The Equivalent Principles in General Relativity

Zhonghao Lu
University of Pittsburgh, HPS
21st Feb, 2023

University of Pittsburgh

an advertisement

Textbook | May 2023

Differential Geometry and General Relativity

Volume 1

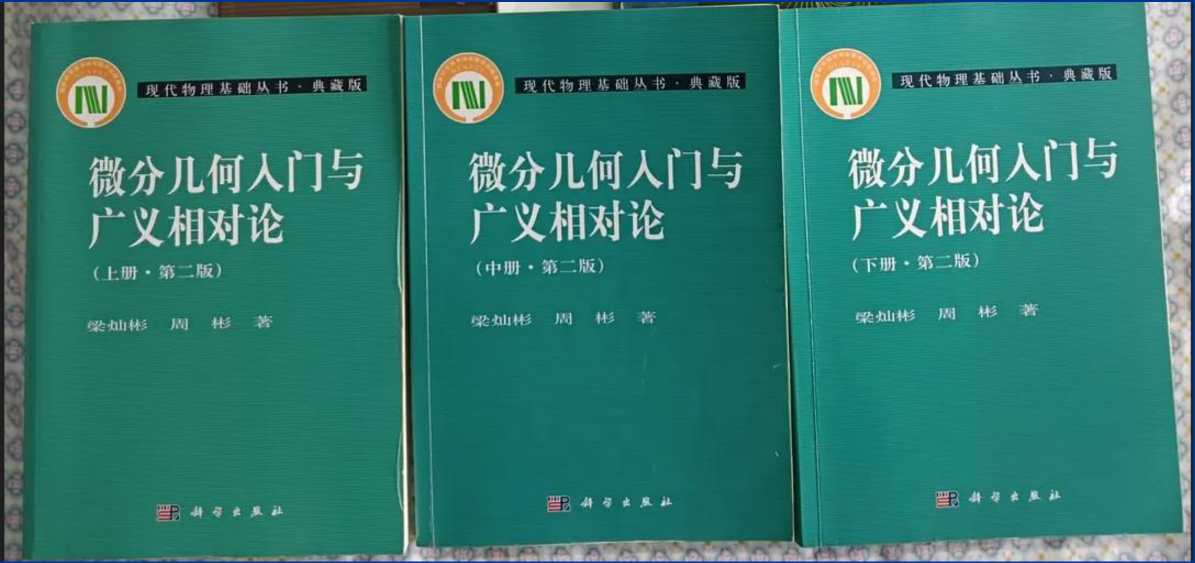
Authors: Canbin Liang, Bin Zhou

A textbook for graduate-level courses on general relativity

Includes both essential and optional reading sections to meet the needs of readers at various levels

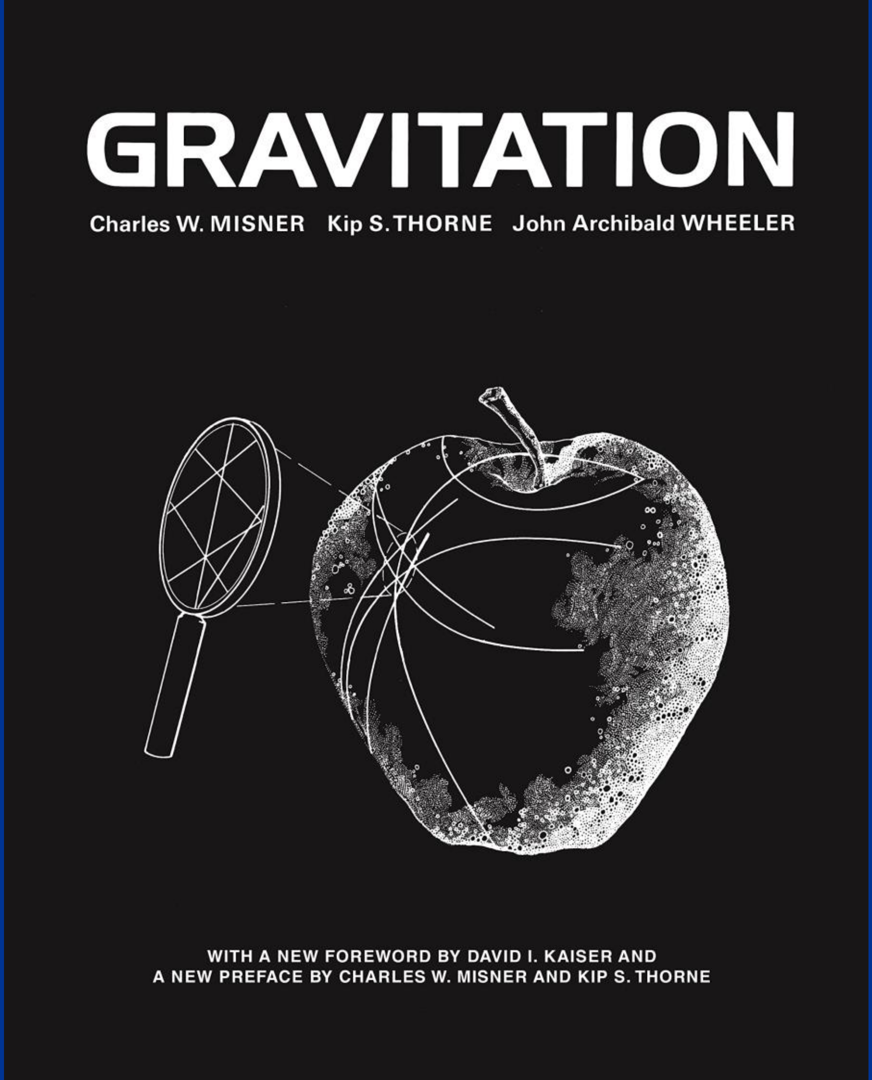
Uses pedagogic features including numerous exercises, examples, and illustrations to elaborate on difficult concepts

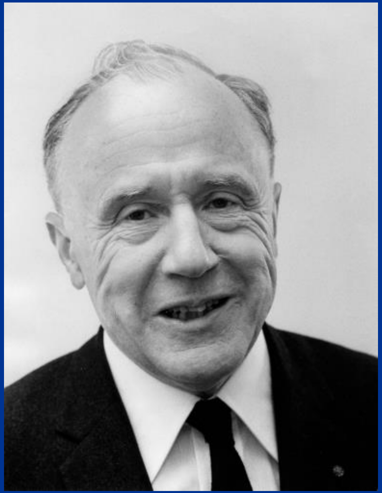
Part of the book series: Graduate Texts in Physics (GTP)



Liang CanBin (梁灿彬)
（1938-2022.2.16）

University of Pittsburgh


GRAVITATION
Charles W. MISNER Kip S. THORNE John Archibald WHEELER
WITH A NEW FOREWORD BY DAVID I. KAISER AND
A NEW PREFACE BY CHARLES W. MISNER AND KIP S. THORNE



The answer is simple: *in any and every local Lorentz frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar special-relativistic forms.* Equivalently: there is no way, by experiments confined to infinitesimally small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime from any other local Lorentz frame in the same or any other region. This is Einstein's principle of equivalence in its strongest form—a principle that is compelling both philosophically and experimentally. (For the relevant experimental tests, see §38.6.)

The principle of equivalence has great power. With it one can generalize all the special relativistic laws of physics to curved spacetime. And the curvature need not be small. It may be as large as that in the center of a neutron star; as large as that at the edge of a black hole; arbitrarily large, in fact—or almost so. Only at the endpoint of gravitational collapse and in the initial instant of the "big bang," i.e., only at "singularities of spacetime," will there be a breakdown in the conditions needed for direct application of the equivalence principle (see §§28.3, 34.6, 43.3, 43.4, and chapter 44). Everywhere else the equivalence principle acts as a tool to mesh all the nongravitational laws of physics with gravity.

Synge:

I have never been able to understand this Principle [of Equivalence]. [...] Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property; it has nothing to do with any observer's world-line. Space-time is either flat or curved, and in several places in the book I have been at considerable pains to separate truly gravitational effects due to curvature of space-time from those due to curvature of the observer's world-line … . The Principle of Equivalence performed the essential office of midwife at the birth of general relativity … . I suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced.

Three Versions of the Equivalence Principle

* Einstein Equivalence Principle (EEP)
(equivalence between gravitational and inertial effects or between essence of gravity and inertia)
"Homogeneous gravitational fields, or certain gravitational effects, can be 'transformed away' by changing the coordinate system" (Lehmkuhl 2022, 125)

University of Pittsburgh

Three Versions of the Equivalence Principle

* Strong Equivalence Principle (SEP)

Spacetime geometry is locally Minkowski. (I shall elaborate later)

Three Versions of the Equivalence Principle

* Weak Equivalence Principle (WEP)

[WEP1:] [I]f an uncharged test body is placed at an initial event in spacetime and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition.

[WEP2:] For any body the gravitational mass of the body is equal to its inertial mass.

Three Versions of the Equivalence Principle

* Weak Equivalence Principle (WEP)

It is usually acknowledged that WEP is weaker than both EEP and SEP.

A role of equivalence principles: to choose among competent theories of gravity

(For example, Brans-Dicke theory violates WEP)

University of Pittsburgh

Three Versions of the Equivalence Principle

* Einstein Equivalence Principle (EEP)
(equivalence between gravitational and inertial effects or between essence of gravity and inertia)
“Homogeneous gravitational fields, or certain gravitational effects, can be ‘transformed away’ by changing the coordinate system” (Lehmkuhl 2022, 125)

Synge: 👎

I have never been able to understand this Principle [of Equivalence]. [...] Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property; it has nothing to do with any observer's world-line. Space-time is either flat or curved, and in several places in the book I have been at considerable pains to separate truly gravitational effects due to curvature of space-time from those due to curvature of the observer's world-line … . The Principle of Equivalence performed the essential office of midwife at the birth of general relativity … . I suggest that the midwife be now buried with appropriate honours and the facts of absolute space-time faced.

Critisim of EEP

Synge: EEP is false.
Whether gravity exists or not, represented by Riemann tensor field, is an absolute property.

“The effects of a gravitational field are indistinguishable from the effects of an observer’s acceleration.”?

Critisim of EEP

Lehmkuhl:
Rather it is that
"The effects of a *homogenous* gravitational field are indistinguishable from the effects of an observer's *uniform* acceleration."?

But how to define homogenous gravitational field and uniform acceleration in terms of general relativity?

University of Pittsburgh

EEP

Restate the EEP:

Gravity and inertia are the same in their very essence.

Midwife from Newtonian Gravity to General Relativity

Critisim of EEP

Kottler:

Einstein introduces Newtonian “real gravitational force” in general relativity again.

$$\frac{d^2x^\gamma}{ds^2} - \Gamma^{\gamma}{}_{\alpha\beta}(x)\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = 0$$

Critisim of EEP

It seems that Einstein does operate with a coordinate-dependant concept of "gravitational field"

But the claim is that the "gravitational force" is *equivalent* to "inertial force".

Critisim of EEP

von Laue:

So in the general theory of relativity, *g*-brackets are supposed to represent something like the field strength of the gravitational field. Now let us look at the normal pseudo-Euclidean metric of the special theory of relativity, and transform to spatial cylinder coordinates, or indeed any other curved coordinates, without changing the temporal coordinate. Success: *g*-brackets show up in the geodesic equation. But surely it does not make physical sense to say that one has created a gravitational field by this purely mathematical operation.

Critisim of EEP

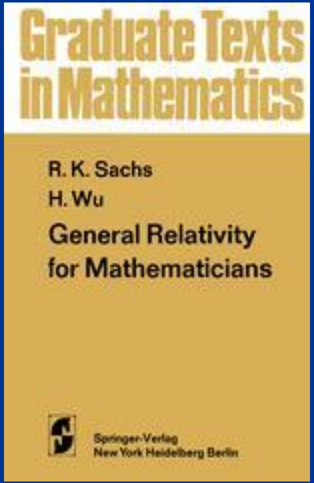
Einstein:

This is only a manner of speaking…

There is a gravitational field even in Minkowskian spacetime…

Critisim of EEP

My comments to von Laue's example: the difference between coordinate system / frame of reference



2.3 Reference frames

A single observer is so local that only cooperation between observers gives much information. In this section, we give the mathematical definition of a family of observers. Let (M, g, D) be a spacetime.

Definition 2.3.1. A *reference frame* Q on a spacetime M is a vector field each of whose integral curves is an observer.

von Laue's example belongs to the same reference frame.
The confusion may suggest that we had better adopt EEM is a coordinate-free fashion as well.

Three Versions of the Equivalence Principle

* Strong Equivalence Principle (SEP)

Spacetime geometry is locally Minkowski.

point SEP and

neighborhood SEP

(the coordinate system whose existence is postulated is taken to exist in some neighborhood of a point *p* of the spacetime manifold)

Question: what's the significant difference?

(geodesic coordinate system)

Three Versions of the Equivalence Principle

* Strong Equivalence Principle (SEP)

SEP as restriction of *non-gravitational* rules

To any required degree of approximation, given a sufficiently small region of spacetime, it is possible to find a reference frame with respect to whose coordinates $[K_0]$ the metric field takes Minkowskian form, and the connection and its derivatives do not appear in any of the fundamental field equations of matter.

An counter-example?

$$F^{c}_{ab;c} = 2\left(F^{e}_{[b}R_{a]e} - R_{abcd}F^{cd} + J_{[a;b]}\right).$$

Ite, missa est.