Engraved by E. Mackenzie. from an Original.

Carnot.

THE

PHILOSOPHICAL MAGAZINE:

COMPREHENDING

THE VARIOUS BRANCHES OF SCIENCE,

THE LIBERAL AND FINE ARTS,

AGRICULTURE, MANUFACTURES,

AND

COMMERCE.

BY ALEXANDER TILLOCH,

HONORARY MEMBER OF THE ROYAL IRISH ACADEMY, &c. &c. &c.

"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Monit. Polit.* lib. i. cap. 1.

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consonances, and of *Mr. Maxwell* on the system of *perfect consonancy,* are in danger of falling into utter contempt.

I beg here to mention, respecting the new notation for musical intervals, which I have explained vol. xxviii. p. 140, that the Octave, happening to contain just 12 of the *lesser fractions* f, and one of these to fall near each note of the equal temperament ; in almost all calculations respecting *Douzeaves,* the temperaments or results, are free of f, and *two* only of the *three* independent or *prime* terms, of which every *accurate* notation must consist, are in general found at last ; while the smallness of the most *minute,* m, it being less than the $\frac{1}{127}$th part of the *Schisma,* Σ, which is itself but a very trifle more than $\frac{1}{11}$th part of a *Comma,* c, (or $\frac{1}{11}\Sigma + \frac{1}{11}$ m) render it allowable in most practical cases to neglect m, and to consider the Σ s as *elevenths* of a comma, in the results; although I would advise the previous calculations to be always carried on strictly, in Σ, f and m, especially, as the number of f s will generally point out, to what finger-key or number of half notes, any step in the process answers.

I am, sir, your obedient servant,

JOHN FAREY.

12, Upper Crown-Street, Westminster,
February 1, 1808.

II. *Essay upon Machines in General. By M.* CARNOT, *Member of the French Institute, &c. &c.**

Preface.

ALTHOUGH the theory to be discussed be applicable to every subject which concerns the communication of motion, I have given to this work the title of Essay upon Machines in General ;—in the first place, because it is principally machines I purpose to treat of, as being the most important

* For a Translation of Carnot's "Reflections on the Theory of the Infinitesimal Calculus," see Phil. Mag. vol. viii. p. 222, and 335 ; and vol. ix. p. 39.

branch

branch of mechanics; and in the second place, because I do not mean to treat of any machine in particular, but solely of the properties which are common to all.

This theory is founded upon three principal definitions: the first regards certain movements which I call *geometrical*, because they may be determined by the principle of geometry alone, and are absolutely independent of the rules of dynamics. I have not thought that we could easily pass over them without leaving some obscurity in the elucidation of the principal propositions, as I have particularly shown with respect to the principle of Descartes.

By the second of my definitions, I endeavour to fix the signification of the terms *force soliciting* and *force resisting*: we cannot, in my opinion, perspicuously compare causes with effects in machinery without a marked distinction between these different forces; and this is the distinction upon which I think something vague and indeterminate has been always left.

Lastly, my third definition is that by which I give the name of *moment of activity* of a power, to a quantity in which a power is mentioned which is really in activity or in movement, and where we also take account of each of the instants employed by this force, *i. e.* of the time during which it acts. Whatever it be, we cannot refuse to allow that this quantity, under whatever denomination we designate it, is not to be continually met with in the analysis of machines in movement.

With the assistance of these definitions, I arrive at propositions which are very simple: I deduce all of them from one same fundamental equation, which, containing a certain indeterminate quantity, to which we may attribute different arbitrary values, will give successively in each particular case, all the determinate equations required for the solution of the problem.

This equation, which possesses the greatest simplicity, generally extends to all imaginable cases of equilibrium and movement, whether the movement changes hastily, or varies by insensible degrees: it is even applied to all bodies, whether hard, or endowed with a certain degree of elasticity;

and

and if I am not deceived, it is sufficient of itself, and independently of every other mechanical principle, to resolve all the particular cases to be met with.

I easily draw from this equation a general principle of equilibrium and movement in machines properly so called, and from the latter naturally flow other principles more or less general, several of which are already known and very celebrated, but which have been hitherto either inexactly or vaguely explained, rather than rigorously demonstrated.

Without departing from general principles, I have united in a scholium, and as clearly as possible, the most useful remarks for practice, and which, from their importance, appeared to me to merit a particular development. Every person repeats, that in machines in movement, we always lose in time or in velocity what we gain in power; but after perusing the best elements of mechanics, which seem to be the true place where the proofs and explanation of this principle should be found,—Is its extent or even its true signification easy to seize? Has its generality, with most readers, that irresistible evidence which should characterize mathematical truths? If they exhibit this striking conviction, ought we not to see mechanics instructed in these works, incessantly renounce their chimerical projects? Would they not cease to believe, in spite of every thing that has been taught them, that there is something of magic in machines? The proofs given them of the contrary only extend to simple machines: now they do not think these capable of any great effect, and they cannot be brought to believe that it must be the same in every case imaginable; they only speak of that where there are solely two forces in the system, and they are contented with an analogy: this is the reason why these mechanicians always hope that their sagacity will make them discover some unknown resource, some machine which is not comprehended within the ordinary rules; they think themselves so much the more certain of meeting with it, the further they remove from every thing which seems to have any relation with machines in use, because they imagine that the theory established with respect to the latter, cannot be extended to constructions which do not

seem

seem to have any connection with them. It is in vain to tell them that every machine may be reduced to the lever: this assertion is too vague and too wire-drawn to be admitted without a profound examination; they cannot persuade themselves that machines which appear to have nothing in common with those denominated simple ones, are subject to the same law, nor that we can pronounce upon the inutility of a secret which has not been communicated to any person: thence it happens that the most absurb ideas, and the furthest removed from the simplicity so advantageous to machines, are those which furnish them the most hopes.

The method of rooting out this error is certainly to attack it in its very source, by showing that not only in all the machines known, but also in all possible machines, it is an invariable law—that *we always lose in time or in velocity what we gain in power*,—and to explain clearly what this law signifies; but to this effect we must raise ourselves to the greatest generality possible, and not stop at any particular machine, or resort to any analogy. In the last place, there must be a general demonstration, deduced immediately and geometrically from the first axioms in mechanics: this is what I have attempted in this Essay. I have strongly insisted upon this fundamental point, and I do not know if I have succeeded in placing it in a sufficiently clear light; but on attacking error we are compelled to substitute truth in its place;—I have shown what is the true end of machinery: if it be unreasonable to expect prodigies from them beyond all probability, we shall still find there is plenty of utility in them for exercising the most lively imagination.

The reflexions I propose upon this law lead me to say a word of perpetual motion: and I have shown not only that every machine abandoned to itself must infallibly stop, but I assign the very instant when this must happen.

There will also be found among these reflections one of the most interesting properties of machines, which I think has not yet been remarked; it is, that in order to make them produce the greatest possible effect, it must necessarily happen that there be no percussion, *i. e.* that the movement

Movement by imperceptible degrees for greatest effect.

ment should always change by imperceptible degrees; which occasions, among other things, some remarks upon hydraulic machines.

Finally, I terminate this production by some reflections upon the fundamental laws of the communication of movement, which, if they be not agreeable to every body, have at least the merit of brevity.

I repeat that this Essay has merely for its object machinery in general; each machine has its peculiar properties: here we have only to do with those which are common to all; these properties, although sufficiently numerous, are in some measure all comprehended in one very simple law: it is this law I purpose to explain, to demonstrate, and develop, always regarding machines under the most general and direct point of view.

Introduction.

Machines in general, not particular machines.

I. There is no want of excellent treatises upon machinery: the properties peculiar to those in frequent use, and particularly to those called simple, have been inquired after and expounded with all possible sagacity. In my opinion, however, too little attention has been bestowed in the development of those properties which are common to machinery in general, and which for this reason no more belong to the cords of a machine than to the lever, the vice, or any other machine, whether simple or compound.

It is not, however, because geometricians have neglected to ascend to the general principles of equilibrium or movement; but it is only, as it were, *en passant* that they have spoken of their application to the theory of machines properly so called: and perhaps there is none of these principles to be found which unites to a rigorous demonstration a sufficient generality, to make it answer solely and independently for the solution of the various questions which may be proposed, as well upon the equilibrium as upon the movement of machines, *i. e.* for reducing every question to a business of geometry and calculation;—this is the true object of mechanics.

II. Among

II. Among the principles more or less general which have been hitherto proposed, we shall only mention two very celebrated ones, and upon which we shall have some observations to offer.

The first is that which assigns for the general law of equilibrium in weighing machines, that the centre of gravity of the system is then at the lowest possible point; but although this antient principle be very simple and general, it does not seem that all the attention it deserves has been paid to it: it is certainly, first, because it is subject to some expressions, like all these where a *maximum* and *minimum* is mentioned: second, because it has no relation except to a particular species of force, which is gravity: thirdly and lastly, because it appears difficult to give a general and rigorous demonstration of it. But first, we shall show that by a small change in the display of this principle, we may make of it a very precise, geometrical, and true proposition, without any exception whatever. Secondly, although it has no relation except to gravity, yet it is easy to apply it to all imaginable cases: for this purpose it is only requisite to substitute a weight in the place of each of the powers which are of a different genus; this is very easy by means of a line passing upon a return pulley, in such a manner that there now remains no other defect to this principle than that of being indirect. Thirdly and lastly, although we cannot demonstrate it rigorously without ascending to the first principles of mechanism, it is, however, easy to account for it so as to remove every doubt, if we had even no other proofs, as we shall show when we come to the exact demonstration which we shall endeavour to give of it in the course of this Essay.

Let us imagine therefore a machine to which there are no other forces except weights applied; I suppose it, besides, to be of any arbitrary form, but that no movement has been given to it; this being done, whatever be the disposition of the bodies of the system, it is clear that if there be equilibrium, the sum of the resistances of the fixed points or any obstacles, estimated in the vertical direction, contrary to the gravity, will be equal to the total weight of the system;

but if a movement is given, a part of the gravity will be employed to produce it, and it is only with the surplus that the fixed points will be charged; thus, in this case the sum of the vertical resistances of the fixed points will be less at the first instant than the total weight of the system: thus from these two forces combined (the gravity of the system and the vertical charge of the fixed points) there will result from it a single force equal to their difference, and which will push the system from top to bottom as if it were free: thus the centre of gravity will descend necessarily with a velocity equal to this difference divided by the total mass of the system. Again, if the centre of gravity of the system does not descend, there will necessarily be an equilibrium.

In general therefore—*For ascertaining that several weights applied to any given machine should make a mutual equilibrium, it is sufficient to prove that if we abandon this machine to itself, the centre of gravity of the system will not descend.*

III. The immediate consequence of this principle, which is true without exception, is, that if the centre of gravity of the system is at the lowest possible point, there will necessarily be an equilibrium; for, according to this proposition, it is sufficient, in order to prove it, to show that the centre of gravity will not descend: Now, how could it descend, when upon this hypothesis it is at the lowest point possible?

IV. In order to give another application of this principle, I suppose that it is required to find the general law of equilibrium between two weights, A and B, applied to a given machine: I say then, that in consequence of the preceding principle, there will be an equilibrium between these two weights A and B, if by supposing that one of the two has to bear it, and the machine has to take a small movement, it would happen that one of these bodies would ascend while the other descended; and that at the same time these weights were in the reciprocal rates of their estimated velocities in the vertical direction: in fact, if we suppose that A then descends with the vertical velocity V, while the velocity of B, also estimated in the vertical direction would be

u, we shall have by hypothesis, $A : B :: u : V$, or $AV = Bu$, therefore $\frac{AV - Bu}{A + B} = o$. This being done, since the bodies are supposed to be in motion, the one from top to bottom, and the other *vice versâ*, it is evident that the first member of this equation is the vertical velocity of the centre of gravity of the system: thus this centre of gravity will not descend, and therefore by the preceding position there must be an equilibrium.

[To be continued.]

III. *Additional Memoir upon living and fossil Elephants.* *By* M. Cuvier.

[Concluded from vol. xxix. p. 254.]

Article VII.

Comparison of the Crania of the Elephant of India and that of Africa—External Characters taken from the Ears—Parts of the Cranium susceptible of Variation in one and the same Species.

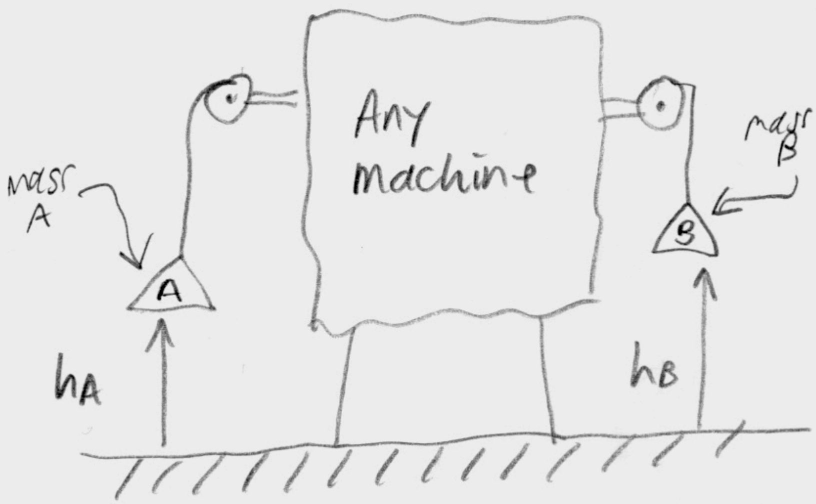
I had the good fortune to be the first to remark, in 1795, the distinctive characters presented by the crania of the two elephants, and which are so much the more interesting, as they may be applied to living, or entire individuals, without being obliged to examine their jaws*. I was able to recognise them at first only by the comparison of a cranium of each species; I have now verified these observations by inspecting seven real crania, (five of which are Indian, and two African,) and several drawings.

When these crania are separated from their lower jaws and placed upon the grinders, and upon the edges of the alveoli of the tusks, the zygomatical arcades are nearly horizontal in both species.

If we next view them laterally, what is very striking is, that

* Plate II. was long ago engraved from my own drawings. I gave a proof impression of it several years ago to M. Wiedeman of Brunswick, who copied it into his Archives de Zootomie, tome ii. cah. I. pl. I.—The Author.

p. 15



center of gravity $X = \dfrac{A h_A + B h_B}{A + B}$

Center of gravity is immobile

$$0 = \frac{dX}{dt} = \frac{A \overbrace{\frac{dh_A}{dt}}^{\text{“}-v\text{”}} + B \overbrace{\frac{dh_B}{dt}}^{\text{“}u\text{”}}}{A + B} = -\frac{Av - Bu}{A + B}$$

XXIX. *Essay upon Machines in general.* By M. Carnot, *Member of the French Institute, &c. &c.*

[Continued from p. 15.]

V. The second principle upon which we purpose making some observations, is the celebrated law of equilibrium of Descartes. It comes to this, that two powers in equilibrium are always in reciprocal ratio to their velocity, estimated in the direction of these forces, when we suppose that one of the two comes to take it from the other in an infinitely small degree; so that a small movement arises from it.

But although this proposition be very beautiful, and we generally regard it as the fundamental principle of equilibrium in machines, it is nevertheless infinitely less general than that which has been quoted in the first place: because it is applied solely to the case where there are only two powers in the system: and besides, it is very easily deduced from what has been said upon the subject of the two weights A and B, since we evidently approximate the one case to the other by substituting, by means of pulleys, weights in place of the forces which we wish to value.

Moreover, it is to be remarked, that this principle does not express the conditions of the equilibrium between two powers so completely as that which has been quoted in the first place; for it only gives the account of the quantities of force composing equilibrium, at the place where the latter also gives, in some sort, the account of their directions:—for example, in the case of equilibrium between two weights, the principle of Descartes solely teaches that the weights should be in the reciprocal ratio to their vertical velocities; but it does not indicate, like the first, that one of these bodies should necessarily ascend, while the other descends. In order that an axle, for instance, to the wheel and cylinder of which weights are suspended by cords, should remain in equilibrium, it is not sufficient that the weight applied to the wheel be to that of the cylinder as the radius of the cylinder is to the radius of the wheel:—it must also happen that these weights tend to make the machine turn in a contrary direction to each other; *i. e.* that they are placed in different sides with respect to the axis; else their efforts, being mutual, will put the machine in motion. It is therefore evident that what renders the principle of Descartes incomplete is, that by determining the reference of the powers, as to their values or intensities, he does not express that these powers should make opposite efforts, nor in what consists this opposition of efforts: it is clear, in fact, that for an equilibrium one of the forces must resist while the other solicits: now, this is not what happens in the case of the example of the axletree;—But what is it in general that distinguishes soliciting from resisting forces? This in my opinion has not yet been determined. We shall see in this essay that the characteristic difference of these forces consists in the angle they form with the directions of their velocities, so that the one form always acute angles with their velocities, while the others form obtuse ones with theirs.

Lastly. One fault with which we may reproach the principle of Descartes, as well as all those where we are discussing the small movement which would arise in the system if the equilibrium was disturbed, is, that they do not indicate the method of determining this small movement. Now, if for this purpose we must have recourse to some new mechanical principle, the former is not sufficient; and if we can determine it by pure geometry, What is the method of doing so? This is what the principle does not say: and let us not say that the proportion indicated by the principle always takes place whatever the movement is, provided it is possible, *i. e.* compatible with the impenetrability of bodies; for this would be an error: and we shall by and by show that these movements are subjected to certain conditions, in consequence of which I think it right to give them the name of *geometrical movements.*

We may make the same remark upon all the principles upon which we propose to consider a machine in two states infinitely near each other; for, in order to determine what are those two states; *i. e.* what movement the machine should take in order to pass from the one to the other, we must either employ new mechanical principles conjunctly with that proposed, which would render the latter insuffi-

cient;

cient; or else geometry is sufficient; and in this case it is a defect in the principle, not to make known the geometrical conditions to which this movement is subjected.

VI. The two laws mentioned are confined to the case of equilibrium. We pass easily from this case to that of the movement by M. D'Alembert's principle in dynamics. But we have found several others which are immediately applied to the case of movement; such as that of the preservation of living powers under the shock of perfectly elastic bodies; which is so much the more general, as it extends even to the case of the movement passing rapidly from one state to the other: but it would seem that people have little dreamed of the use that might be made of it in the theory of machines properly so called. It is, however, evident, that this law should have its analogy in the shock of hard bodies: and as we generally take the latter to use it as a term of comparison, this principle, transferred to hard bodies with the modification which the difference of their nature requires, cannot fail to be more useful than the preservation in question. We shall show, in fact, that we may deduce from it several capital truths with the greatest facility, and particularly the preservation of living powers in a system of hard bodies, the movement of which changes by insensible degrees; a principle of well-known utility in the theory of machines. We shall thereby see, at the same time, an intimate relation between these two preservations of living powers;—we draw from it also the principle of Descartes; and even, by generalizing it, the law of equilibrium in machines with weights above mentioned. This principle, in short, after having given to it the extension of which it is susceptible, appeared to us to contain all the laws of equilibrium and of movement: and we have not found a better for the basis of our theory.

Perfectly elastic bodies contrasted with "hard" bodies. Hard = inelastic

VII. This essay will be divided into two parts: In the first we shall treat of the general principles of equilibrium and of movement in machines; and in the second we shall examine the properties of machines properly so called, without ever stopping at any particular machine.

PART

PART FIRST.

General Principles.

When one body acts upon another, it is always immediately, or by the agency of some intermediate body: This intermediate body is generally what is called a machine: the movement lost every instant by bodies applied to this machine is partly absorbed by the machine itself, and partly received by the other bodies in the system; but as it may happen that the object of the question is simply to find the reciprocal action of bodies applied to intermediate bodies, without having any occasion to know the effect of it upon the intermediate body itself, it has been thought, in order to simplify the question, to make an abstraction of the very mass of this body, preserving to it on the other hand all the other properties of matter. Hence the science of machines has become in some measure an isolated branch of mechanics, in which it is required to consider the reciprocal action of the different parts of a system of bodies; among which there are found things which, when deprived of the inertness common to all parts of matter such as exists in nature, have retained the name of machines.

IX. This abstraction may simplify in certain particular cases, where circumstances indicate those of bodies, the mass of which it is convenient to neglect, in order more easily to attain our object; but we conceive that the theory of machines in general has really become more complicated than formerly: for this theory was once contained in that of the movement of bodies, such as nature presents them to us; but at present we must consider at once two kinds of bodies, the one as they really exist, and the other as deprived in part of their natural properties. Now, it is clear, that the first of these problems is a particular case of the latter; therefore the latter is more complicated: further, although we easily succeed by similar hypotheses, in finding the laws of equilibrium and of movement in each particular machine, such as the lever, the axle, and the vice, there results an assemblage of facts, the connection of which is perceived with difficulty, and solely by a kind of analogy; which should ne-

cessarily

cessarily happen, as often as we have recourse to the particular figure of each machine, in order to demonstrate a property which is common to it with all others. These common properties being those which we have in view in this essay, it is clear that we shall only succeed in finding them by the abstraction of particular forms. Let us begin, therefore, by simplifying the state of the question, by ceasing to consider under one and the same system, bodies differing in their nature. Finally, let us restore to machines their *vis inertiæ.* It will be easy, after this, to neglect their mass in the result: we shall have the choice of doing so or not; and in setting out, the solution of the problem will be equally general, at the same time that it will be simpler.

[To be continued.]

XXX. *On Caloric, and the Heat evolved during Combustion.* *By* JAMES SCHOLES, *Esq., Manchester.*

To Mr. Tilloch.

SIR,

HAVING been induced to pay particular attention to combustion for some time past, I have insensibly imbibed principles different from the generally received theory. I very soon began to suspect caloric as a compound substance, and six months ago had recognised two fluids of electricity for its component parts. The only demonstrative grounds I then had for my ideas was the production of light and heat, particularly the latter, and for the purpose of measuring the quantity thereof I had an apparatus constructed. But when Mr. Davy's recent experiments were noticed in your Magazine, I immediately saw them as an additional support of my peculiar principles, and prepared a lecture, which was delivered to a Society in this town on the 29th of January, laying down the whole system, as supported by facts deduced from electricity and the experiments of Mr. Davy, which I intended to publish when more matured; but on looking over the monthly publications yesterday, I found a communication in Mr. Nicholson's to a similar purport, which has induced

me

consist, will remain unsupported; for if the above-mentioned fits have no existence, the whole foundation on which the theory of the size of such parts is placed, will be taken away, and we shall consequently have to look out for a more firm basis on which a similar edifice may be placed. That there is such a one we cannot doubt, and what I have already said will lead us to look for it in the modifying power which the two surfaces, that have been proved to be essential to the formation of rings, exert upon the rays of light. The second part of this paper, therefore, will enter into an examination of the various modifications that light receives in its approach to, entrance into, or passage by, differently disposed surfaces or bodies; in order to discover, if possible, which of them may be the immediate cause of the coloured rings that are formed between glasses.

XLIII. *Essay upon Machines in General. By M.* Carnot, *Member of the French Institute, &c. &c.*

[Continued from p. 158.]

X. The science of machines in general is therefore reduced to the following question:

"*Being acquainted with the virtual movement of any system of bodies* (*that is to say, that movement which each of these bodies would take if it were free*), *find the real movement which will take place the instant following, on account of the reciprocal action of bodies, by considering them such as they exist in nature,* i. e. *as endowed with all the inertness common to all the particles of matter.*"

XI. Now, as this question evidently contains the whole of mechanics, we must, in order to proceed with precision, go back to the first laws which nature observes in the communication of movements. We may reduce them in general to two, which are the following:

FUNDAMENTAL LAWS OF EQUILIBRIUM, AND MOTION.

First Law.—*Action and Reaction are always equal and contrary.*

This

This law consists in this, that every body which changes its state of repose or uniform and rectilinear motion, never does so except by the influence or action of some other body, upon which it impresses, at the same time, a quantity of motion equal and directly opposite to that which it receives from it; that is to say, that the velocity it assumes the instant afterwards is the force resulting from that which this other body impresses upon it, and from that which it would have had without this last force. Every body therefore resists its change of state; and this resistance, which is called *vis inertiæ*, is always equal and directly opposite to the quantity of motion it receives, *i. e.* to the quantity of motion which combined with that which it had immediately before the change, produces, as the result, the quantity of motion which it should really have immediately afterwards. This is also expressed by saying, that in the reciprocal action of bodies, the quantity of motion lost by the one is always gained by the others, in the same time and in the same ratio.

= Momentum

First law is conservation of momentum.

Characterizes fully inelastic collisions.

Below: "hard" = inelastic.

SECOND LAW.—*When two hard bodies act upon each other, by shock or pressure,* i. e. *in virtue of their impenetrability, their relative velocity, immediately after the reciprocal action, is always null.*

In fact, we constantly observe, that if two hard bodies give a shock to each other, their velocities, immediately after the shock, estimated perpendicularly to their common surface at the point of contact, are equal, in the same way as if they were drawn by inextensible wires, or pushed by incompressible rods; their velocities, estimated in the ratio of this wire or rod, would necessarily be equal: whence it follows that their relative velocity, *i. e.* that by which they approach or recede from each other, is in every case null at the first instant.

From these two principles it is easy to draw the laws of the shock of hard bodies, and consequently to conclude the two other secondary principles, the use of which is continual in mechanics, viz.

1. That the intensity of the shock, or of the action which is

is exercised between two bodies which meet, does not depend upon their absolute *movements, but solely upon their relative movements.* 2. *That the force or quantity of movement which they exercise upon each other, by the shock, is always directed perpendicularly to their common surface at the point of contact.*

XII. Of the two fundamental laws, the *first* generally agrees with all the bodies of nature, as well as the two secondary laws which we have seen; and the *second* solely regards hard bodies; but as those which are not hard have different degrees of elasticity, we generally refer the laws of their movement to those of the hard bodies, which we take for a term of comparison, *i. e.* we regard the elastic bodies as composed of an infinity of hard corpuscles separated by small compressible rods, to which we attribute all the elastic virtue of these bodies; so that, properly speaking, we do not consider in nature any other than bodies endowed with different moving forces. We shall follow this method as the simplest: we shall therefore reduce the question to the investigation of the laws observed by hard bodies, and shall afterwards make some applications of them to cases in which bodies are endowed with different degrees of elasticity.

Hard = inelastic

XIII. This essay upon machines not being a treatise upon mechanics, my object is not to explain in detail, nor to prove the fundamental laws I have related; these are truths which all the world knows, as to which they are generally agreed, and which are most strongly manifested in all the phænomena of nature. This is sufficient for my object, which is merely to draw from these laws a simple and exact method for finding the state of rest or of movement which results from them in any given system of bodies, *i. e.* to present the same laws under a form which may facilitate their application to each particular case.

Problem posed

XIV. Let us suppose therefore any system of hard bodies, the virtual given movement of which is changed by their reciprocal action into another which we wish to find; and in order to embrace the question in all its extent, let us suppose that the movement may either change suddenly, or vary by insensible degrees: finally, as fixed points or some obstacles may be met with, let us consider them as they really are in fact, that is to say, as ordinary bodies of themselves, making part of the system proposed, but firmly arrested in the spot where they are placed.

Pefectly hard is incompressible, inextendible

XV. In order to attain the solution of this problem, let us first observe, that, all the parts of the system being supposed perfectly hard, *i. e.* incompressible and inextensible, we may visibly, whatever it may be, regard it as composed of an infinity of hard corpuscles, separated from each other either by small incompressible rods, or by small inextensible wires; for when two bodies strike, push, or tend in general to approach each other without being able to do it, on account of their impenetrability, we can conceive between the two a small incompressible rod, and suppose that the movement is transmitted from the one to the other according to this rod: and in the same way, if two bodies tend to separate, we may conceive that the one is attached to the other by a small inextensible wire, according to which the movement is propagated: this being done, let us consider successively the action of each of these small corpuscles upon all those which are adjacent to it, *i. e.* let us examine two by two all these small corpuscles separated from each other by a small incompressible rod, or by a small inextensible wire, and we shall see what ought to result in the general system of all these corpuscles. Let us name for this purpose,

m' and m'' The masses of the adjacent corpuscles.

V' and V'' The velocities they ought to have the following instant.

F' The action of m'' upon m', that is to say, the force or quantity of movement which the first of these corpuscles impresses upon the other.

F'' The reaction of m' upon m''.

q' and q'' The angles formed by the directions of V' and F' and by those of V'' and F''.

This being done, the real velocity of m' being V', this velocity estimated in the direction of F' will be V' cosine q'; in the same manner the velocity of m'' estimated in the direction of F'' will be V'' cosine q''. Therefore, since by the

the second fundamental law bodies should go in company, we shall have V′ cosine $q' + V''$ cosine $q'' = 0$ (A): thus by the first fundamental law, we shall also have F′ V′ cosine $q' + F'' V''$ cosine $q'' = 0$ (B): for if m' and m'' are both moveable, it is clear, by this law, that we have $F'' = F''$; therefore on account of the equation (A) we shall also have the equation (B); and if one of the two, m' for instance, be fixed, or form part of an obstacle, we shall have V′ cosine $q' = 0$; therefore on account of the equation (A) we shall also have V″ cosine $q'' = 0$; therefore the equation (B) will still take place: therefore this equation (B) is true for all the corpuscles of the system taken two by two. Imagining therefore a similar equation for all these bodies taken in fact two by two, and adding together all these equations, or, what comes to the same thing, the integral equation (B), we shall have for the whole system,

s F′ V′ cosine q' + *s* F″ V″ cosine $q'' = 0$: that is to say, the sum of the products of the quantities of movement which are reciprocally impressed by the corpuscles separated by each of the small inextensible wires or incompressible rods; from these quantities, I say, each of them multiplied by the velocity of the corpuscle on which it is impressed, estimated in the direction of this force, is equal to zero.

This being done, abandoning the preceding denominations, let us name

The mass of each of the corpuscles of the system - m

Its virtual velocity, *i. e.* that which it would assume if it were free, - - - - - - - W

Its real velocity - - - - - - V

The velocity which it loses in such a manner that W is the result of V and of this velocity - - U

The force or quantity of movement which each of the adjacent corpuscles impresses upon m, and by the intermedium of which it evidently receives all the movement that is transmitted to it from the different parts of the system, - - - - - - - - F

The angle comprehended between the directions of W and V - - - - - - - - X

The angle comprehended between the directions of W and U - - - - - - - Y

The angle comprehended between the directions of V and U - - - - - - - - - Z

The angle comprehended between the directions of V and F - - - - - - - - - *q*

We shall therefore have for the whole system *s* F V cosine *q* = 0, or *s* V F cosine *q* = 0 (C): at present we must observe that, the velocity of *m* before the reciprocal action being W, this velocity estimated in the direction of V will be W cosine X: therefore V — W cosine X is the velocity gained by *m* in the direction of V: therefore *m* (V — W cosine X) is the sum of the forces F which act upon *m*, estimated each in the direction of V: therefore *m* and V (V — W cosine X) is the same sum multiplied by V. Now to each molecule a similar sum answers; and further, the sum total of all these particular sums is visibly for the whole system *s* V F cosine *q*; therefore *s m* V (V — W cosine X) = *s* F V cosine *q*: adding to this equation the equation (C), there comes *s m* V (V — W cosine X) = 0 (D); but W resulting from V and U, it is clear that we shall have W cosine X = V + U cosine Z: substituting therefore this value of W cosine X in the equation (D), it will be reduced to *s m* V U cosine Z = 0 (E); *first fundamental equation.*

I'm lost. Secondary literature identifies this as equivalent to conservation of mv^2. i.e. of kinetic energy.

XVI. Let us imagine that at the moment when the shock is about to be given, the actual movement of the system is at once destroyed, and that we make it take instead of it successively two other arbitrary movements, but equal and directly opposite to each other, *i. e.* let us make it set out successively from its actual position, with two movements, such that, in virtue of the second, each point of the system has at the first instant a velocity equal and directly opposed to that which it would have had in virtue of the first of these movements: this being done, it is clear, 1st, That the figure of the system being given, this may be done in an infinity of different ways, and by operations purely geometrical; this is the reason why I shall call these movements *geometrical movements*; i. e. that *if a system of bodies sets out from a given position with an arbitrary movement, but yet of such a nature that it is possible to make it take another in every respect equal and directly opposite, each of these*

movements

Geometrical movements defined

They are reversible.

movements will be named a geometrical movement *. 2dly, I say that in virtue of this geometrical movement, the adjacent corpuscles, which may be regarded as being pushed by a rod, or drawn by a wire, will not approach nor recede from each other at the first instant, *i. e.* at the first instant of this geometrical movement the relative velocity of these adjacent corpuscles will be nothing: in fact, it is clear, in the first place, that if *m* be separated from an adjacent corpuscle by an incompressible rod, it will not be able to approach it; and that if it be separated from it by an inextensible wire, it will not be able to recede from it: secondly, I say that if it be separated from it by an incompressible

Footnote makes clear that "geometrical motion" = motion admitted by the geometry of the rigid members, independent of force considerations..

* In order to distinguish by a very simple example those movements called *geometrical* from those which are not so, let us imagine two globes which push each other, but in other respects free and disengaged from every obstacle: let us impress upon these globes equal velocities, and moved in the same direction according to the line of the centres;—this movement is *geometrical*, because the bodies could even be moved in a contrary direction with the same velocity, as is evident: but let us now suppose that we impress upon these bodies movements equal, and directed in the line of the centres, but which, in place of being, as formerly, moved in the same direction, tend on the contrary to recede from each other; these movements, although possible, are not what I mean by *geometrical movements*; because if we wished to make each of these moveable powers to assume a velocity equal and contrary to that which it receives in this first movement, we should be hindered from doing so by the impenetrability of bodies.

In the same way if two bodies are attached to the extremities of an inextensible wire, and if we make the system assume an arbitrary movement, but so as that the distance of the two bodies may be constantly equal to the length of the wire, this movement will be *geometrical*, because the bodies may assume a similar movement in quite a contrary direction; but if these moveable bodies approach to each other, the movement is not *geometrical*, because they could not take a movement equal and contrary without receding from each other; which is impossible on account of the inextensibility of the wire.

In general it is evident, that whatever be the figure of the system and the number of bodies, if we can make it assume a movement so as there should result no change in the respective position of the bodies, this movement will be geometrical; but it does not follow from this that there is no other method of satisfying this condition, as we shall show from several examples.

Let us imagine an axle, to the wheel and cylinder of which are attached weights suspended by cords: if we turn the machine in such a manner that the weight attached to the wheel should descend from a height equal to its circumference, while that of the cylinder will ascend from a height equal to its circumference, this movement will be *geometrical*, because it is equally

possible

compressible rod, it cannot recede from it any more; for, if it receded, it is clear that in virtue of the equal and directly opposite movement, which is also possible by hypothesis, it would approach it; which could not be on account of the incompressibility of the rod: for the same reason finally it is obvious, that if it be a wire which separates *m* from the adjacent corpuscle, it will not approach, because then it would be possible to remove it by an equal and directly opposite movement: now this cannot be, on account of the inextensibility of the wire: therefore, whatever may be the geometrical movement impressed upon the

possible to make the weight attached to the cylinder descend from a height equal to its circumference, while the weight attached to the wheel would mount from an equal height to its circumference; but if while we cause the weight attached to the wheel to descend from a height equal to its circumference, we should cause the weight attached to the cylinder to ascend from a height greater than its circumference, the movement would not be *geometrical*, because the equal and contrary movement would be visibly impossible.

If several bodies be attached to the extremities of different wires united by the other extremities to one and the same knot, and if we make the system assume such a movement as that each of the bodies remains constantly removed from the knot of one and the same quantity at the length of the wire to which it is attached, this movement will be *geometrical*, even when the different bodies approach to each other; but if some of them approach the knot, the movement would not be geometrical, because, the wires being supposed to be inextensible, the equal and contrary movement would be visibly impossible.

If two bodies are attached to the extremities of a wire into which is introduced a moveable particle, it will be sufficient, in order that the movement be geometrical, that the sum of the distances from the moveable particle to each of the two other bodies is constantly equal to the length of the wire; so that if these two bodies are fixed, the moveable particle will not depart from an elliptical curve.

If a body be moved by a curved surface, for instance, in the concavity of a spherical shell, the movement will be geometrical, while the body will move in a tangent form to the surface; but if it be separated the movement will cease to be geometrical, because the equal and contrary movement is visibly impossible.

From all this it is evident, that although on giving to a system a *geometrical* movement, the different bodies of this system may be brought near to each other, yet we may say that the adjacent corpuscles, considered two by two, do not tend at the first instant either to approach or recede, as I shall prove at length in the text. Bodies therefore exercise no action upon each other in virtue of a similar movement: these movements are therefore absolutely independent of the rules of dynamics, and it is for this reason that I have called them *geometrical.*

system,

system, the relative velocity of all these adjacent corpuscles which act upon each other, taken two by two, will be nothing at the first instant. This being granted, let us call u the absolute velocity which m will have in the first instant, in virtue of this geometrical movement, and z the angle comprehended between the directions of u and U; it is clear that the corpuscles m will not tend to approach or recede from each other in virtue of the velocities u, if we suppose them animated at the same time with these velocities u and velocities U; nor will they tend more to approach or recede if animated with the mere velocities U: therefore the reciprocal action exercised among the different parts of the system will be the same, whether each molecule be animated with the single velocity U, or with the two velocities u and U: but if each molecule was animated with the single velocity U, it is plain that there would be equilibrium: thus, if it was animated at once with the two velocities U and u, or with a single velocity the result of both, U will still be the velocity lost by m; and u will be the real velocity after the reciprocal action: thus, by the same reasoning by which we had the fundamental equation (E) we shall also have $s\, m\, u$ U cosine $z = 0$ (F); *second fundamental equation.*

P. 313. (F) is identified as a statement of the conservation of momentum of an isolated, rigidly connected system of masses..

It is very easy at present to resolve the problem which we propose for the preceding equation necessarily taking place, whatever be the value of u and its direction, provided the movement to which it refers be geometrical: it is clear that by successively attributing to that indeterminate different values and arbitrary directions, we shall obtain all the necessary equations among the unknown quantities, upon which depends the solution of the problem and of quantities either given or taken at pleasure.

XVII. In order to place this solution in the clearest light, it will be sufficient to give an example of it.

Let us suppose therefore that the whole system is reduced to an assemblage of bodies united to each other by inflexible rods, in such a manner that all the parts of the system should be forced always to preserve their same respective

 positions;

positions; but that there is no fixed point or any obstacle; the equation (F) gives us the solution of this problem on attributing successively to u different values and directions.

1st. As the velocities u are not subjected to any condition, unless the movement of the system in virtue of which the corpuscles m have these velocities be geometrical, it is evident that we can at first suppose all of them equal and parallel to one given line: then u being constant, or the same with respect to all the points of the system, the equation (F) will be reduced to $s\, m$ U cosine $z = 0$; which informs us that the sum of the forces lost by the reciprocal action of the bodies in the arbitrary sense of u is null, and that consequently that which remains is the same as if each body had been free; *this is a well-known principle.*

2dly. Let us now imagine that we make the whole system turn round a given axis, so that each of the points will describe a circumference round this axis, and in a plane which shall be perpendicular to it; this movement is visibly geometrical; therefore the equation (F) takes place: but then on calling R the distance from m to the axis, it is clear that we have $u = AR'$, A being the same for all the points; therefore the equation (F) is reduced to $s\, m$ R U cosine $z = 0$; that is to say, that the sum of the *momenta* of the forces lost by the reciprocal action relatively to any axis is null; *this is another well-known principle.*

3dly. We might also attribute to u other values; but this would be useless, and might lead to equations already contained in the preceding; for we know that the latter are sufficient for resolving the question, or at least for reducing it to a matter of pure geometry.

First Remark.

XVIII. The object we propose by giving a geometrical movement is to change the state of the system, without altering however the reciprocal action of the bodies which compose it, in order thereby to procure relations between these exercised and unknown forces and the arbitrary velocities which bodies assume in virtue of these different geometrical

metrical movements: but it must be remarked that there is a case where geometrical movements are not the only ones which can answer the same purpose, and where some other movements may be employed in the same way, in order to extract from the general equation (F) determinate equations: this happens when these other movements, without being absolutely geometrical, become so, nevertheless, merely on suppressing some of the small wires or rods we have supposed to be interposed between the adjacent particles of the system, at the time, I say, when these rods or wires supposed to transmit the movement from one corpuscle to another, transmitted none at all in fact; *i. e.* when the tension of some of these wires, or the pressure of some of these rods, is equal to zero; for then by suppressing these wires or rods, the tensions or pressures of which are null, we evidently change nothing at all of the reciprocal action of the bodies, and nevertheless it is possible that we may thereby render the system susceptible of some geometrical movements, which could not otherwise take place: there is nothing therefore to prevent us from regarding these rods and wires as annihilated, since they have no influence upon the state of the system; and as we consequently employ as geometrical the movements which, without being so effectively, become so nevertheless by this suppression.

Further, when two bodies are contiguous to each other, it is evidently the same thing to suppress the small rod which we have imagined to be interposed between two, to hinder them from approaching, or to suppose that these bodies are permeable to each other, *i. e.* that they may be penetrated as easily as the empty space is penetrated by all bodies; whence it evidently follows, that in general, in any system of bodies acting upon each other, immediately or by wires and rods, *i. e.* by the intermedium of any machine, if there be any wire, rod, or other part of the machine which exercises no action upon bodies applied to it, *i. e.* which may be annihilated without any change resulting in the reciprocal action of these bodies, we shall be able to treat as geometrical all the movements which, without being so effectively, would become so by this suppression,

in

in the same way as those which would become so also, regarding as freely permeable to each other, those of the bodies among which no pressure is exercised, although they are adjacent. The following, however, shows the utility of this observation :

If, when we undertake the solution of any problem, we know beforehand that a certain part of the machine does not exercise any action upon the other parts of the system, we shall be able to suppose that this part of the machine is totally annihilated, and ascertain the movement of the system according to this hypothesis, *i. e.* by treating as geometrical all the movements which would really become so by this supposition; and in the same way, if one of the given conditions of the problem is, that certain adjacent bodies do not exercise any pressure upon each other, we shall express this condition by regarding these two bodies as permeable to each other, *i. e.* by treating as geometrical the movements which would in fact become so by this supposition.

But if it happens that we are ignorant whether this pressure be real or null, we must ascertain the movement of the system, by first supposing the one or the other at pleasure: we shall suppose therefore, for example, that this pressure is real: then, if on inquiring, according to this hypothesis, the value of this pressure, we find it real and positive, we shall conclude that the hypothesis is legitimate, and the exact result; or else we shall be assured that the pressure in question is null, and that we may consequently treat as geometrical, motions which would become so in fact, if the two bodies in question were freely permeable to each other.

Further, if there was a machine in the system, a wire for example, and that we were ignorant if the tension of this wire is null or real, we might make the calculation by at first supposing that there really is tension ; then, if we find for the value of this tension a real and positive quantity, we shall conclude that the supposition is legitimate, and that the result is exact; or else, we must recommence the calculation, setting out from the contrary supposition, *i. e.* supposing that the tension of the wire is equal to zero ;

which

which will be done by supposing the wire annihilated, *i. e.* by treating as geometrical the motions which would be so effectively if the wire in question did not exist.

From this it follows, that in order to extract in each particular case from the general equation (F) all the determinate equations which it can give, we must first make the system assume all the geometrical movements of which it is susceptible; secondly, to treat also as such all those which would become so by suppressing some machine or part of a machine, the action of which upon the rest of the system is null, or by regarding as permeable to each other, the bodies among which, although adjacent, no pressure is exercised. 3dly. In the last place, if we are in doubt whether a certain wire, rod, or any part of the machine has or has not a real action upon the other parts of the system, or that there was a real pressure between two adjacent bodies, we must first clear up this doubt, by supposing the thing in question as we have above explained it, and by treating as geometrical the movements which these suppositions shall have discovered as being capable of being taken for such.

According to this remark, it seems proper therefore to extend the name of geometrical to all the movements, which, without being so effectively, become so on suppressing some machine or part of a machine which has no influence upon the state of the system, and on regarding also as perfectly permeable to each other, bodies in contact, without any pressure being exercised among them, *i. e.* without there being any thing except a simple juxtaposition: thus we shall presently comprehend all these movements, under the title of geometrical movements, since in fact they are equally well determined by operations purely geometrical, and are employed in the same way for extracting from the general equation (F) determinate equations, while the general and exclusive property* of these movements

* It is evident that this property belongs successively to the movements which I here call geometrical, and that it would consequently be a very false idea of them to regard them as movements simply possible, *i. e.* compatible with the impenetrability of matter: for, supposing, for instance, that all the system

ments is to change the state of the system, without altering the reciprocal action of the bodies which compose it. To leave, however, some distinction between them, we may call the first *absolute geometrical movements,* and the others *geometrical movements by supposition:* but when I speak simply of geometrical movements, without otherwise designing them, I shall imply both indifferently.

This being done,—since we have explained how we may determine, without the assistance of any mechanical principle, all the geometrical movements of which a given system is susceptible, it follows that the general problem which we proposed is entirely reduced by the general equation (F) to operations purely geometrical and analytical: we must, however, observe, that it is not sufficient to attribute to the *arbitraries u* different values, but we must also attribute to them different relations or directions; for, if we are contented to attribute different values to them without changing any thing in the relations or directions, we should obtain different equations, quite true and correct, but which would be evidently reduced to the same on multiplying them by different *constants.*

Second Remark.

XIX. As we are only speaking of hard bodies here, it is clear that among the different values which we may attribute to *u,* the velocity V is itself comprehended; *i. e.* that the real movement of the system is itself one of the geometrical movements of which it is susceptible: the first equation (E) is therefore contained in the indeterminate equation (F), and consequently we may reduce to this single equation (F) all the laws of equilibrium and of movement in hard bodies.

Now we have seen, that this equation is nothing else than the first (E), to which we have succeeded in giving more

system be reduced to two adjacent globes, and pushing each other, it is clear, that if we force these bodies to separate or to move in a direction contrary to each other, this movement will not be impossible, but that at the same time bodies cannot assume it without ceasing to act upon each other. This movement, therefore, is not proper for attaining the object proposed, which is to change nothing in the reciprocal action of bodies.

more extension by means of the geometrical movements; but as we shall soon see (XXIV) the analogy of this equation (E) with the principle of the preservation of the moving powers in the shock of perfectly elastic bodies becomes striking by a slight transformation; and we shall see (XXVI), that in fact it is nothing else than this principle itself transferred to hard bodies, with the modification required by the different nature of these bodies: it is therefore this preservation of moving powers which will serve, as we have premised, as a basis to the whole of our theory of machines, whether at rest or in motion.

According to these remarks we shall briefly recapitulate the solution of the preceding problem, in order to show at one glance the course of the operations indicated.

[To be continued.]

XLIV. *Processes employed for finishing the Inside of the Palaces of the Native Princes in some Parts of the East Indies**.

THE principal workman employed by colonel Close in repairing the palace in the *Laul Baug*, gave me the following account of the processes used for finishing the inside of the palaces at *Seringapatam*.

At first sight, one would imagine that much gilding is used in the ornaments; but, in truth, not a grain of gold is employed. The workmen use a paper covered with false gilding. This they cut into the shape of flowers, and paste these on the walls or columns. The interstices are filled up with oil colours, which are all of European preparation.— The manner of making this false gilded paper is as follows:

Take any quantity of lead, and beat it with a hammer into leaves, as thin as possible. To twenty-four parts of these leaves add three parts of English glue, dissolved in water, and beat them together with a hammer, till they be

* From Buchanan's *Journey from Madras through the Mysore, Canara, and Malabar.*

thoroughly

soft to the touch, and has yellow ochrey spots in it, apparently proceeding from grains of altered cupreous and ferruginous pyrites. M. Ansaldo informed me, that this pyrites was formerly wrought for the sake of its sulphate of copper, but abandoned on account of its poverty.

LIX. *Essay upon Machines in General. By M.* Carnot, *Member of the French Institute, &c. &c.*

[Continued from p. 221.]

Problem.

XX. *The virtual movement being known of any given system of hard bodies,* (i. e. *that which it would assume if each of the bodies were free,*) *to find the real movement which it should have the following instant.*

Solution. Let us denominate each molecule of the system, - - - - - m

Its virtual given velocity, - - - W

Its real velocity sought, - - - - V

The velocity it loses, in such a manner that W is the result of V and of this velocity, - - U

Let us now imagine that we make the system assume an arbitrary *geometrical movement,* and let the velocity which m will then have be - - - u

The angle formed by the directions of W and V, X

The angle formed by the directions of W and U, Y

The angle formed by the directions of V and U, Z

The angle formed by the directions of W and u, x

The angle formed by the directions of V and u, y

The angle formed by the directions of U and u, z

This being done, we shall have the equation $s\, m\, u$ U cosine $z = 0$ (F); by means of which we shall find in all cases the state of the system, by attributing successively to the indeterminates u different relations and arbitrary directions.

Definitions.

XXI. Let us imagine a system of bodies in movement in any

any given manner: let m be the mass of each of these bodies, and V its velocity; let us now suppose that we make the system assume any geometrical movement, and let u be the velocity which m will then have, (and what I shall call its geometrical velocity,) and let y be the angle comprehended between the directions of V and u; this being done, the quantity $m\,u\,V$ cosine y will be named the momentum of the quantity of movement mV, with respect to the geometrical velocity u; and the sum of all these quantities, namely $s\,m\,u\,V$ cosine y, will be called the momentum of the quantity of movement of the system with respect to the geometrical movement which we have made it assume: thus *the momentum of the quantity of movement of a system of bodies, with respect to any geometrical movement, is the sum of the products of the quantities of movement of the bodies which compose it, multiplied each by the geometrical velocity of this body, estimated in the ratio of this quantity of movement.* In such a manner that by preserving the denominations of the problem, $s\,m\,u\,W$ cosine x is the momentum of the quantity of movement of the system before the shock; $s\,m\,u\,V$ cosine y is the momentum of the quantity of movement of the same system after the shock; and $s\,m\,u\,U$ cosine z is the momentum of the quantity of movement lost in the shock (all these momenta being referred to the same geometrical movement). Thus, from the fundamental equation (F) we may conclude, that *in the shock of hard bodies, whether these bodies be all moveable, or some of them fixed, or, what comes to the same thing, whether the shock be immediate, or made by means of any machine without spring, the momentum of the quantity of movement lost by the general system is equal to zero.*

W being the result of V and U, it is clear that we have W cosine $x =$ cosine $y +$ U cosine z, or $m\,u\,W$ cosine $x = m\,u\,V$ cosine $y + m\,u\,U$ cosine z, or lastly, $s\,m\,u\,W$ cosine $x = s\,m\,u\,V$ cosine $y + s\,m\,u\,U$ cosine z: now we have found $s\,m\,u\,U$ cosine $z = 0$; therefore $s\,m\,u\,W$ cosine $x + s\,m\,u\,V$ cosine y, that is to say, *in respect to any geometrical movement, the momentum of*

U 4 *the*

the quantity of movement of the system, immediately after the shock, is equal to the momentum of the quantity of movement immediately before the shock.

When we decompose the velocity which a body would assume if it were free, into two, one of which is the velocity it actually assumes, and the other the velocity it loses; and reciprocally if we decompose the velocity it loses, into two, one of them being that which it would have taken if it had been free, the other will be the velocity it gains: whence it visibly follows, that what we understand by the velocity gained by a body, and what we understand by its velocity lost, are two quantities equal and directly opposite: this being done, the momentum of the quantity of movement lost by m, with respect to the geometrical velocity u, being, according to the preceding definition, $m\,u$ U cosine z, the momentum of the quantity of movement gained by the same body will be $-\,m\,u$ U cosine z; for there is no difference between these two quantities, except in this, that the angle comprehended between u and the velocity gained is the supplement of that comprehended between u and U; so that one of these angles being sharp, the other will be obtuse, and its cosine equal to the cosine of the other, taken negatively.

Hence it follows, that the momentum of the quantity of movement lost by the general system, with respect to any geometrical movement, (which is null, as we have seen above,) is the same thing as the difference between the momentum of the quantity of movement lost by any part of the bodies which compose it, and the momentum of the quantity of movement gained by the other bodies of the same system: thus this difference is equal to zero, and thus the one of these two quantities is equal to the other; that is to say, *the momentum of the quantity of movement lost in the shock by any part of the bodies of the system, with respect to any geometrical movement, is equal to the momentum of the quantity of movement gained by the other bodies of the same system.*

We may, therefore, from the preceding definition, collect the three propositions contained in the following

THEOREM.

THEOREM.

Momentum is conserved in inelastic collisions.

XXII. *In the shock of hard bodies, whether this shock be immediate, or whether it be made by means of any machine without spring, it is clear that in respect to any geometrical movement,—*

1*st. The momentum of the quantity of movement lost by the whole system is equal to zero.*

2*d. The momentum of the quantity of movement lost by any part of the bodies of the system, is equal to the momentum of the quantity of movement gained by the other part.*

3*d. The momentum of the quantity of real movement of the general system, immediately after the shock, is equal to the momentum of the quantity of movement of the same system, immediately before the shock.*

It is clear, from the preceding definition, that these three propositions are radically the same, and are nothing else than the same fundamental equation (F) expressed in different ways.

We may also remark that these propositions bear a great relation to those we extract from the consideration of the momenta relatively to different axes; but the latter are less general, and are easily inferred from those established in XVII.

There is, therefore, as we see by the third proposition of this theorem, in every percussion or communication of movement, whether immediate, or caused by the intermedium of a machine, a quantity which is not altered by the shock: this quantity is not, as Descartes thought, the sum of the quantities of movement; nor is it the sum of the active forces, because the latter is only preserved in the case where the movement changes by insensible degrees, as we shall see lower down, and it always diminishes when there is percussion, as will be proved in the second corollary. When the system is free, the quantity of movement estimated in any ratio, is in truth the same before and after the percussion; but this preservation does not take place if there are obstacles, any more than that of the momenta of quantity of movements referred to different axes: all these quantities

tities are therefore altered by the shock, or at least are only preserved in some particular cases. But there is another quantity, which neither the various obstacles opposed to the movement, nor the machines which transmit it, nor the intensity of the different percussions can change; it is the momentum of the quantity of movement of the general system, with respect to each of the geometrical movements of which it is susceptible; and this principle contains in itself alone all the laws of equilibrium and of movement in hard bodies: we shall even see in corollary IV, that this law equally extends to other kinds of bodies, whatever be their nature and degree of elasticity.

If the shock destroyed all the movements, we should have $V = 0$: therefore the equation would be reduced to $s\,m\,W\,u$ cosine $x = 0$; which shows us that this case happens; namely, that all the movements are reciprocally destroyed by the shock, in the case where, immediately before this shock, the *momentum of the quantity of movement* of the general system is null, relatively to all the geometrical movements of which it is susceptible.

First Corollary.

Resultant of inelastic collisions is that geometrically possible motion that minimizes sum mv^2, i.e. kinetic energy.

XXIII. *Among all the movements of which any system of hard bodies acting upon each other is susceptible, whether by an immediate shock, or by any machines without spring, that movement which shall really take place the instant afterwards will be the geometrical movement, which is such that the sum of the products of each of the masses, by the square of the velocity which it will lose, is a* MINIMUM, i. e. *less than the sum of the products of each of these bodies, by the velocity it would have lost, if the system had taken any other geometrical movement.*

Here it must be remarked, that, by giving for the minimum the sum of the products of each mass, by the square of its velocity lost, I understand solely that the differential of this sum is null; *i. e.* that its difference from what it would be if the system had a geometrical movement infinitely little different from the first, is equal to zero: thus

thus this sum may be sometimes a *maximum*, or even neither a *maximum* nor a *minimum*; and I have only to establish $d\,s\,m\,U^2 = 0$.

Demonstration.—It is at first evident that the true movement of the system after the shock should be geometrical; for geometrical movements being those which do not alter the action which is exercised among bodies, it is clear that the first in order is the same movement as assumed by the system: it is therefore required to know, which, among all possible geometrical movements, is the one that should take place. Now, supposing that it should take another infinitely little different from that which we are seeking, the velocity of each molecule m would then have been V'; let us decompose V' into two, one of which shall be V, *i. e.* the real velocity, and the other V'': this being done, it is evident that if the bodies had not other velocities than these last V'', the movement would be still geometrical, for V'' is visibly the result of V' and of a velocity equal and directly opposite to V: now, by hypothesis, the molecules taken two by two do not tend, either in virtue of V', or in virtue of $-V$, to approach or recede, since in these two cases the movement is geometrical: thus, by supposing that the molecules m have at once the velocities V' and $-V$, or their result V'', they will neither tend to approach nor to recede; and therefore the movement will then be geometrical: thus, if we call z'' the angle comprehended between the directions of V'' and U, we shall have by means of the fundamental equation (F) $s\,m\,U\,V''$ cosine $z = 0$: on the other side, let us call U' the velocity which m would lose if its effective velocity were V', so that W would be the result of V' and U', it would necessarily follow that U' would be composed of U and of a velocity equal and directly opposite to V''; whence it evidently follows, that $U'-U$ or $d\,U = -V''$ cosine z''; therefore the equation $s\,m\,U\,V''$ cosine $z'' = 0$, found above, becomes $s\,m\,U\,d\,U = 0$ or $d\,s\,m\,U^2 = 0$.

Suppose, for example, two globes A and B striking each other obliquely, I demand their movements after the shock.

Suppose

Suppose that the velocity of A, estimated according to the line of the centres, was before the shock a, and after the shock V; that the velocity of B, also estimated according to the line of the centres, was before the shock b, and after the shock u; that the velocity of A, estimated perpendicularly to the same line, was before the shock a, and after the shock V'; finally, that the velocity of B, also estimated perpendicularly to this line of the centres, was before the shock b', and after the shock u'; this being done, by our proposition, the movement being necessarily geometrical, we must at first have $V=u$; thus the velocity lost by A, according to the line of the centres, will be $a-u$, and that lost by B, in the same direction, will be $b-u$: besides, in the direction perpendicular to the line of the centres, the velocity lost by A will be $a'-V'$, and that lost by B will be $b'-u'$; therefore $\sqrt{(a-u)^2+(a'-V')^2}$ will be the absolute velocity lost by A, and that lost by B will be $\sqrt{(b-u)^2+(b'-u')^2}$: therefore, according to the proposition, we should have $d\left[A(a-u)^2+A(a'-V')^2+B(b-u)^2+B(b'-u')^2\right]=0$, or $A(a-u)\,du+A(a'-V')\,dV'+B(b-u)\,du+B(b'-u')\,du'=0$, an equation which should generally take place, *i. e.* whatever be the values of du, dV', and du': therefore the co-efficient of each of these differentials must be equal to zero; which gives $V'=a'$, $u'=b'$, and $u=\frac{Aa+Bb}{A+B}$. Q. E. D.

It is clear that this proposition contains all the laws of the shock of hard bodies, whether this shock be immediate, or be made by means of any machine, since it assigns the character under which we recognise, among all possible movements, that which should really take place at each instant: this principle has a considerable analogy with that found by M. Maupertuis, and by him called *principe de la moindre action.* See his "Essai de Cosmologie."

Second Corollary.

XXIV. *In the shock of hard bodies, whether some of them are fixed, or all moveable, or (what comes to the same thing) whether*

whether the shock be immediate, or given by means of any machine without spring, the sum of the active forces before the shock is always equal to the sum of the active forces after the shock, plus the sum of the active forces which would take place if the velocity which remains to each moveable body were equal to that which it has lost in the shock.

That is to say, we must prove the following equation $s\,m\,W^2 = s\,m\,V^2 + s\,m\,U^2$. Now it is easily deduced from the fundamental equation (E); for W being the result of V and U, it is clear that W V and U are proportional to the three sides of a certain triangle: thus by trigonometry we have $W^2 = V^2 + U^2 + 2\,V\,U$ cosine z: therefore $s\,m\,W^2 = s\,m\,V^2 + s\,m\,U^2 + 2\,s\,m\,V\,U$ cosine Z: now by the equation (E) we have $s\,m\,V\,U$ cosine $Z = 0$; therefore the preceding equation is reduced to $s\,m\,W^2 = s\,m\,V^2 + s\,m\,U^2$. Q. E. D.

We see, therefore, as has been said (XXI), that by this transformation the analogy of the equation (E) with the preservation of the active forces becomes striking: we may also easily demonstrate the one by the other, as we shall see in XXVI.

The analogy of this same equation with the preservation of the active forces in a system of hard bodies the movement of which changes by insensible degrees, is still more evident, since it then regards a case peculiar from that we have examined; it is in fact visibly the particular case where U is infinitely small, and therefore U^2 is infinitely small of the second order; this reduces the equation to $s\,m\,W^2 = s\,m\,V^2$: but this preservation will be explained more at length in the following corollary.

Third Corollary.

XXV. *When any system of hard bodies changes its movement by insensible degrees; if, for a moment, we call* m *the mass of each of the bodies,* V *its velocity,* p *its vis motrix,* R *the angle comprehended between the directions of* V *and* p, u *the velocity which* m *would have if we made the system take any geometrical movement,* r *the angle formed by* u *and* p, y *the angle formed by* V *and* u, d t *the element of the time; we shall have these two equations:*

$$s\,m V p\,d\,t \text{ cosine } R - s\,m V\,d\,V = 0:$$

$$s\,m\,u\,p\,d\,t \text{ cosine } r - s\,m\,u\,d\,(V \cos y.) = 0.$$

Demonstration.—In the first place, $p\,d\,t$ cos R is visibly the velocity which the vis motrix p would have impressed upon m in the direction of V, if this body had been free: besides, d V is the velocity which it would in reality receive in the same direction; therefore $p\,d\,t$ cosine R $-$ d V is the velocity lost by m in the direction of V, in virtue of the reciprocal action of the bodies: it is therefore this quantity that we must put for U cos. Z in the fundamental equation (E), which becomes by this substitution $s\,m\,V\,p\,d\,t$ cosine R $-$ $s\,m\,V\,d\,V = 0'$, being the first of the two equations which we had to demonstrate.

Secondly, $p\,d\,t$ cosine r is the velocity which the vis motrix p would have impressed upon m in the direction of u, if this body had been free; besides, V cosine y being the velocity of m in the direction of u, d (V cosine y) is the quantity which this velocity estimated in the same direction augments: therefore $p\,d\,t$ cosine r $-$ d (V cosine y) is the velocity lost by m in the direction of u, in virtue of the reciprocal action of the bodies: it is therefore this quantity which we must put for U cosine z in the second equation (F), which becomes by this substitution $s\,m\,u\,p\,d\,t$ cosine r $-$ $s\,m\,u\,d$ (V cosine y) $= 0$, which is the second of the two equations we had to demonstrate.

These equations are therefore nothing else than the fundamental equations (E) and (F) applied to the case where the movement changes by insensible degrees, and therefore they contain all the laws of this movement: we may remark also, that the first of these two equations is only a particular case of the second, for the same reason that the equation (E), whence it is extracted, is contained in that (F) whence the second is extracted. But this first equation $s\,m\,V\,p\,d\,t$ cosine R $-$ $s\,m\,V\,d\,V = 0$ deserves particular attention; because it contains the famous principle of the preservation of active forces in a system of hard bodies the movement of which changes by insensible degrees: thus:

Let

Let us first call $d\ s$ the element of the curve described by the corpuscle m during $d\ t$; this being done, we shall have $V\ d\ t = d\ s$; and therefore the preceding equation assumes this form $s\ m\ p\ d\ s$ cosine $R - s\ m\ V\ d\ V = 0$. Now let us suppose for a moment that the curve described by m is an inflexible line, that m is a moveable grain interwoven with this curve, that it traverses it freely, *i. e.* without being pressed by the re-actions of the other parts of the system, that it experiences at each point of this curve the same vis motrix as that with which it was animated in the first case; and that, finally, in this first case the initial velocity of m is K, while in the second it will be null at the first instant, and V' after an indeterminate time t: this being done, by integrating the preceding equation, in order to have the state of the system at the end of the time t, we shall have for the first case $s'\ s\ m\ p\ d\ s$ cosine $R - s'\ s\ m\ V\ d\ V = 0$, s' designating the sign of integration relative to the duration of the movement, while s is the sign of integration relative to the figure of the system: now, $s'\ s\ m\ V\ d\ V = \frac{s\ m\ V^2}{2}$: therefore the equation may be placed in this form $s'\ s\ m\ p\ d\ s$ cosine $R - s\ m\ V^2 + C = 0$; C being a constant added to complete the integral: in order to determine it, we shall observe that at the first instant we have $V = K$ and $s'\ s\ m\ p\ d\ s$ cosine $R = 0$; therefore $C = \frac{s\ m\ K^2}{2}$; therefore $2\ s'\ s\ m\ p\ d\ s$ cosine $R - s\ m\ V^2$ $s\ m\ K^2 = 0$: by the same reasons we have for the second case $2\ s'\ s\ m\ p\ d\ s$ cosine $R - s\ m\ V'^2 = 0$, without a constant, because we suppose V' as null at the first instant: taking away therefore this equation from the preceding one, reducing and transposing, we have $s\ m\ V^2 = s\ m\ K^2 + s\ m\ V'^2$; that is to say, *in any system of hard bodies the movement of which changes by insensible degrees, the sum of the active forces at the end of any given time is equal to the sum of the initial active forces, plus the sum of the active forces which would take place if each moveable particle had for its velocity that which it would have acquired by freely traversing the curve it had described, and supposing besides that it had*

been

been animated at each point of this curve, with the same vis motrix which it there really experiences, and that its velocity at the first instant had been null.

It is this proposition which we call the principle of the preservation of active forces; and whence we may conclude that,

In a system of hard bodies the movement of which changes by insensible degrees, and which are not animated with any vis motrix, the sum of the active forces is a constant quantity, i. e. *the same for every instant.*

For in this case we have by hypothesis $p = 0$, which gives $V' = 0$, and therefore $s\, m\, V^2 = s\, m\, K^2$; an equation besides which is extracted immediately from that $s\, m\, p\, V\, d\, t$ cosine $R - s\, m\, V\, d\, V = 0$, found in XXIV, which, on account of $p = 0$, is reduced to $s\, m\, V\, d\, V = 0$, the integral of which completed is $\frac{1}{2} s\, m\, V^2 = \frac{1}{2} s\, m\, K^2 = 0$; whence follows the equation $s\, m\, V^2 = s\, m\, K^2$. Q. E. D.

[To be continued.]

LX. *On Chemical Nomenclature. By a Correspondent.*

To Mr. Tilloch.

SIR,

The importance of an accurate and scientific nomenclature being now admitted by every lover of chemistry, I shall make no apology for suggesting what I consider an improvement. The metalline salts are named after a plan which is extremely defective. It proceeds upon the supposion that no more than two oxides of any metal can combine with the same acid. The salt whose base is the first of these oxides is named as if there were no oxide present: thus, the protoxide of iron and sulphuric acid form what is called sulphate of iron. On the other hand, the salt which contains the second of these oxides is known by *oxy* being prefixed, as in the oxy-sulphate of iron.

This mode of nomenclature is objectionable on several accounts.

1st. It is extremely deficient in the extent of application,

as

THE

PHILOSOPHICAL MAGAZINE:

COMPREHENDING

THE VARIOUS BRANCHES OF SCIENCE,

THE LIBERAL AND FINE ARTS,

AGRICULTURE, MANUFACTURES,

AND

COMMERCE.

BY ALEXANDER TILLOCH,

HONORARY MEMBER OF THE ROYAL IRISH ACADEMY, &c. &c. &c.

" Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Monit. Polit.* lib. i. cap. 1.

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The analysis leaving nothing further to be done, the contents of this water may be stated as follows, viz.

	Contents in one Gallon.	*In one Pint.*
	Grains.	Grains.
Muriate of soda - -	219·75	27·46875
Sulphate of magnesia	98·25	12·28125
Sulphate of soda -	80·01	10·00125
Muriate of magnesia	40·00	5·
Muriate of lime - -	36·00	4·5
Carbonate of iron -	7·15	·89375
Sulphate of lime -	85·01	10·62625
	566·17	70·77125
	Cubic inches.	Cubic inches.
Carbonic acid gas -	12·07	1·50875
Oxygen gas - - -	4·03	·50375
Atmospheric air - -	1·21	·15125
	17·31	2·16375

[To be continued.]

V. *Essay upon Machines in General. By M.* Carnot, *Member of the French Institute, &c. &c.*

[Concluded from vol. xxx. p. 320.]

Fourth Corollary.

XXVI. I have proved (XIX) that the indeterminate equation (F) contains all the laws of equilibrium and of movement in hard bodies: I now go further, and I say that this equation agrees equally with bodies which are not so, and consequently this general law extends indiscriminately to all bodies in nature. In fact, when several bodies, which are not hard, act upon each other, in any given manner, if we conceive the movement that each particle would have taken, if it had been free, as decomposed into two, one of which is what it would have really taken, the other will be destroyed; whence it evidently follows, that if the bodies had been hard, and had not had other movements

ments than the latter, there would have been an equilibrium: these destroyed movements are therefore subjected to the same laws, have the same relations to each other, and, lastly, may be determined in the same manner as if the bodies were hard, *i. e.* by the general equation (F). This equation (F) is not confined therefore to hard bodies, it also belongs to all the bodies in nature, and consequently contains all the laws of equilibrium and of movement, not only for the first, but even for all the others, whatever may be their degree of compressibility: but the difference consists in this; that we may, with respect to hard bodies, suppose $u = V$; in such a manner that $s\,m\,V\,U$ cosine $Z = 0$ becomes one of the determinate equations of the problem, whereas this does not take place when the bodies are of a different nature: it is therefore this determinate equation, which is the same with the first fundamental equation (E), it is, I say, this determinate equation which characterizes hard bodies, and consequently it is absolutely necessary to employ it at least implicitly in all questions concerning these bodies; and with respect to any other kind of bodies, we must, besides the determinate equations, which we may obtain by ascribing to u in the indeterminate equation (F) different known values— we must, I say, also extract from it one which is analogous to the equation (E), and which expresses in some measure the nature of these bodies, in the same way as the latter (E) expresses that of hard bodies. But as this inquiry has but a very indirect relation to machines properly so called, we shall at present confine ourselves to examining the case where the degree of elasticity is the same with respect to all bodies, *i. e.* Let us suppose, that in virtue of elasticity the bodies exercise upon each other, pressures n times as great as if the bodies were hard, n being the same for all the bodies of the system; let us next suppose that the pressure and the restitution are made in an indivisible instant, although in strictness that would be impossible. This being done:

The reciprocal pressures F becoming n F, will have among them the same relations as if the bodies were hard; therefore their results m U will not have changed their directions,

rections, but will merely have become n times as great as they would have been if the bodies had been hard: this being done, since W is the result of V and U, we have V cosine Z = W cosine Y − U: thus the equation (E), for which we are seeking one analogous, may be put under this form $s\,m\,W\,U$ cosine $Y - s\,m\,U^2 = 0$. Now, according to what has been said, we must, in order to apply this equation to the case in question, place $\frac{U}{n}$ in place of U, without any change upon Y: therefore in the case we are examining the equation will be $s\,m\,W\,\frac{U}{n}$ cosine $Y - s\,\frac{m\,U^2}{n\,2} = 0$; or by multiplying by n^2, $n\,s\,m\,W\,U$ cosine $Y - s\,m\,U^2 = 0$; or on account of W cosine Y = V cosine Z + U, we shall have $\frac{n}{1-n}\,s\,m\,V\,U$ cosine $Z = s\,m\,U^2$: thus this equation will be, with respect to the bodies in question, what the equation (E) is with respect to hard bodies; and even the latter is the particular case where we have $n = 1$, as is evident.

When $n = 2$, it is the case of bodies perfectly elastic, and the equation becomes $2\,s\,m\,V\,U$ cosine $Z + s\,m\,U^2 = 0$; but this equation relative to bodies perfectly elastic may be expressed in a known and more simple manner, as follows: Since W is the result of V and U, we have by trigonometry $W^2 = V^2 + U^2 + 2\,V\,U$ cosine Z; and therefore $s\,m\,W^2 = s\,m\,V^2 + s\,m\,U^2 + 2\,s\,m\,V\,U$ cosine Z. Adding to this equation that found above, and reducing, we have $s\,m\,W^2 = s\,m\,V^2$, which is precisely the principle of the preservation of active forces, *i. e.* this preservation is, with respect to perfectly elastic bodies, what the equation (E) is with respect to hard bodies, as we undertook to prove.

First Remark.

XXVII. I shall not dwell on the particular consequences which I might draw from the solution of the preceding problem; but shall merely remark, that the velocities W, V, U, being always in proportion to the three sides of a triangle, trigonometry may furnish the means of giving a great number of different forms to the fundamental equations (E) and (F); and I shall content myself with indicating one of them, which is remarkable on account of the method contrived by geometricians, of referring movements to three plans perpendicular to each other; which gives a great deal of elegance and simplicity to the solutions.

Let us imagine, therefore, at pleasure, three axes perpendicular to each other; and let us conceive that the velocities W, V, U and u, are each of them decomposed into three others parallel to these axes. This being done, let us call

Those which answer to W,	W' W'' W'''.
Those which answer to V,	V' V'' V'''.
Those which answer to U,	U' U'' U'''.
Those which answer to u,	u', u'', u'''.

Now if we pay a little attention, we shall easily see that the first fundamental equation (E) may be placed under this form, $s\,m\,V'\,U' + s\,m\,V''\,U'' + s\,m\,V'''\,U''' = 0$; and the second (F) under the latter $s\,m\,u'\,U' + s\,m\,u''\,U'' + s\,m\,u'''\,U''' = 0$; because in general every quantity which is the product of two velocities A and B, by the cosines of the angle comprehended between them, is equal to the sum of three other products $A'\,B' + A''\,B'' + A'''\,B'''$; $A'\,A''\,A'''$, being the estimated velocity A of these three axes, and $B'\,B''\,B'''$ being the estimated velocity B in the ratio of these same axes: *i. e.* A' being the velocity A, and B' the velocity B, estimated parallel to the first of these axes; A'' and B'' the same velocities A and B' estimated parallel to the second axis, A''' and B''' the same velocities estimated parallel to the third axis: this is easily proved by the elements of geometry.

In the case of equilibrium, the first of these transformed equations is reduced to $0 = 0$; and the second, because in this case $W = U$, becomes $s\,m\,u'\,W' + s\,m\,u''\,W'' + s\,m\,u'''\,W''' = 0$; which expresses all the conditions of the equilibrium.

When the movement changes by insensible degrees, we have found (XXV.) that the fundamental equations become $s\,m\,V\,p\,t$ cosine $R - s\,m\,V\,d\,V = 0$, and $s\,m\,u\,p\,d\,t$ cosine

$r - s\,m\,u\,d$

$r - s\,m\,u\,d$ (V cosine y) $= 0$; therefore by decomposing p into three other forces parallel to the three axes, if these component forces are designated by p', p'', p''', the preceding equations will become, the first, $s\,m\,V'\,p'\,d\,t + s\,m\,V''\,p''\,d\,t + s\,m\,V'''\cdot p'''\,d\,t = s\,m\,V'\,d\,V' + s\,m\,V''\,d\,V'' + s\,m\,V'''\,d\,V'''$; and the second, $s\,m\,u'\,p'\,d\,t + s\,m\,u''\,p''\,d\,t + s\,m\,u'''\,p'''\,d\,t = s\,m\,u'\,d\,V' + s\,m\,u''\,d\,V'' + s\,m\,u'''\,d\,V'''$: finally, in the case of equilibrium, the first will vanish, and the second will be reduced to $s\,m\,u'\,p' + s\,m\,u''\,p'' + s\,m\,u'''\,p''' = 0$.

Second Remark.

XXVIII. Hitherto I have regarded wires, rods, levers, &c. as bodies making of themselves part of the system. And this hypothesis entirely conforms to nature; but one thing indispensably necessary to observe is, that, strictly speaking, there is probably no absolutely fixed point in the universe, no obstacle absolutely immoveable; the fulcrum of a lever is not so, because it is supported upon the earth, which is not fixed itself; but the mass of which is almost infinitely great in comparison of those the action and reaction of which upon each other we generally consider in machines: in order to move the hypomochlion of a lever, we must also put in motion the globe of the earth; and it is so in fact, however feeble be the powers which act upon the machine*: the quantity of movement which they produce upon it, is equal to the resistance of the hypomochlion; but this finite quantity of movement distributing itself into a mass almost infinitely great, there results to this mass a velocity almost infinitely small, and this is the reason why this movement is not sensible, and may be neglected in practice.

Hence it follows, that what we call immoveable obstacles in mechanics, are nothing else than bodies the mass of which is so considerable, and consequently the velocity so small, that their movement cannot be observed. We shall therefore approach nearer nature, by considering the obstacles,

* M. Carnot does not exhibit here his usual accuracy. If the power applied to the lever belonged to any other system than that of the earth, the earth would be moved; but in the case of the fact here assumed, it is not moved, even in an infinitely small degree.—*Edit.*

or fixed points, as moveable bodies, as well as all the others ; but of a mass infinitely large, or, what comes to the same thing, as bodies of an infinite density, and which do not diffef from all the other bodies of the system except in this point. Hence a considerable advantage will result, as we shall be able to make the system into which these bodies enter, take any given geometrical movements; for the instant we suppose these obstacles moveable like all other bodies, they will become susceptible of assuming any movements, and the general system must be regarded as an assemblage of bodies perfectly moveable: consequently, the quantities of movements absorbed by the obstacles may be estimated as with respect to all the other parts of the system ; in such a manner, that if we call R the resistance of any given fixed point, this quantity R will be in the equation (F), with respect to the point in question, what m U is with respect to the body m: we shall therefore find by this equation, this same quantity R like all the other forces m U, which could not be the case by considering the obstacles as absolutely immoveable, without having recourse to some new mechanical principle, which we must have made concur with the general equation (F), in order to attain the complete solution of each particular problem. Thus this method of considering the fixed points is not only the most conformable to nature, as we have said before, but also the simplest and the easiest.

As to the wires, rods, or any other portions of the system, the masses of which may be supposed to be infinitely small, we may neglect, *i. e.* suppose each of their molecules m equal to zero, or, what comes to the same thing, regard their density as infinitely small, or as nothing : our equation (F) will therefore become independent of these quantities, *i. e.* the same as if we had abstracted these masses from the bodies; and it is thus that we shall easily find the mathematical theory of each machine, *i. e.* by making the abstractions spoken of (VIII.)

XXIX. From this remark it results, that although there is only a single kind of bodies in nature, we distinguish them however, for the facility of calculations, into three different classes, which are, 1st. Those which we consider as what they really are, and as nature presents them to us, *i. e.* which are of a finite density. 2d. Those to which we ascribe a density infinitely great, and which, for this reason, must be regarded as sensibly fixed and immoveable. 3d. Those to which we ascribe a density infinitely small, or null, and which, consequently, by their inertness, oppose no resistance to their change of state. In practice we generally regard as such, wires, rods, and generally all bodies which do not influence sensibly, by their proper mass, the changes which happen in the system; but which are solely regarded as means of communication between the different agents which compose it.

Third Remark.

XXX. After having treated of equilibrium and of movement in general, as much as my principal object permitted, I shall pass to what regards more particularly what we commonly understand by machines; for although the theory of every kind of equilibrium and movement always enters into the preceding principles, since there are only, according to the first law, bodies which can destroy or modify the movement of other bodies; nevertheless there are cases where we make abstraction of the mass of these bodies, merely for the purpose of considering the effort they make: for example, when a man draws a body by a wire, or pushes it by a rod, we do not introduce into the calculation the mass of this man, nor even the effort of which he is capable, but solely that which he exerts upon the point to which he applies it; *i. e.* the tension of the wire, if it is by drawing that it acts, or the pressure, if it be by pushing; and without considering whether it be a man, an animal, a weight, a spring, or a resistance occasioned by any obstacle, or by the *vis inertiæ* of a moveable body*, a friction, an impulse caused by

* Any body which we force to change its state of repose, or of movement, resists (XI.) the agent which produces the change; and it is this resistance which we call *vis inertiæ*. In order to find the value of this force, we must decompose the actual movement of the body into two, one of which is that which it will have the instant afterwards; for the other will be evidently that which must be destroyed, in order to force the body to change its state; *i. e.* the

by a current of air, or water, &c. We give in general the name of power to the effort exerted by the agent, *i. e.* to that pressure or tension by which it acts upon the body to which it is applied; and we compare these different efforts without regarding the agents which produce them, because the nature of the agents cannot change the forces which they are obliged to exercise in order to fulfil the different objects for which machines are destined: the machine itself, *i. e.* the system of fixed points, obstacles, rods, levers, and other intermediate bodies, which serve to transmit these different efforts from one agent to another; the machine, itself, I say, is considered as a body stripped of its inertness: its proper mass (when it is necessary to have regard to it, whether on account of the movement which it absorbs, or on account of its gravity or of other motive forces with which it may be animated,) is regarded as a foreign power applied to the system; in a word, a machine properly so called, is an assemblage of immaterial obstacles, and of moveable particles incapable of reaction, or deprived of inertness, *i. e.* (XXIX.) a system of bodies the densities of which are infinite or nothing. To this system we imagine that different external agents, in the number of which we comprehend the mass of the machine, are applied, and transmit their reciprocal action by the intermedium of this machine. It is the pressure or other effort exercised by each agent upon this intermediate body, which we call force or power; and the relation which exists between these different forces, forms the subject of the inquiry, which has for its object the theory of machines properly so called. Now, it is in this point of view that we proceed to treat of equilibrium and of movement; but a force taken in this sense is not the less a quantity of movement lost by the agent which exercises it, whatever this agent may be in other respects, whether it acts upon the machine by drawing it by a cord or by pushing it by a rod; the tension of this cord, or the pressure of this

i. e. the resistance which it opposes to this change, or its *vis inertiæ;* whence it is easy to conclude, that the *vis inertiæ of any body is the result of its actual movement, and of a movement equal and directly opposed to that which it should have the instant afterwards.*

rod,

rod, expresses both the effort which it exercises upon the machine, and the quantity of movement which it loses itself by the reaction it undergoes: if, therefore, we call F that force, this quantity F will be the same thing with that which is expressed by m U in our equations*. Thus, if we call Z the angle comprehended between this force F, and the velocity u, which the point would have where we suppose it applied, if we make the system assume any geometrical movement, the general equation (F) will become s F u cosine Z = 0 (AA). It is therefore under this form that we shall immediately employ this equation, by means of which we may apply whatever we can mention, to any imaginable kind of force; and the principles exposed in this first part will serve us to develop the general properties of machines properly so called, which are the object of the ensuing division of the present work.

[To be continued.]

VI. *On the Stratification of Matlock in Derbyshire, pointing out a Mistake of the late Mr.* John Whitehurst, *relative thereto; and on the Transmutation of Lime to Silex. By Mr.* John Farey, *Mineralogical Surveyor.*

To Mr. Tilloch.

SIR,

The late Mr. *John Whitehurst*, in his "Inquiry into the original State and Formation of the Earth," has given sections of the strata to be found in various parts of Derbyshire,

* It is evident that the quantity of movement lost m U, is the result of the movement which the body m would have had the instant afterwards, if it had been free, and of the movement equal, and directly opposed to that which it will really assume: now the first of these two movements is itself the result of the actual movement of m, and of its absolute motive force; therefore m U is the result of three forces, which are: its absolute motive force, its actual quantiy of movement, and the quantity of movement equal and directly opposed to that which it should have the instant after: but according to the preceding note, these two last quantities of movement have for their result the *vis inertiæ*: therefore m U or F is the result of the motive force of m and of its *vis inertiæ*; i. e *the force exercised by any given body, at each instant, is the result of its absolute motive force and of its* vis inertiæ.

which,

consequence to the pleasure of the gentleman, as well as to the profit of the gardener.

Old as I am, I certainly intend this year to commence experiments on the myrtle and the laurel: I trust, therefore, it will not be thought presumptuous in me to invite those of my brethren of this most useful Society, who are younger than I am, and who of course will see the effect of more generations than I shall do, to take measures for bringing to the test of experiment the theory I have ventured to bring forward, I hope not without some prospect of success.

The settlement lately made at New Holland gives a large scope to these experiments: many plants have been brought from thence which endure our climate with very little protection; and some of these arrive at puberty at an early period; we have already three from the south point of Van Diemen's Island, where the climate cannot be wholly without frost; mimosa verticillata, eucalyptus hirsuta, and obliqua. The first of these appears to have produced flowers within eight years of its first introduction; but as a settlement is now made very near the spot where the seeds of these shrubs were collected, we may reasonably hope to receive further supplies, and, among them, the Winterana aromatica, an inhabitant of the inhospitable shore of Terra del Fuego, which Mr. Brown has discovered on the south part of Van Diemen's Island also.

XXVII. *Essay upon Machines in General. By M.* Carnot, *Member of the French Institute, &c. &c.*

[Continued from p. 36.]

Part II. [*Of Machines properly so called**.]

Definitions.

XXXI. Among the forces applied to a machine in motion, some are of such a nature that each of them forms an acute angle with the velocity of the point at which it is applied,

* Vide p. 36 of the present volume.

applied, while others form obtuse angles with their points. This being granted, I shall call the former *moving* or *soliciting forces;* and the others *resisting forces*: for instance, if a person raises a weight by means of a lever, a pulley, a screw, &c., it is clear that the weight and the velocity of the weight necessarily form by their concurrence an obtuse angle; otherwise it is evident that the weight would descend in place of ascending; but the *vis motrix* and its velocity form an acute angle: thus, according to our definition, the weight will be the *resisting force*, and the effort of the person will be the *soliciting force:* it is evident, in short, that the latter tends to favour the actual movement of the machine, while the other opposes it.

We shall observe that the soliciting forces may be directed in the same ratio with their velocities, since then the angle formed by their concurrence is null, and consequently acute, and the resisting forces may act in the direction precisely opposite to that of their velocities, since then the angle formed by their concurrence is 180°, and consequently obtuse.

It is also to be remarked, that any force which is soliciting might become resisting if the movement should change; that any force which is resisting at a certain instant, may become soliciting at another instant; and lastly, in order to judge of it at each instant, we must consider the angle which it makes with the velocity of the point where we suppose it applied: if this angle be acute, the force will be soliciting; and if it be obtuse, it will be resisting, until the angle in question changes. We see from this, that if we make any system of power assume a geometrical movement, each of them will be soliciting or resisting in respect of this geometrical movement, accordingly as the angle formed by this force and by its geometrical velocity shall be acute or obtuse.

XXXII. If a force P be moved with the velocity u, and the angle formed by the concurrence of u and P be z, the quantity P cosine $z\, u\, d\, t$, in which $d\, t$ expresses the element of time, will be named *momentum of activity*, consumed by the force P during $d\, t$; i. e. the momentum of activity consumed

Moment of activity = work done by force P on body moving at velocity u in time dt.

consumed by a force P, in a time infinitely short, is the produce of this force estimated in the ratio of its velocity, by the path described in this infinitely short time by the point to which it is applied.

Carnot's surrogate for later notion of energy.

I shall call the *momentum of activity*, consumed by this force, in a given time, the sum of the *momenta of activity* consumed by it at each instant, in such a manner that s P cosine $z u d t$ is the *momentum of activity*, consumed by it in an indeterminate time:. for instance, if P be a weight, the *momentum of activity* consumed in an indeterminate time t will be $\mathrm{P} s u d t$ cosine z; let us suppose, therefore, that after the time t, the weight P has descended from the quantity H, we shall clearly have $d\mathrm{H} = u d t$ cosine z; therefore the *momentum of activity* consumed during $d t$ will be $\mathrm{P} s d \mathrm{H} = \mathrm{PH}$.

XXXIII. When we are speaking of a system of forces applied to a machine in movement, I shall call *momentum of activity*, consumed by all the forces of the system, the sum of the momenta of activity consumed at the same time by each of the forces which compose it: thus, the *momentum of activity* consumed by the soliciting forces, will be the sum of the momenta of activity consumed at the same time by each of them: and the momentum of activity consumed by the resisting forces will be the sum of the momenta of activity consumed by each of these forces: and as each resisting force makes an obtuse angle with the direction of its velocity, the cosine of this angle is negative; the momentum of activity consumed by the resisting forces is therefore also a negative quantity; and therefore the *momentum of activity* consumed by all the forces of the system, is the same thing as the difference between the *momentum of activity* consumed by the soliciting forces, and the *momentum of activity* consumed at the same time by the resisting forces considered as a positive quantity.

A force estimated in a sense directly opposite to that of its velocity, and multiplied by the path described in an infinitely short time by the point where it is applied, will be called *the momentum of activity produced* by this force in this

this infinitely short time: in such a manner *that the momentum of activity consumed*, and *the momentum of activity produced*, are two equal quantities, but of contrary signs; and there is a difference between them analogous to that which we find (XXI) between the *momenta of the quantity of movement gained and lost*, by a body, in respect of any geometrical movement.

I shall also give the name of *momentum of activity exercised by a force*, to what I have called its momentum of activity *consumed*, if it be soliciting, and to what I have called its *momentum of activity produced*, if it be resisting: thus, the *momentum of activity exercised* by any given force in an infinitely short time is in general the produce of this force, by the path which it describes in this infinitely short time, and by the cosine of the smallest of the two angles formed by the directions of this force and of its velocity; whence it clearly follows, that this *momentum of activity exercised* is always a positive quantity.

We shall make, with respect to the quantities which we call *momenta of activity produced* and *momenta of activity exercised*, the same remarks with those we have made above, upon the subject of *momentum of activity consumed* by a force or system of powers in a given time.

These definitions being admitted, I shall proceed to the general principle of equilibrium and of movement, in machines properly so called; and the inquiry into which has been the principal object of this essay.

FUNDAMENTAL THEOREM.

General Principle of Equilibrium and of Movement in Machines.

XXXIV. *Whatever is the state of repose or of movement in which any given system of forces applied to a machine exists, if we make it all at once assume any given geometrical movement, without changing these forces in any respect, the sum of the products of each of them, by the velocity which the point at which it is applied will have in the first instant, estimated in the direction of this force, will be equal to zero.*

That

Sum over components of a machine. Force dot product velocity = 0

Force . velocity = rate at which force does work. This is statement of the conservation of energy.

THE CONCEPT OF ENERGY IS NOT PRESENT IN CARNOT'S WRITINGS!

I use energy talk here only for modern convenience.

That is to say, by calling F each of these forces *, u the velocity which the point where it is applied will have at the first instant, if we make the machine assume a geometrical movement, and z the angle comprehended between the directions of F and of u, it must prove that we shall have for the whole system $s\,F\,u$ cosine $z = 0$. Now this equation is precisely the equation (AA) found (XXX), which is nothing else in the end but the same fundamental equation (F), presented under another form.

It is easy to perceive that this general principle is, properly speaking, nothing else than that of Descartes, to which a sufficient extension is to be given, in order that it may contain not only all the conditions of the equilibrium between

* It will not perhaps be useless to anticipate an objection which might occur to those who have not paid sufficient attention to what has been said (XXX) upon the true meaning we ought to attach to the word *force*: Let us imagine, for instance, they will say, a wheel and axle to the cylinder of which weights are suspended by means of cords; if there be equilibrium, or if the movement be uniform, the weight attached to the wheel will be to that of the cylinder as the radius of the cylinder is to the radius of the wheel; which is conformable to the proposition. But the case is not the same when the machine assumes an accelerated or a retarded movement: it seems, therefore, that here the forces are not in reciprocal ratio of their velocities estimated in the direction of these forces, as would follow from the proposition. The answer to that is, that in the case where this movement is not uniform, the weights in question are not the only forces exercised in the system; for the movement of each body changing continually, it also opposes at each instant, by its *vis inertiæ*, a resistance to this change of state: we must, therefore, keep an account of this resistance. We have already said (XXX. see the note,) how this force should be estimated, and we shall see further on (XLI), how we should make it enter into the calculation. In the mean time it is sufficient to remark, that the forces applied to the machine in question are not the weights, but the quantities of movement lost by these weights (XXX), which should be estimated by the tensions of the cords to which they are suspended: now whether the machine be at rest or in motion, whether this motion be uniform or not, the tension of the cord attached to the wheel is to that of the cord attached to the cylinder, as the radius of the cylinder is to the radius of the wheel, *i. e.* these tensions are always in reciprocal ratio of the velocities of the weights they support: this agrees with the proposition. But these tensions are not equal to the weights; they are (XXX. see the note) the results of these weights and of their *vis inertiæ*, which are themselves (XXX. see the note) the results of the actual movements of these bodies, and of the movements equal and directly opposed to those which they will really assume the instant afterwards.

two forces, but also all those of equilibrium and of movement, in a system composed of any number of powers: thus the first consequence of this theorem will be the principle of Descartes, rendered complete by the conditions which we have seen were wanting in it (V).

First Corollary.

General Principle of Equilibrium between two Powers.

XXXV. *When any two agents applied to a machine form a mutual equilibrium; if we make this machine assume any arbitrary geometrical movement: 1st, The forces exercised by the agents will be in a reciprocal ratio to their velocities estimated in the direction of these forces: 2d, One of these powers will make an acute angle with the direction of its velocity, and the other an obtuse angle with its velocity.*

For if the forces exercised by the agents are named F and F′; their velocities u and u', the angles formed by these powers and their velocities z and z', we shall have by the preceding theorem, $F u \text{ cosine } z + F' u' \text{ cosine } z' = 0$: therefore $F : F' :: - u' \text{ cosine } z' ; u \text{ cosine } z$, which is the proportion announced by the first part of this corollary, and by which we see at the same time that the relation of cosine z to cosine z' is negative; whence it follows that one of these angles is necessarily acute, and the other obtuse.

Second CololLary.

General Principle of Equilibrium in Weighing Machines.

XXXVI. *When several weights applied to any given machine mutually form an equilibrium, if we make this machine assume any geometrical movement, the velocity of the centre of gravity of the system, estimated in the vertical direction, will be null at the first instant.*

For if we call M the total mass of the system, m that of each of the bodies which compose it, u the absolute velocity of m, V′ the velocity of the centre of gravity estimated in the vertical ratio, g the gravity, z the angle formed by u and by the direction of the weight, we shall have, according to the theorem, $s\, m\, g\, u \text{ cosine } z = 0$; but by the geometrical properties of the centre of gravity we have $s\, m\, u\, d\, t$ cosine

sine $z = \mathrm{M\,V}\,d\,t$, or $s\,m\,g\,u$ cosine $z = \mathrm{M\,V}\,g$; therefore, since the first member of this equation is equal to zero, the second is so also: therefore $\mathrm{V} = 0$. Q. E. D.

In order to have all the conditions of the equilibrium in a weighing machine, it is only necessary to make the machine successively assume different geometrical movements, and to equal in each of these cases the vertical velocity of the centre of gravity at zero.

Third Corollary.

General Principle of Equilibrium between two Weights.

XXXVII. *When two weights form a mutual equilibrium, if we make the machine assume any geometrical movement: 1st, The velocities of these bodies, estimated in the vertical ratio, will be in a reciprocal ratio to their weights: 2d, One of these bodies will necessarily ascend, while the other will descend.*

This proposition is a manifest consequence of the preceding corollary, and is still more evidently deduced from the first corollary.

We may remark by the way, how essential it is for the precision of all these propositions, that the movements impressed upon the machine should be geometrical, and not simply possible; for the slightest attention will show by some particular example, that without this condition all these propositions would be absurd.

Remark.

XXXVIII. We generally take the principle of equilibrium in weighing machines when the centre of gravity of the system is at the lowest possible point; but we know that this principle is not generally true; for besides that this point would be in certain cases at the highest point, there is an infinity of others where it is neither at the highest nor at the lowest point: for instance, if the whole system be reduced to a weighing body, and this moveable article be placed upon a curve which has a point of inflexion, the tangent of which is horizontal, it will remain visibly in equilibrium, if we place it upon this point of inflexion, which nevertheless

nevertheless is not the lowest weight, nor the highest point possible.

We may also take for the principle of equilibrium in a weighing machine the proposition which we have already given (II), and which we shall repeat, in order to give a rigorous demonstration of it.

In order to ascertain that several weights applied to any given machine should mutually form an equilibrium, it is sufficient to prove, that if we abandon this machine to itself, the centre of gravity of the system will not descend.

In order to prove it, let us name M the total mass of the system, m that of each of the weights which compose it, g the gravity; and suppose that if the machine did not remain in equilibrium, as I assert that it should, the velocity of m after the time t would be V, the height from which the centre of gravity would have descended at the end of the same time H, and that from which the body would have descended $m\,h$; we shall then have (XXIV) $s\,m\,g\,d\,h - s\,m\,V\,d\,V = 0$: therefore by integrating $M\,g\,H = \frac{1}{2}\,s\,m\,V^2$; but by hypothesis $H = 0$, therefore $s\,m\,V^2 = 0$; besides, V^2 is necessarily positive, as is evident: therefore the equation $s\,m\,V^2 = 0$ cannot take place, unless we have $V = 0$, *i. e.* unless there be equilibrium. Q. E. D.

Hence it follows, as we have said (III), that there is necessarily equilibrium in a system of weights, the centre of gravity of which is at the lowest possible point; but we have seen (XXXVIII) that the inverse is not always true, *i. e.* that every time there is equilibrium in a system of weight, it does not always follow that the centre of gravity is at the lowest point possible.

Fourth Corollary.

Particular Laws of Equilibrium in Machines.

XXXIX. *If there be equilibrium between several powers applied to a machine, and having decomposed all the forces of the system, as well those which are applied to the machine as those which are exercised by the obstacles or fixed points which form part of it; if we decompose them, I say,* each

each into three others parallel to any three axes perpendicular to each other :

1st. The sum of the component forces, which are parallel to one and the same axis, and conspiring towards one and the same side, is equal to the sum of those which, being parallel to this same axis, conspire towards the opposite side:

2d. The sum of the momenta of the component forces which tend to turn around one and the same axis, and which conspire in one and the same ratio, is equal to the sum of the momenta of those which tend to turn around the same axis, but in a contrary direction.

In order to demonstrate this proposition, let us begin by imagining, that in place of each of the forces exercised by the resistance of obstacles, we substitute an active force equal to this resistance, and directed in the same ratio: this change does not alter the state of equilibrium, and makes of the machine a system of powers perfectly free, *i. e.* freed from every obstacle. This being granted, if we make this system assume any geometrical movement, we shall have by the fundamental theorem $s \mathrm{F} u$ cosine $z = 0$, by calling F each of these forces, u its velocity, and z the angle comprehended between F and u: thus,

1st. If we suppose that u is the same with respect to all the points of the system, and parallel to any one of the axes, the movement will be geometrical, and the equation, on account of u constant, will be reduced to $s \mathrm{F}$ cosine $z = 0$: *i. e.* the sum of the forces of the system estimated in the ratio of the *velocity* u, impressed parallel to this axis, will be null; which evidently reverts to the first part of the proposition.

2d. If we make the whole system turn round any one of the axes, without changing in any respect the respective position of the parts which compose it, this movement will still be geometrical; u will be proportional to the distance of each power from the axis: and therefore might be expressed by A R, R expressing this distance, and A a constant: thus the equation will be reduced to $s \mathrm{F} \mathrm{R}$ cosine $z=0$; which, as may easily be seen, reverts to the second part of the proposition.

[To be continued.]

XXVIII. *Re-*

XL. *Essay upon Machines in General. By M.* Carnot, *Member of the French Institute, &c. &c.*

THE CONCEPT OF ENERGY IS NOT PRESENT IN CARNOT'S WRITINGS!

I use energy talk here only for modern convenience.

[Continued from p. 144.]

Fifth Corollary.

Particular Law concerning Machines, the Movement of which changes by insensible Degrees.

XL. *In a machine, the movement of which changes by insensible degrees,* the momentum of activity consumed *in a given time by the soliciting forces, is equal* to the momentum of activity exercised *at the same time by the resisting forces.*

Conservation of energy.

That is to say (XXXIII) that the *momentum of activity consumed* by all the forces of the system, during the time given, is equal to zero: this will be clear (XXXII) if we prove that the *momentum of activity* consumed at each instant by these forces is null: now F expressing each of these forces, V its velocity, Z the angle comprehended between F and V, and $d\,t$ the element of time, *the momentum of activity consumed* by all the forces of the system during $d\,t$, (XXXIII) s F V cosine Z $d\,t$; we must therefore prove that we have s F V cosine Z $d\,t = 0$; or s F V cosine $z = 0$: now this is clear by the fundamental theorem: ergo &c.

The particular law here in question is certainly the most important of the whole theory of the movement of machines properly so called: we shall give some peculiar applications when we enter upon the detail of the subject, in the scholium which will succeed to the following corollary, and which will conclude this essay.

XLI. Let us suppose, therefore, for instance, that the powers applied to the machine are weights: let us call m the mass of each of these bodies, m the total mass of the system, g the gravity, V the actual velocity of the body m, K its initial velocity, t the time which has gone past since the commencement of the movement, H the height from which the centre of gravity of the system has descended during

during the time t, and lastly, W the velocity due to the height H.

This being done, we must consider that there are two sorts of forces applied to the machine, viz. those which proceed from the gravity of the bodies, and those which proceed from their *vis inertiæ*, or from the resistance which they oppose to their change of state (note to XXX): now (XXXII) the momentum of activity consumed during the time t by the first of these forces, is, with respect to the whole system, $M\,g\,H$, or $\frac{1}{2}\,M\,W^2$. Let us now see what is the momentum of activity consumed by the *vis inertiæ*: the velocity of m being V, and becoming the instant afterwards $V + d\,V$, it is clear (note to XXX) that its *vis inertiæ* estimated in the direction of V, is $m\,d\,V$, or rather $m\,\frac{d\,V}{d\,t}$; therefore (XXX) the momentum of activity, exercised by this force during $d\,t$, is $m\,\frac{d\,V}{d\,t}V\,d\,t$, or $m\,V\,d\,V$: therefore the momentum of activity, consumed by this *vis inertiæ* during the time t, is $s\,m\,V\,d\,V$, or, by integrating and completing the integral, $\frac{1}{2}\,m\,V^2 - \frac{1}{2}\,m\,K^2$: therefore the momentum of activity, consumed at the same time by the *vis inertiæ* of all the bodies of the system, will be $\frac{1}{2}\,s\,m\,V^2 - \frac{1}{2}\,s\,m\,K^2$: now this *vis inertiæ* is a resisting force, since it is by it that bodies resist their change of state: and the weight is here a soliciting force, since the centre of gravity is supposed to descend: thus, by the proposition of this corollary, we should have $M\,W^2 = s\,m\,V^2 - s\,m\,K^2$, or $s\,m\,V^2 = s\,m\,K^2 + M\,W^2$; i. e.

Body of mass M falls through height H releases energy MgH that manifests as kinetic energy (1/2)MW^2

In a machine with weights, the movement of which changes by insensible degrees, the sum of the active forces of the system is, after any given time, equal to the sum of the initial active forces, plus the sum of active force which would take place if all the bodies of the system were animated with a common velocity, equal to that which is owing to the height from which the centre of gravity of the system has descended.

XLII. If the movement of the machine be uniform, we shall continually have $V = K$, and therefore $W^2 = 0$, or $H = 0$: this teaches us that

In

In a weight machine, the movement of which is uniform, the centre of gravity of the system remains constantly at the same height.

XLIII. Since $\frac{1}{2}$ M W² or M g H is (XXXII) the momentum of activity produced by a weight M g, which we make to ascend to the height H, it follows evidently that

Whatever method we take to raise a certain weight to a given height, the forces employed to produce this effect consume a momentum of activity equal to the produce of this weight, by the height to which we should raise it.

XLIV. In the same manner since (XLI) the momentum of activity produced in a given time by the *vis inertiæ* of any body is equal to the half of the quantity by which its active force augments during this time, we may conclude also, that

In order to make any given movement arise by insensible degrees in a system of bodies, or to change that which has arisen, it must follow that the powers destined to this effect do consume a momentum of activity equal to the half of the quantity by which the sum of the active forces of the system will have been augmented by this change.

XLV. It follows evidently from these two last propositions, that in order to elevate a weight M g to a height H, and make it assume at the same time a velocity V, it must happen, supposing this body in repose at the first instant, that the forces employed to produce this effect consume of themselves a momentum of activity equal to M g H + $\frac{1}{2}$ M V².

XLVI. We have supposed in all that has been said, as the title of this corollary announces, that the movement changes by insensible degrees; but if, when proceeding, any sudden shock or change happens in the system, what we have mentioned would not take place. Let us suppose, for instance, that at the moment of this shock the centre of gravity of the system has descended from the height h; that at this same instant the sum of the active forces is X immediately before the shock, and Y immediately after the shock: let us call Q the momentum of activity, which the

the moving forces will have to consume during the whole time of the movement, and g that which they will have to consume from the commencement to the epoch of the percussion: let us suppose finally, for the sake of more simplicity, that the system is at rest at the first instant, and at the last, it is clear (XLV) that we shall have $q = \text{M}\,g\,h + \frac{1}{2}\text{X}$; and that, by the same ratio, the momentum of activity to consume by the forces moving after the shock, *i. e.* $\text{Q} - q$, will be $\text{M}\,g\,(\text{H} - h) - \frac{1}{2}\text{Y}$; therefore $\text{Q} = \text{M}\,g\,\text{H} + \frac{1}{2}\text{X} - \frac{1}{2}\text{Y}$: now (XXIII), it is clear that $\text{X} > \text{Y}$: thus the momentum of activity to consume in order to raise in this case M to the height H, is necessarily greater than if there had been no shock, since in this case we should have simply had $\text{Q} = \text{M}\,g\,\text{H}$ (XLIII).

Hence it follows, that without consuming a greater momentum of activity, the moving forces may, by avoiding all shock, raise the same weight to a greater height H, for then we shall have (XLV) $\text{Q} = \text{M}\,g\,\text{H}$, or $\text{H} = \frac{\text{Q}}{\text{M}\,g}$, while in the present case we have $\text{H} = \frac{\text{Q} - \frac{1}{2}(\text{X} - \text{Y})}{\text{M}\,g}$: whence we see, that X being greater than Y, we must necessarily have also $\text{H}' > \text{H}$.

Sixth Corollary.

Of Hydraulic Machines.

XLVII. We may regard a fluid as an assemblage of an infinity of solid corpuscles detached from each other; we may therefore apply to hydraulic machines all that we have said of other machines: thus, for example, from the first corollary (XXXV) we may conclude, that if a fluid mass without gravity, be enclosed completely in a vessel, and, that, having made two equal apertures in this vessel, we apply pistons to it; the forces which will act upon the fluid mass on pushing these pistons must be equal, if they mutually form an equilibrium; *i. e.* that in a fluid mass the pressure spreads equally in every direction: this is the fundamental principle of the equilibrium of fluids, which we generally regard as a truth purely experimental. We shall even

even prove (XXV), that the conservation of the active forces takes place in incompressible fluids, the movement of which changes by insensible degrees; and in short, generally every thing which we have proved of a system of hard bodies is equally true with respect to a mass of incompressible fluid.

Scholium.

XLVIII. This scholium is destined for the development of the principle laid down in the fifth corollary: this proposition, in fact, contains the principal part of the theory of machines in a state of motion, because most of them are moved by agents which can only exercise dead forces, or those of pressure: of this description are all animals, springs, weights, &c., which is the cause why the machine generally changes its state by insensible degrees. It also most frequently happens, that this machine passes very quickly to uniformity of motion, for the following reason:

The agents which move this machine being at first a little above the resisting forces, give rise to a small movement which is afterwards gradually accelerated; but, whether as a necessary consequence of this acceleration, the soliciting force diminishes, whether the resistance increases, or, lastly, if there happens any variation in the directions, it almost always happens that the relation of the two forces is brought nearer and nearer to that in virtue of which they could mutually form equilibrium: these two forces are then destroyed, and the machine is no longer moved, except in virtue of the acquired movement, which, on account of the inertness of the matter, generally remains uniform.

XLIX. In order to understand still better how this happens, it is only necessary to attend to the motion of a ship which has the wind directly on her poop: this is a kind of machine animated by two contrary forces, which are the impulse of the wind, and the resistance of the fluid upon which it swims: if the first of these two forces, which may be regarded as soliciting, is greatest, the movement of the ship will be accelerated: but this acceleration necessarily has limits, for two reasons; because, the more the movement of the vessel is accelerated, 1st, the more is it subtracted from the

the impulse of the wind; 2d, on the other hand, the resistance of the water increases: consequently these two forces tend to equality: when they have attained this point they will be mutually destroyed; and therefore the vessel will be moved as a free body, *i. e.* its velocity will be constant. If the wind fell, the resistance of the water would surpass the soliciting force; the movement of the vessel would slacken; but, as a necessary consequence of this slackening, the wind would act more efficaciously upon the sails; and the resistance of the water would at the same time diminish: these two forces would still tend therefore to equality, and the machine would at the same time attain an uniformity of movement.

L. The same thing happens when the moving forces are men, animals, or other agents of this kind: at first the mover is a little above the resistance; thence arises a small movement, which is gradually accelerated by the repeated efforts of the moving power; but the agent itself is obliged to assume an accelerated movement, in order to remain attached to the body upon which it impresses motion. This acceleration, which it procures for itself, consumes a part of its effort, in such a manner that it acts less efficaciously upon the machine; and the movement of the latter, accelerating less and less, finishes by soon becoming uniform. For instance: a man who could make a certain effort in the case of equilibrium, would make a much less one if the body he applies his strength to should yield, and if he was obliged to follow it in order to act upon it: it is not because the absolute labour of this man is less; but it is because his effort is divided into two, one of which is employed in putting the man himself in motion, and the other is transmitted to the machine. Now it is from this last alone that the effect is manifested in the object proposed.

I shall nevertheless continue to consider machines under a more general point of view: thus, I shall place in this scholium several reflections applicable to the varied movement. I shall only suppose that this variation takes place by insensible degrees; and I shall prove that this should in

 fact

fact be the case, when we wish to employ them in the most advantageous manner possible.

Important for later analysis.

Q = energy supplied by activating forces.

q = energy absorbed by resisting forces.

LI. Let us therefore designate by Q, the momentum of activity consumed by the soliciting forces in a given time t, and by q, the momentum of activity exercised at the same time by the resisting forces: this being done, whatever be the movement of the machine, we shall always have, by the fifth corollary, $Q = q$; in such a manner, for example, that if each F of the soliciting forces be constant, its velocity V uniform, and the angle Z formed by the directions of F and V always null, we shall have at the end of the time t s F V $t = q$; and if all the soliciting forces are reduced to a single one, we shall consequently have F V $t = q$ (XXXII and XXXIII).

LII. We may in general regard the momentum of activity q, exercised by the resisting forces, as the effect produced by the soliciting forces: for instance, when it is requisite to raise a weight P to a given height H, it is very easy to regard the effect produced by the moving force as being in a compound ratio of the weight, and the height to which we have to raise it; so that P H is what we then naturally understand by the effect produced. Now, on the other hand, this quantity P H is precisely what we have called the momentum of activity exercised by the resisting force P; therefore this momentum of activity, or q, is what we naturally understand in this case by the effect produced.

Now, in the other cases, it is evident that q is always a quantity analogous to that just mentioned: this is the reason why I shall frequently, in the course of my subsequent observations, call this quantity q the *effect produced:* thus, by the terms *effect produced*, I shall mean the momentum of activity exercised by the resisting forces; in such a manner that, in virtue of the equation $Q = q$, we may establish as a general rule, that *the effect produced in a given time by any system of moving forces, is equal to the momentum of activity consumed at the same time by all these forces.*

LIII. We see by the equation F V $t = q$, found in the preceding article, that it is of no use to be acquainted with the

the figure of a machine, in order to know what effect any power applied to it can produce, when we are acquainted with that which it would produce without the machine: let us suppose, for example, that a man is capable of exercising a continual effort of 25^{tt}, by moving his own body continually with a velocity of three feet in the second: this being granted, when we apply it to a machine, the momentum of activity $F V t$, which this man will exercise, will be (XXXII) $25^{tt}\ 3\ p^i$ (3 feet) t, *i. e.* we shall have $FV^t = 25^{tt}\ 3\ p^i\ t$, t expressing the number of seconds: therefore, on account of $FV^t = q$, we shall have $q = 25^{tt}\ 3\ p^i\ t$, whatever be the machine: therefore the effect q is absolutely independent of the figure of this machine, and can never surpass that which the power is in a state to produce naturally, and without a machine.

Thus, for example, if this man with his effort of 25^{tt}, and his velocity of three feet in the second, is in a state with a given machine, or without a machine, to raise, in a given time, a weight p to a height H, we cannot invent any machine by which it is possible, with the same labour, (*i. e.* the same force, and the same velocity as in the first case,) to raise, in the given time, the same weight to a greater height, or a greater weight to the same height, or, finally, the same weight to the same height, in a shorter time: this is evident: since then q being (XXXII) equal to $P H$, we have, by the preceding article, $P H = 25^{tt}\ 3\ p^i\ t$.

LIV. The advantages resulting from machines do not therefore consist in producing great effects from small causes, but in affording the means of choosing, among different methods which may be called equal, that which is most convenient in the existing circumstances. In order to force a weight P to ascend to any height proposed, a spring to close together in a given quantity, a body to assume any given movement by insensible degrees, or, finally, any other given agent to produce any given momentum of activity, the moving forces employed must of themselves consume a momentum of activity equal to the first: no machine can dispense with it: but as this momentum results from several terms or factors, we may vary them at pleasure, by dimi-

 nishing

nishing the force at the expense of the time, or the velocity at the expense of the force; or rather by employing two or more forces instead of one: this gives an infinity of resources for producing the momentum of activity necessary: but, whatever we do, these means must always be equal, *i. e.* the momentum of activity consumed by the soliciting forces, is equal to the effect or momentum exercised at the same time by the resisting forces.

[To be continued.]

XLI. *On the Planet Vesta.* *By* S. Groombridge, *Esq.* *To Mr. Tilloch.*

Sir,

The discovery of the planet Vesta, on the 29th of March 1807, having been communicated to this country by Dr. Olbers; on the 26th of April I found its place, and observed the same on the meridian. I obtained a series of observations to the 20th of May; after which, from the increase of daylight, it was no longer visible on the meridian. The observations which were afterwards made were with equatorial instruments; and these cannot be depended on, for sufficient accuracy in calculating the elements. I have, however, used some of these, from the 29th of March to the 22d of June, to determine the eccentricity; those which were made on the meridian producing nearly the same radius. I thence discovered, that the planet was decreasing in radius, and therefore conjecture that it was in aphelio about the time it was first seen. When the planet was discovered by Dr. Olbers on the 29th of March, it appears to have been about seven days past the opposition; and it is well known, not having that point of the orbit for a datum, the difficulty of calculation is increased. I was therefore anxious to observe the planet before the ensuing opposition, to obtain sufficient materials for ascertaining all the elements. For this purpose, I assumed a mean radius of the extreme observations; which, if I was right in my conjecture of the aphelium, would prove too great; and therefore the planet should be further advanced in the ecliptic. On the 30th of July, the evening being clear, and the moon not risen, I observed the difference of right ascension of several stars of the

"I now proceed, in conformity with your suggestion, to make a brief recapitulation of the most material parts of this paper, and to endeavour to enumerate, and to place in one point of view, those articles recommended above, which appear to me to be best calculated to answer the desired purpose:

"Caustic potash, train oil, waste salt, mixture of salt and oil, urine, oil of hartshorn, linseed oil, sea water, assafœtida, chaff and refuse oil.

"Any of these, in my opinion, might be employed with perfect safety to the revenue.

"I have the honour to be, gentlemen,

"your faithful and obedient humble servant,

"SAMUEL PARKES."

LIII. *Essay upon Machines in General. By M.* CARNOT, *Member of the French Institute, &c. &c.*

[Continued from p. 228.]

LV. THESE reflections should seem sufficient for undeceiving those who think that with machines charged with levers arranged mysteriously, we may put an agent, though never so feeble, in a condition to produce the greatest effects: the error proceeds from persuading ourselves, that it is possible to apply to machines in movement what is not true except with respect to the case of equilibrium: from the circumstance of a small power holding a very great weight in equilibrium, many persons think that it could in the same way raise this weight as quickly as they please: now this is a very striking mistake, because, in order to succeed, the agent must procure for itself a velocity beyond its faculties, or which would at least make it lose so much the greater part of its effort upon the machine as it would be obliged to move itself more quickly. In the first case the agent has no other object to attain than to make an effort capable of counterbalancing the weight; in the second case, besides this effort, there must be also another to overcome the inertia, both of the body on which it impresses the movement and of its own proper mass: the total effort which in the first

first case would be employed entirely in conquering the weight of the body, is here divided into two, the first of which continues to make an equilibrium in the weight, and the other produces the movement. We therefore cannot augment one of these efforts except at the expense of the other; and this is the reason why the effect of machines in motion is always so limited that it can never surpass the momentum of activity exercised by the agent which produces it.

It is, without doubt, for want of paying sufficient attention to these different effects of one and the same machine, considered sometimes in a state of repose and sometimes in movement, that some persons not unacquainted with sound theory frequently abandon themselves to the most chimerical ideas, while we see simple workmen turning to advantage, as it were by instinct, the real properties of machines, and judging very accurately of their effects. Archimedes only wanted a lever and a fixed point, in order to move the globe of the earth; how did it happen then, it may be said, that so great a man as Archimedes could not, even when furnished with the best machine in the world, raise a weight of one hundred pounds in one hour to a small given height? It is because the effect of a machine at rest and of one in movement are two very different things, and somewhat heterogeneous: in the first case it is requisite to destroy and to hinder the movement; in the second, the object is to produce it and to keep it up; now it is clear that this last case requires more consideration than the first: viz. the real velocity of each point of the system;—but we shall better perceive the reason of this difference by the following remark.

Any given fixed points or obstacles are forces purely passive, which may absorb a movement however great it may be, but which can never produce one, let it be never so small, in a body at rest: now it is very improperly that in the case of equilibrium we say of a small power, that it destroys a great one: it is not by the small power that the great one is destroyed; it is by the resistance of the fixed points: the small power in reality destroys but a small part of the great, and the obstacles do the rest. If Archimedes had possessed what he wished for, it would not have been he who would

have

have supported the globe, it would have been his fixed point: all his art would have consisted not in redoubling his efforts to contend against the mass of the globe, but to put in opposition two great forces, the one active, and the other passive, which he would have had at his disposal: if, on the contrary, it had been requisite to produce an effective movement, in this case Archimedes would have been obliged to draw it entirely from his own proper person; and yet it would have been very small, even after several years: let us not attribute therefore to active forces, what is owing to the resistance of obstacles only, and the effect will not appear more disproportioned to the cause in machines at rest than in machines in motion.

LVI. What is the true object therefore of machines in motion? We have already said, that it is to procure the faculty of varying at pleasure the terms of the quantity Q, or the *momentum* of activity which should be exercised by the moving forces. If time be precious, if the effect must be produced in a very short time, and if we have only a power capable of very little velocity, but of a great effort, we may find a machine capable of supplying the velocity necessary for the force: if, on the contrary, we must raise a very considerable weight, and we have but a weak power, although capable of great velocity, we may contrive a machine with which the agent will be in a condition to compensate by its velocity the force of which it is deficient. Lastly, if the power is neither capable of a great effort nor of a great velocity, we may still, with a proper machine, make it produce the effect desired, but then it will require much time; and herein consists the well-known principle, *that in machines in movement, we always lose in time or in velocity what we gain in force.*

Machines are therefore very useful, not by augmenting the effect of which powers are naturally capable, but by modifying this effect: it is true we shall never succeed by means of them in diminishing the expense or *momentum* of activity necessary for producing an effect proposed; but they will assist us in making a proper division of this quantity for attaining the design in view: it is by their assistance that

that we shall succeed in determining, if not the absolute movement of each part of the system, at least in establishing among these different particular movements the relations which are most proper: it is by them, lastly, that we shall give to the moving forces the most convenient situations and directions, the least fatiguing, and the most proper for employing their faculties in the most advantageous manner.

THE BIG QUESTION

LVII. This naturally leads us to the following interesting question—Which is the best method of employing any given powers, the natural effect of which is known, on applying them to machines in motion? In other words, What is the method of making them produce the greatest possible effect?

The solution of this problem depends upon particular circumstances; but we may hereupon make some general observations applicable to all cases. The following are among the most essential.

q = energy absorbed by resisting forces

The effect produced being the same thing (LII.) with the momentum of activity exercised by the resisting forces, the general condition is, that q is a *maximum*: now q never being able to surpass Q, 1st, The quantity Q must itself be the greatest possible; 2dly, All this momentum Q must be solely employed in producing the effect proposed.

In order to make Q a *maximum*, we must consider that it depends upon four things, viz.: upon the quantity of force exercised by the agent which should produce the effect q, upon its velocity, upon its direction, and upon the time during which it acts. Now, 1st, As to what regards the direction of the force, it is evident that this power should be in every thing, besides being equal, directed in the same ratio with its velocity, for the momentum of activity which during $d\,t$ a power F exercises, the velocity of which is V, and the angle comprehended between F and V, Z, being (XXXII.) F V $d\,t$ cosine z, it is clear that this produce will never be greater than when cosine z will be equal to the total sinus, *i. e.* when the force and its velocity shall be directed in the same ratio: 2dly, As to what regards the intensity of the force exercised, its velocity, and the time during which it is exercised; we should not determine these things

things in an absolute manner, but solely place them in the relations in which experience has shown they will be of most advantage: for instance, I shall suppose that a man attached or eight hours in a day to a winch of one foot radius, might make continually an effort of 25 tons by making one turn every two seconds, which nearly amounts to the velocity of three feet per second; but if we forced this man to go quicker, thinking thereby to hasten the business, we should retard it, because he would not be in a condition to make an effort of 25 tons, or could no longer work at the rate of eight hours a day. If, on the contrary, we diminished the velocity, the force would augment, but in a less degree, and the momentum of activity would also diminish: thus, according to experience, in order that this momentum should be a *maximum*, we must proportion the machine so as to preserve to the power the velocity of three feet per second, and not let it work more than eight hours a day. It is well known that each kind of agent has, in respect of its physical nature or constitution, a *maximum* analogous to that of which we have spoken, and that this *maximum* can in general only be found by experience.

LVIII. This first condition being fulfilled, nothing remains to be done, to produce with any given machine the greatest effect possible, but to manage matters so as that the whole quantity Q is employed in producing this effect; for if this be done, we shall have $q = Q$; and this is all we can expect, since Q can never be less than q.

Now in order to fulfil this condition, I say, in the first place, that we should avoid every shock or sudden change whatever; for it is easy to apply to all imaginable cases the reasoning which has been laid down (XLVII.) as to machines with weights; whence it follows, that every time there is a shock, there is at the same time a loss of momentum of activity on the part of the soliciting forces; a loss so real that the effect of it is necessarily diminished, as we have shown with respect to machines with weights in the above article: it is therefore with reason that we have advanced (LI.), that in order to make machines produce the greatest effect possible, they must of necessity never change their movement,

movement, except by insensible degrees;—we must solely except those which by their very nature are subject to undergo different percussions, like most kinds of mills; but even in this case, it is clear that we should avoid every sudden change which is not essential to the constitution of the machine.

Maximum effect water wheel.

LIX. We may conclude from this, for example, that the method of producing the greatest possible effect in a hydraulic machine moved by a current of water, is not to adapt a wheel to it, the wings of which receive the shock of the fluid. In fact, two good reasons prevent us from producing in this way the greatest effects: the first is, as we have already said, because it is essential to avoid every kind of percussion whatever; the second is, because after the shock of the fluid there is still a velocity which remains to it as a pure loss, since we should be able to employ this remainder in still producing a new effect to be added to the first. In order to make the most perfect hydraulic machine, *i. e.* capable of producing the greatest possible effect, the true difficulty lies, 1st, In managing so as that the fluid may lose absolutely all its movement by its action upon the machine, or at least that there should only remain precisely the quantity necessary for escaping after its action; 2d, Another difficulty occurs in so far as it loses all this movement by insensible degrees, and without there being any percussion, either on the part of the fluid, or on the part of the solid parts among themselves: the form of the machine would be of little consequence; for a hydraulic machine which will fulfil these two conditions will always produce the greatest possible effect: but this problem is very difficult to resolve in general, not to say impossible; it may even happen that in the physical state of things, and in respect of their simplicity, there can be nothing better than wheels moved by shocks: and in this case as it is impossible to fulfil at once the two conditions most desirable, the more we wish to make the fluid lose of its movement in order to attain the first condition, the stronger will be the shock; the more, on the contrary, we wish to moderate the shock in order to approach the second, the less will the fluid lose of its movement.

Form machine irrelevant when most efficient.

Most efficient machine is unachievable.

Seek optimum tradeoff of two opposing processes.

ment. We perceive that there is a medium, by means of which we shall determine, if not in an absolute manner, at least, having regard to the nature of the machine, that method which will be capable of the greatest effects.

LX. Another general condition, which is not less important when we wish that machines should produce the greatest possible effect, is, to contrive that the soliciting forces should give rise to no movement inapplicable to the object in view. If my object, for example, is to raise to a given height the greatest quantity of water possible, whether with a pump or otherwise, I should contrive that the water on flowing into the upper reservoir should only have precisely as much velocity as was necessary and no more, for all beyond this quantity would uselessly consume the effort of the motive power. It is clear in fact (XLV.), that in this case this power would have to consume an useless momentum of activity, and which would be equal to the half of the real force with which the water would have arrived in the reservoir.

Raising water by pump or other means.

It is not less evident, that in order to give the machines the greatest effect possible, we should avoid or diminish, at least as much as possible, the passive powers, such as friction, rubbing of cords, the resistance of the air, which are always, in whatever direction the machine moves, among the number of the forces I have called resisting*.

Avoid friction etc

It would be easy to extend these particular remarks, but my object is not to enter at present into any larger detail.

LXI. It may be concluded, from what has been said on the subject of friction and other passive bodies, that perpetual motion is a thing absolutely impossible, by only employing in order to produce it bodies which would not be solicited by any motrix force, and even heavy bodies; for

New topic. Impossibility of perpetual motion.

What is not explicit here, but is in S Carnot: idea of running a machine in reverse.

Earlier: geometric motions are reversible. Any connection?

* We often hear of passive forces; but where is the difference between an active and a passive force? I think this question has never yet been answered. Now it appears to me that the distinctive character of passive forces consists in this, that they never can become soliciting forces, whatever may be the movement of the machine, while active forces can act sometimes in the quality of soliciting and sometimes as resisting forces. In this view, obstacles and fixed points are evidently passive forces, since they can neither act as soliciting nor as resisting forces (XXXI).

these

these passive forces from which nothing can be subtracted being always resisting, it is evident that the movement must continually slacken: and from what we have said (XLV.), we see that if bodies are not solicited by any motrix force, the amount of the active forces will be reduced to nothing; *i. e.* the machine will be reduced to a state of rest, when the momentum of activity, produced by the friction since the commencement of the motion, will have become equal to half the amount of the initial active forces: and if the bodies are heavy, the motion will finish when the momentum produced by the frictions shall be equal to half the amount of the initial active forces, plus the half of the active force which would take place if all the points of the system had one common velocity, equal to that which is owing to the height of the point where the centre of gravity was at the first instant of the motion, above the lowest point to which it can descend: this is evident from (XLII).

It is easy to apply the same reasoning to the case of springs, and in general to all cases in which the friction being subtracted, the soliciting forces are obliged, in order to make the machine pass from one position to another, to exercise a momentum of activity as great as that which is produced by the resisting forces when the machine returns from this last position to the former.

The motion would end much sooner if some percussion took place, since the sum of the active forces is always diminished in such cases (XXIII).

It is therefore evident, that we ought entirely to despair of producing what is called a perpetual motion, if it be true that all the moving powers which exist in nature are nothing else than attractions, and that this force, as it should seem, has a general property, that of being always the same at equal distances between given bodies, *i. e.* of being a function which only varies in cases where the distance of these bodies itself varies.

LXII. One general observation resulting from all that has been said, is, that the kind of quantity to which I have given the name of *momentum of activity*, performs a very conspicuous part in the theory of machines in a state of motion; for

for it is in general this quantity which we must œconomize as much as possible, in order to draw all the effect we can from one agent.

If it be required to raise a weight, water for example, to a given height; you will be able to raise more in a given time, not from having exhausted a greater quantity of power, but in proportion as you have exercised a greater momentum of activity (XLIV).

If it be required to turn a mill, either by water, or wind, or animals, it is not necessary that the shock of the water, the wind, or the effort of the animal be greater; but these agents should be made to consume the greatest momentum of activity possible.

If we wish to make a vacuum in the air in any way whatever, we must, in order to succeed, consume a *momentum of activity* as great as that which would be necessary for raising to the height of 30 feet a volume of water equal to the vacuum which we wish to produce.

If it be a vacuum in an indefinite mass of water like the sea, we must consume the same *momentum of activity* as if the sea were a vacuum; as if the vacuum which we wish to make were a volume of sea water, and as if we must raise this volume to the height of the level of the sea.

If it be required to produce a vacuum in a vessel of a given figure, it is evident that we cannot succeed without causing to ascend the centre of gravity of the total mass of the fluid in a quantity determined by the figure of the vessel; we must therefore consume a *momentum of activity* equal to that which would be necessary to raise all the water in the vessel in a quantity equal to that from which the centre of gravity of the fluid must ascend.

In a machine at rest, where there is no other force to overcome except the *vis inertiæ* of the bodies, if we wish to produce any movement by insensible degrees, the *momentum of activity* which we have to consume will be equal to half the amount of the active forces we wish to produce; and if it be merely required to change the movement it has already, the *momentum of activity* to be produced will only be the quantity

quantity in which this half amount will be increased by the change (XLV).

Finally, supposing we have any system of bodies, that these bodies attract each other, on account of any function of their distances; even supposing, if we please, that this law is not the same with respect to all the parts of the system, *i. e.* that this attraction follows any law we please, (providing that, between two given bodies, it only varies when the distance of these bodies in itself varies,) and it be required to make the system pass from any given position to another: this being done, whatever be the path that we wish each of the bodies to take, in order to attain this object, whether we put all these bodies in motion at once, or the one after the other, whether we conduct them from one place to another by a rectilinear or curvilinear motion, and varied in any manner (providing no shock nor rapid change occur); lastly, whether we employ any kind of machines whatever, even by a spring, providing that in this case we ultimately replace the springs in the same state of tension in which they were at the first moment, the *momentum of activity* which they will have to consume, in order to produce this effect, the external agents employed to move this system, will always be the same, supposing the system to be at rest at the first instant of the movement, and at the last also.

And if, besides all this, it be necessary to produce in the system any given movement, or if it be already in motion at the first moment; and if it be requisite to modify or change this movement, the *momentum of activity* which the external agents will have to consume will be equal to that which it would be necessary to consume if it were merely requisite to change the position of the system, without impressing any motion upon it (*i. e.* considered as at rest at the first and last instants,) plus the half of the quantity by which we must augment the sum of the active forces.

It is of very little importance therefore, as to the expenditure or *momentum of activity* to be consumed, that the forces employed are great or small, that they employ such

and

and such machines, or that they act simultaneously or not: this *momentum of activity* is always equal to the produce of a certain force, by a velocity, and by a time, or the sum of several products of this nature; and this sum should always be the same, in whatever way we take it: the agents therefore will gain nothing on the one hand, which they do not lose on the other.

To conclude, let us suppose that in general we have any system of animated bodies, of any motrix forces, and that several external agents, such as men or animals, are employed to move this system in various and different ways, either by themselves or by machines:—This being granted,

Whatever be the change occasioned in the system, the momentum of activity consumed during any time by the external powers, will be always equal to the half of the quantity by which the sum of the active forces will have augmented during this time, in the system of bodies to which they are applied: minus the half of the quantity by which this same sum of active forces would have augmented, if each of the bodies were freely moved upon the curve it has described, supposing that it had then undergone at each point of this curve the same motrix force as that which it really undergoes: providing always that the motion changes by insensible degrees, and that if we employ machines with springs, we leave these springs in the same state of tension in which we found them. [To be continued.]

LIV. *Memoirs of the late* Erasmus Darwin, *M. D.*

[Continued from vol. xxx. p. 115.]

DARWINIANA.

Having laboured under a severe illness, the author of this memoir must apologize for so long delaying the continuation of the remarkable medical opinions of the great Dr. Darwin, whose powers of mind, fully bent upon one important subject, namely health, and the causes of disease, and the remedies to be applied, with the rationale of each, cannot fail to interest the philosophic world.