Cosmology and Time

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Seminar Philosophy of Space and Time
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Overview

Physics:

1. General Relativistic Cosmology
2. Inflation

Philosophy:

1. The notion of Cosmic Time
2. Does Time have a beginning?
3. The Cosmic Arrow of Time
4. Cosmology and the Past Hypothesis

General Theory of Relativity

Potential models of the Theory: $\langle M, g, T \rangle$

M space-time manifold

$g/g^{\mu\nu}$ metric field tensor: $ds^2 = g_{\mu\nu}\, d\, x^{\mu}\, d\, x^{\nu}$

$T/T^{\mu\nu}$ momentum energy tensor

Actual models fulfill Einstein's field equations:

$$G + \Lambda g = 8 \pi T$$

Λ Cosmological Constant

G Einstein curvature tensor, derived from g

Cosmological Models

History:

Einstein (1917), Sitzungsberichte: Kosmologische Betrachtungen zur Allgemeinen Relativitätstheorie -> Einstein model/cosmos

De Sitter (1917), KNAW Proceedings 19 II: de Sitter model

Cosmological Models?

Terminology: Matter/energy: “Cosmic fluid”

Constraints on cosmological models:

1. Realistic matter model for cosmic fluid: constrains/fixes T.
   Simplest model: Dust model:

   Every fluid element characterized in terms of density ρ and four-velocity u^μ

   pressure-less fluid: $T^{\mu\nu} = \rho\, u^\mu\, u^\nu$
   Idealization!
2. <M,g> faithfully captures the geometrical structure of the Universe at large scales.

Cosmological *Standard* Models

Obey the Cosmological Principle. First shot:

(CP) The Universe is spatially homogeneous and spatially isotropic around every observer comoving with the cosmic fluid.

Covariance?!

Define spatial homogeneity and spatial isotropy in a covariant way.

Homogeneity and isotropy

< *M*, *g* > is spatially homogeneous, iff there is a foliation of space-time into space-like hypersurfaces H_t with the following property: For each t and each pair of events p, q $\in H_t$ there is an isometry (metric-preserving map) that maps p into q.

< *M*, *g* > is spatially isotropic around each event, iff there is a congruence of time-like curves with tangent vectors u^μ with the following properties: 1. The congruence fills space-time. 2. For each p on curve C in the congruence, for all vectors v, w orthogonal to u^μ there is an isometry that maps v on w but leaves u^μ and p fixed.

Following Wald (1984), pp. 92-3

Alternatively

Introduce Weyl's principle:

Weyl's principle:

The fluid elements of the cosmic fluid move on non-intersecting geodesics orthogonal to space-like hypersurfaces (e.g. Narlikar 1983, p. 91)

That yields a foliation wrt which you can require spatial homogeneity and isotropy.

Notes

1. These definitions are purely geometrical. No momentum-energy tensor needed.

2. Spatial isotropy at each event implies spatial homogeneity.

3. Spatial homogeneity does not imply spatial isotropy (cf. anisotropically expanding Universes)

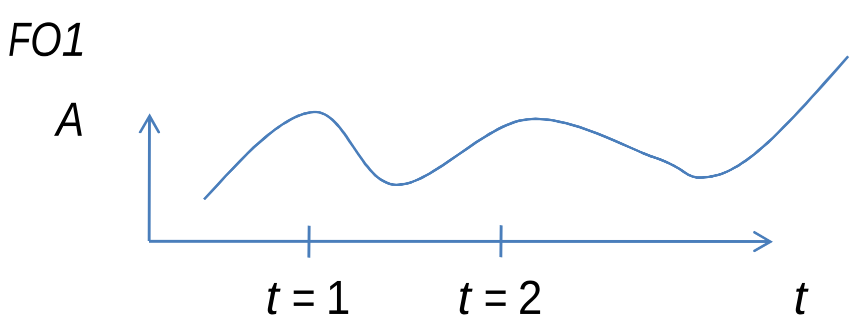
4. Spatial homogeneity and isotropy imply Robertson-Walker metric.

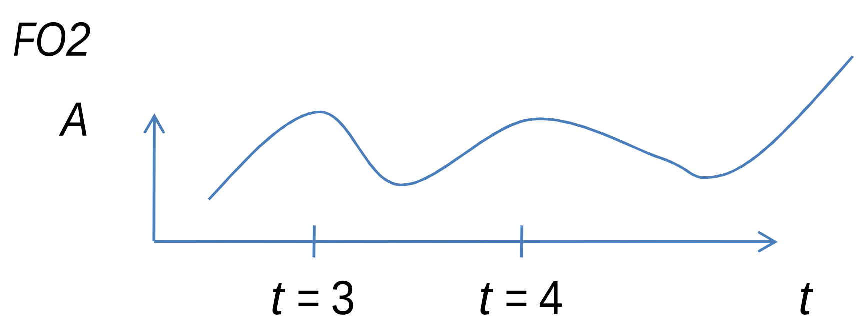
The CP in alternative terms

Fundamental observers = observers that move with the cosmic fluid.

CP:

1. For every fundamental observer and for every time (her time) the Universe looks the same in every direction to her.
2. If different fundamental observers record their observations, they come up with the same history of the Universe.





Cosmological Models

Given the CP, the metric can be written in the Robertson-Walker form (Robertson-Walker line element):

$$ds^2 = c^2\, dt^2 - R^2(t)\, (dr^2/(1 - K r^2) + r^2\, (d\theta^2 + \sin^2(\theta)\, d\varphi^2))$$

$R(t)$ cosmic scale factor

K curvature ($|K| = 1 / k^2$ with k curvature radius)

Cases: $K = 0$: Euclidean space

$K > 0$: spherical space

$K < 0$: hyperbolic space

Cosmological Models

Einstein's field equations: equation for *R* (*t*)

$$\frac{\dot{R}(t)^2}{R(t)^2} = \frac{8\pi G}{3}\rho(t) - \frac{k}{R(t)^2} + \frac{\Lambda}{3}$$

Friedmann equation.

Conservation of energy: for the density ρ:

$$\dot{\rho}(t) = -3(\rho(t) + p(t))$$

e.g.: Matter without pressure:

$$p(t) = 0, \text{thus } \rho(t) \propto 1/a^3(t)$$

Radiation:

$$p(t) = \frac{1}{3}\rho(t), \text{ thus } \rho(t) \propto 1/a^4(t)$$

Cosmological Models

many solutions for scale factor $R(t)$

-> define the cosmological standard models/Friedmann-Lemaître models.

In general: R not constant in time -> Expansion or contraction of the Universe and Hubble flow.

These days: Most popular model: Concordance model

Consequences for Time

„Komisch ist, dass nun endlich doch wieder eine quasi-absolute Zeit und ein bevorzugtes Koordinatensystem erscheint, aber bei voller Wahrung aller Erfordernisse der Relativität.“

Einstein 1917, quoted after Jung (2006), *Philosophia Naturalis*

Consequences for Time

1. In the standard models, there is a preferred foliation of space-time into space-like hypersurfaces (in most cases, the foliation is unique).

2. Time in this preferred foliation is called cosmic time.

3. Meaning: Cosmic time is the proper time of the fundamental observers, which move with the cosmic fluid. Synchronization.

4. In expanding/contracting models: “Density as clock”

The other way round

Question: What are the conditions for a distinguished cosmic time?

Weyl's principle:

The fluid elements of the cosmic fluid move on non-intersecting geodesics orthogonal to space-like hypersurfaces (e.g. Narlikar 1983, p. 91)

Sufficient cosmic time.

The other way round

Mittelstaedt 1989:

Necessary condition: Cosmic time needs rotation = 0

(rotation: local quantity constructed from four-velocity field)

Philosophical Discussion 1

Significance of cosmic time?

Lit.:
Dorato (1995), *Time and Reality*
Bourne (2004), BJPS 55

Cf. also Goedel, in Schilpp (1949)

Philosophical Discussion 2

Does time have a beginning?

Many standard models, including the concordance model start with a space time singularity.

Any event has a finite past (defined in terms of the proper time of the fundamental observer at that event).

In this sense: The Universe has a beginning.

For discussion see: Earman (2006), in: Stadler, Stöltzner Rughes and Zinkernagel (2008), St.H.P.M.Ph.

Inflation

History: Inflationary cosmology introduced in the early 80ies.

Problems with the standard models that inflationary cosmology sets out to solve:

1. The horizon problem
2. The flatness problem
3. …

Horizons

Particle Horizons:

What are the parts of the Universe, from which we can have obtained information at our time t_0? Fastest signal: light.

Light: d s = 0.
Without loss of generality: dθ = 0, d φ = 0

e.g. K = 0

$$\frac{c\mathrm{d}t}{R(t)} = \frac{\mathrm{d}r}{\sqrt{1-Kr^2}}$$

$$\frac{c\mathrm{d}t}{R(t)} = \mathrm{d}r \qquad \int_0^{t_0} \mathrm{d}t\,\frac{c}{R(t)} = r(t_1) < \infty$$

If the Universe is very big, we can only see part of it.

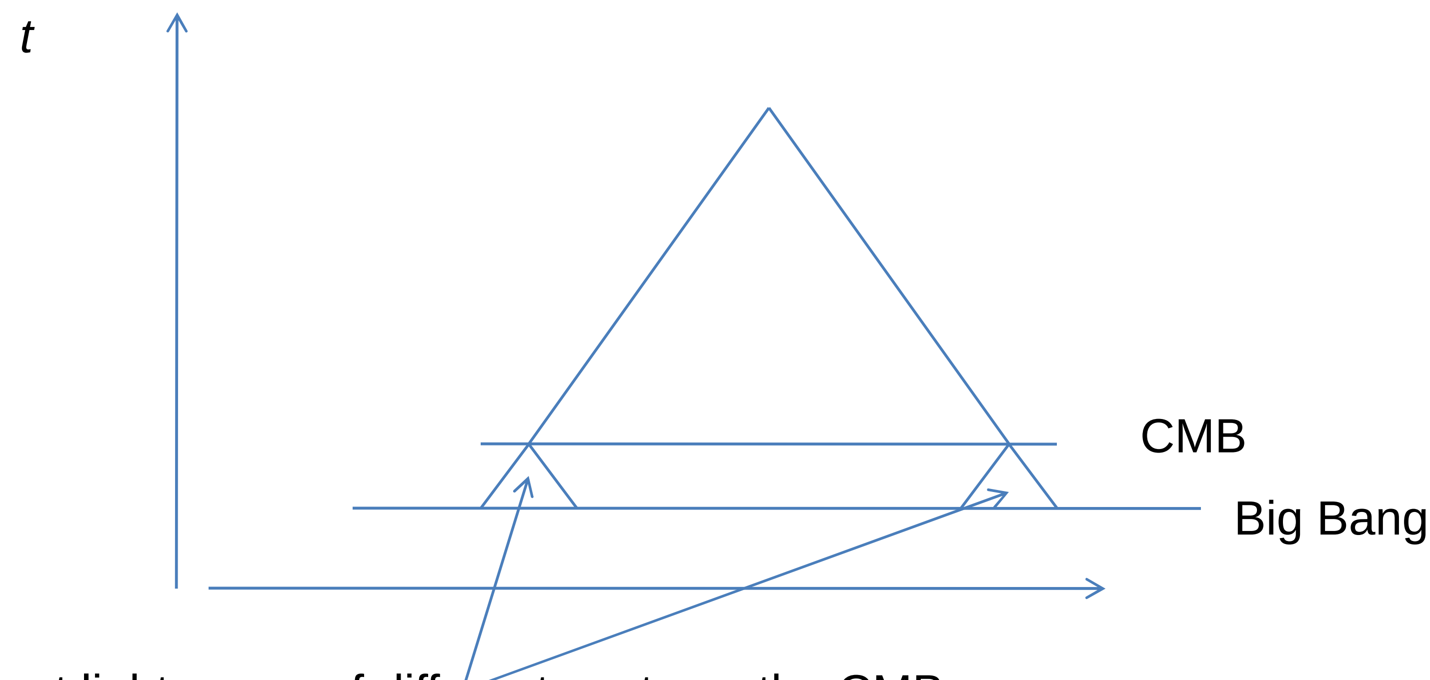
The horizon problem

Observation: The cosmic microwave background (CMB) is very isotropic. Thus regions that are very far send us the same radiation.

Calculate the horizon at the time the CMB radiation originated.

Result: much smaller than the distances between different regions that send us the same radiation.
The homogeneity of the CMB can not have been originated from a causal process.

The horizon problem



Past light cones of different spots on the CMB don’t overlap.

The horizon problem

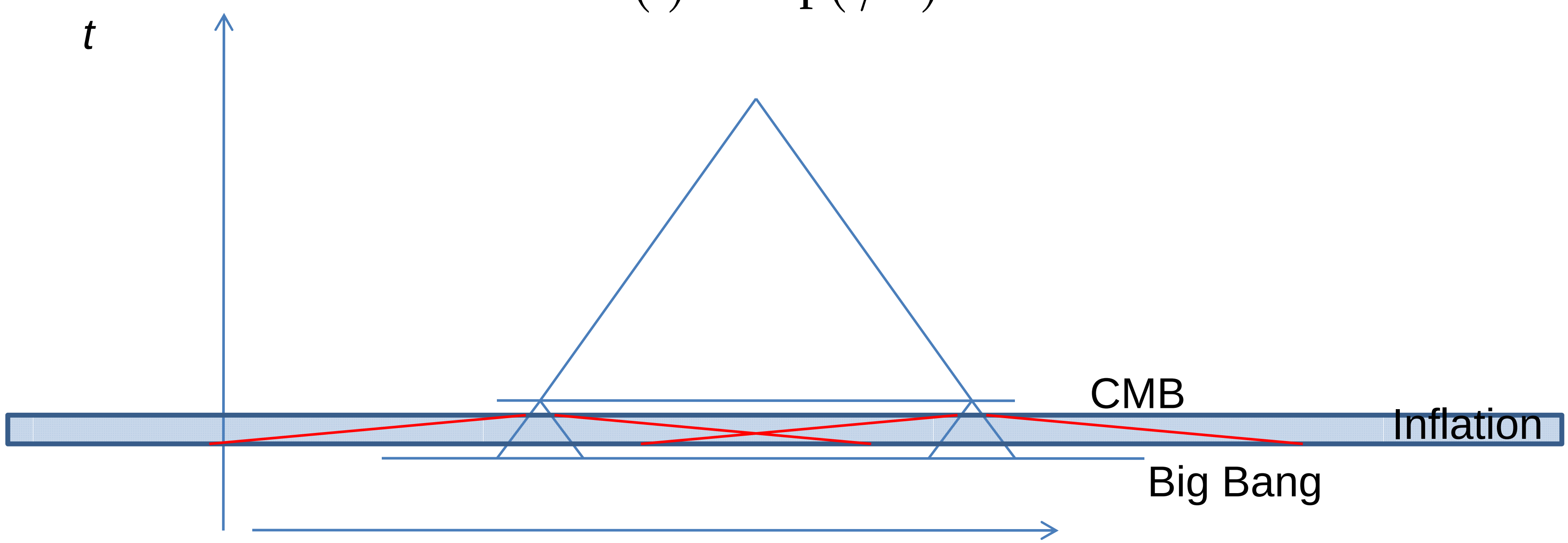
Philosophical question: Do we need an explanation of the homogeneity of the CMB?

Callender (2004), BSPS: “I urge the view that it is not always a serious mark against a theory that it must an ‘improbable’ initial condition.” (p. 195)

Inflation and the horizon

Idea: In the early Universe there was a period in which the Universe expanded very quickly (exponentially).

$$R(t) \propto \exp(t/T)$$



Past light cones do not overlap, because they are larger because of the inflation

The flatness problem

Cosmological parameters:
Rewrite Friedmann's equation:

$$H(t)^2 \equiv \frac{\dot{R}(t)^2}{R(t)^2} = \frac{8\pi G}{3}\rho(t) - \frac{k}{R(t)^2} + \frac{\Lambda}{3}$$

$$1 = \frac{8\pi G}{3H(t)^2}\rho(t) - \frac{k}{R(t)^2 H(t)^2} + \frac{\Lambda}{3H(t)^2} \equiv \Omega_m(t) + \Omega_k(t) + \Omega_\Lambda(t)$$

The flatness problem

For the next discussion, let Λ= 0 (Ω_Λ=0).
Time evolution of Ω_m :

Ω_m at present of the order of .3.
Therefore, Ω_m must have been very close to 1 at early times.
Explanation?

Inflation and the flatness problem

An very fast expansion drives the Universe in a regime in which Ω_m is close to 1 and thus

$$\Omega_k(t) = -\frac{k}{R(t)^2 H(t)^2} = -\frac{k}{\dot{R}(t)^2} \propto -\frac{k}{\exp(t/T)^2} \to 0$$

The physics behind inflation?

How do we get exponential growth of the Universe?

$$\frac{\dot{R}(t)^2}{R(t)^2} = \frac{8\pi G}{3}\rho(t) - \frac{k}{R(t)^2} + \frac{\Lambda}{3}$$

For small times, we can neglect the last two source terms. We need:

$$\rho(t) = const.$$

$$0 = \dot{\rho}(t) = -3(\rho(t) + p(t))$$

Thus

$$p(t) = -\rho(t)$$

Scalar field (inflaton) in slow-roll approximation.

Problems with inflation

Not much predictive power:

e.g. flat Universe (but built in)
Fluctuations of the microwave background are Gaussian
(but pretty generic)

For details see
Earman and Mosterin (1999), PoS
Tegmark (2005), astro-ph 0410281

Arrows of time

Time reversal invariance of micro-physics.

Macro-physics: Certain processes seem only to occur in one
direction,
several arrows of time

Explanation???

How cosmology becomes relevant

1. The expansion of the Universe defines a cosmic arrow of time that accounts for other arrows of time.

2. If the problem is solved by assuming Past Hypothesis, then cosmologists should be able to confirm that hypothesis.

The Cosmic Arrow of Time

The Universe is currently expanding rather than contracting.

That is, the model that describes the Universe does not respect the time-reversal invariance of the underlying equations.

Idea: That explains why the physics at some other levels does not respect the presumed time-reversal invariance of the microphysics.

A problem

What if the Universe will contract at some time?

The Gold Universe.