swans]. Mankind were wrong, it seems, in concluding that all swans were white; are we also wrong when we conclude that all men's heads grow above their shoulders and never below, in spite of the conflicting testimony of the naturalist Pliny? As there were black swans, though civilized people had existed for three thousand years on the earth without meeting them, may there not also be "men whose heads do grow beneath their shoulders," notwithstanding a rather less perfect unanimity of negative testimony from observers? Most persons would answer, No…

I have my own contribution to this literature of counterexamples.[27] Fundamental theories of physics have long treated left and right as equivalent. If some system or process is possible, then so is its mirror image. If we encounter the left handed process and not the right handed mirror image, that is mere happenstance. *Apriori* there seems no reason to expect such even

[26] The Western Australian government website dates the first report of these black swans to January 1697 by the Dutch navigator Vlamingh. Mill's mention of it comes a century and a half later.

[27] Let us not forget two commonly overlooked counterexamples. We routinely accept that the sun will rise tomorrow and that any account of induction must somehow deliver that result--that is unless we live beyond the Arctic or Antarctic circles. Then, as winter approaches, we eventually encounter a sunset not followed by a sunrise, at least for months. (I am grateful to Eric Angner, who is Swedish, for reminding me of this!) Also, everything expands on being heated--except ice, that most familiar of counterexamples.

handedness in the fundamentals of nature. The human form, for example, is only superficially the same as its mirror image; we all have hearts on the left side. What led us to believe in the equivalence of left and right was an enumerative induction. As we uncovered more and more of the laws of fundamental physics, each new law respected this equivalence. There was no deeper justification. By the 1950s, with not a single counterexample known, the expectation of this equivalence was massive. Thus, even with suspicions set in motion in 1955, it came as a shock when processes governed by the so-called "weak" force of particle physics were found to treat left and right differently. This is the famous violation of parity in particle physics. Lee and Yang found no one had checked whether parity was violated by weak interactions. Experiments soon revealed the violation, such as Wu's experiments on the beta decay of radioactive cobalt.[28]

My award for the best contribution to this genre, however, is less elevated scientifically and goes to Russell (1912, pp. 97-98):

> Domestic animals expect food when they see the person who usually feeds them. We know that all these rather crude expectations of uniformity are liable to be misleading. The man who has fed the chickens every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken.

[28] For a brief account, see Segrè (1980, pp. 258-63).

Pages from John D. Norton, “A Survey of Inductive Generalization”

http://www.pitt.edu/~jdnorton/homepage/cv.html#survey_ind_gen