Doc. 47

ON THE RELATIVITY PRINCIPLE AND THE CONCLUSIONS DRAWN FROM IT

by A. Einstein

[*Jahrbuch der Radioaktivität und Elektronik* 4 (1907): 411-462]

Newton's equations of motion retain their form when one transforms to a new system of coordinates that is in uniform translational motion relative to the system used originally according to the equations

$$x' = x - vt$$

$$x' = y$$

$$z' = z \ .$$

As long as one believed that all of physics can be founded on Newton's equations of motion, one therefore could not doubt that the laws of nature are the same without regard to which of the coordinate systems moving uniformly (without acceleration) relative to each other they are referred. However, this independence from the state of motion of the system of coordinates used, which we will call "the principle of relativity," seemed to have been suddenly called into question by the brilliant confirmations of H. A. Lorentz's electrodynamics of moving bodies.[1] That theory is built on the presupposition of a resting, immovable, luminiferous ether; its basic equations are not such that they transform to equations of the same form when the above transformation equations are applied.

After the acceptance of that theory, one had to expect that one would succeed in demonstrating an effect of the terrestrial motion relative to the luminiferous ether on optical phenomena. It is true that in the study cited
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v/c of the relative velocity to the velocity of light in vacuum appears in the first power. But the negative result of Michelson and Morley's experiment[1] showed that in a particular case an effect of the second order (proportional to v^2/c^2) was not present either, even though it should have shown up in the experiment according to the fundamentals of the Lorentz theory.

It is well known that this contradiction between theory and experiment was formally removed by the postulate of H. A. Lorentz and FitzGerald, [4]
according to which moving bodies experience a certain contraction in the direction of their motion. However, this ad hoc postulate seemed to be only an artificial means of saving the theory: Michelson and Morley's experiment had actually shown that phenomena agree with the principle of relativity even where this was not to be expected from the Lorentz theory. It seemed therefore as if Lorentz's theory should be abandoned and replaced by a theory whose foundations correspond to the principle of relativity, because such a theory would readily predict the negative result of the Michelson and Morley experiment. [5]

Surprisingly, however, it turned out that a sufficiently sharpened conception of time was all that was needed to overcome the difficulty discussed. One had only to realize that an auxiliary quantity introduced by H. A. Lorentz and named by him "local time" could be defined as "time" in [6]
general. If one adheres to this definition of time, the basic equations of Lorentz's theory correspond to the principle of relativity, provided that the above transformation equations are replaced by ones that correspond to the new conception of time. H. A. Lorentz's and FitzGerald's hypothesis appears then as a compelling consequence of the theory. Only the conception of a luminiferous ether as the carrier of the electric and magnetic forces does not fit into the theory described here; for electromagnetic forces appear here not as states of some substance, but rather as independently existing things that are similar to ponderable matter and share with it the feature of inertia. [7]

The following is an attempt to summarize the studies that have resulted to date from the merger of the H. A. Lorentz theory and the principle of relativity.

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[1]A. A. Michelson and E. W. Morley, *Amer. J. of Science* 34, (1887): 333. [3]

The first two parts of the paper deal with the kinematic foundations as well as with their application to the fundamental equations of the Maxwell-Lorentz theory, and are based on the studies[1] by H. A. Lorentz (*Versl. Kon.*
[9] *Akad. v. Wet., Amsterdam* (1904)) and A. Einstein (*Ann. d. Phys.* 16 (1905)).

In the first section, in which only the kinematic foundations of the theory are applied, I also discuss some optical problems (Doppler's principle, aberration, dragging of light by moving bodies); I was made aware of the possibility of such a mode of treatment by an oral communication and a paper
[10] by Mr. M. Laue (*Ann. d. Phys.* 23 (1907): 989), as well as a paper (though in
[11] need of correction) by Mr. J. Laub (*Ann. d. Phys.* 32 (1907)).

In the third part I develop the dynamics of the material point (electron). In the derivation of the equations of motion I used the same method as
[12] [13] in my paper cited earlier. Force is defined as in Planck's study. The reformulations of the equations of motion of material points, which so clearly demonstrate the analogy between these equations of motion and those of classical mechanics, are also taken from that study.

The fourth part deals with the general inferences regarding the energy and momentum of physical systems to which one is led by the theory of relativity. These have been developed in the original studies,

[14] A. Einstein, *Ann. d. Phys.* 18 (1905): 639 and *Ann. d. Phys.* 23 (1907): 371, as well as M. Planck, *Sitzungsber. d. Kgl. Preuss. Akad. d. Wissensch.* XXIX (1907),

but are here derived in a new way, which, it seems to me, shows especially clearly the relationship between the above application and the foundations of the theory. I also discuss here the dependence of entropy and temperature on the state of motion; as far as entropy is concerned, I kept completely to the Planck study cited, and the temperature of moving bodies I defined as did Mr. Mosengeil in his study on moving black-body radiation.[2]

The most important result of the fourth part is that concerning the inertial mass of the energy. This result suggests the question whether energy also possesses *heavy* (gravitational) mass. A further question suggesting

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[8] [1]E. Cohn's studies on the subject are also pertinent, but I did not make use of them here.

[15] [2]Kurd von Mosengeil, *Ann. d. Phys.* 22 (1907): 867.

itself is whether the principle of relativity is limited to *nonaccelerated* moving systems. In order not to leave this question totally undiscussed, I added to the present paper a fifth part that contains a novel consideration, based on the principle of relativity, on acceleration and gravitation.

I. KINEMATIC PART

§*1. Principle of constancy of the velocity of light. Definition of time. Principle of relativity.*

To be able to describe a physical process, we must be able to evaluate the changes taking place at the individual points of the space as functions of position and time.

To determine the position of a process of infinitesimally short duration that occurs in a space element (point event) we need a Cartesian system of coordinates, i.e., three mutually perpendicular rigid rods rigidly connected with each other, and a rigid unit measuring rod.[1] Geometry permits us to determine the position of a point, i.e., the location of a point event, by means of three numbers (coordinates x, y, z).[2] To evaluate the time of a point event, we use a clock that is at rest relative to the coordinate system and in whose immediate vicinity the point event takes place. The time of the point event is defined by the simultaneous clock reading.

Imagine that clocks at rest with respect to the coordinate system are arranged at many points. Let all these clocks be equivalent, i.e., the difference between the readings of two such clocks shall remain unchanged if they are arranged next to each other. If these clocks are imagined to be set in some manner, then the totality of the clocks, provided they are arranged sufficiently closely, will permit the temporal evaluation of any point event, say by using the nearest clock.

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[1]Instead of speaking of "rigid" bodies, we could equally well speak, here, as well as further on, of solid bodies not subjected to deforming forces. [16]

[2]For this one also needs auxiliary rods (rulers, compasses).

$$dE = F_x dx + F_y dy + F_z dz - pdV + TdS \tag{28}$$ [90]

$$F_x = \frac{dG_x}{dt}, \text{ etc.} \tag{29}$$

Keeping in mind that

$$F_x dx = F_x \dot{x}\, dt = \dot{x} dG = d(\dot{x} G_x) - G_x d\dot{x} \text{ , etc.}$$ [91]

and

$$Td\eta = d(T\eta) - \eta dT \text{ ,}$$

one obtains from the above equations the relation

$$d(-E + T\eta + qG) = G_x d\dot{x} + G_y d\dot{y} + G_z d\dot{z} + pdV + \eta dT \text{ .}$$

Since the right-hand side of this equation must also be a total differential, and taking into account (29), it follows that

$$\frac{d}{dt}\left(\frac{\partial H}{\partial \dot{x}}\right) = F_x \qquad \frac{d}{dt}\left(\frac{\partial H}{\partial \dot{y}}\right) = F_y \qquad \frac{d}{dt}\left(\frac{\partial H}{\partial \dot{z}}\right) = F_z$$

$$\frac{\partial H}{\partial V} = p \qquad \frac{\partial H}{\partial T} = \eta \text{ .}$$

But these are the equations derivable by means of the principle of least action which Mr. Planck had used as his starting point. [92]

V. PRINCIPLE OF RELATIVITY AND GRAVITATION

§17. *Accelerated reference system and gravitational field*

So far we have applied the principle of relativity, i.e., the assumption that the physical laws are independent of the state of motion of the reference system, only to *nonaccelerated* reference systems. Is it conceivable that the principle of relativity also applies to systems that are accelerated relative to each other?

While this is not the place for a detailed discussion of this question, it will occur to anybody who has been following the applications of the principle of relativity. Therefore I will not refrain from taking a stand on this question here.

We consider two systems Σ_1 and Σ_2 in motion. Let Σ_1 be accelerated
[93] in the direction of its X-axis, and let γ be the (temporally constant) magnitude of that acceleration. Σ_2 shall be at rest, but it shall be located in a homogeneous gravitational field that imparts to all objects an acceleration $-\gamma$ in the direction of the X-axis.

[94] As far as we know, the physical laws with respect to Σ_1 do not differ from those with respect to Σ_2; this is based on the fact that all bodies are equally accelerated in the gravitational field. At our present state of experience we have thus no reason to assume that the systems Σ_1 and Σ_2 differ from each other in any respect, and in the discussion that follows, we shall therefore assume the complete physical equivalence of a gravitational field and a corresponding acceleration of the reference system.

This assumption extends the principle of relativity to the uniformly accelerated translational motion of the reference system. The heuristic value of this assumption rests on the fact that it permits the replacement of a homogeneous gravitational field by a uniformly accelerated reference system, the latter case being to some extent accessible to theoretical treatment.

§18. *Space and time in a uniformly accelerated reference system*

We first consider a body whose individual material points, at a given time t of the nonaccelerated reference system S, possess no velocity relative to S, but a certain acceleration. What is the influence of this acceleration γ on the shape of the body with respect to S?

If such an influence is present, it will consist of a constant-ratio dilatation in the direction of acceleration and possibly in the two directions perpendicular to it, since an effect of another kind is impossible for reasons of symmetry. The acceleration-caused dilatations (if such exist at all) must be even functions of γ; hence they can be neglected if one restricts oneself to the case in which γ is so small that terms of the second or higher power in γ may be neglected. Since we are going to restrict ourselves to that case, we do not have to assume that the acceleration has any influence on the shape of the body.

We now consider a reference system Σ that is uniformly accelerated relative to the nonaccelerated system S in the direction of the latter's X-axis. The clocks and measuring rods of Σ, examined at rest, shall be identical with the clocks and measuring rods of S. The coordinate origin of Σ shall move along the X-axis of S, and the axes of Σ shall be perpetually parallel to those of S. At any moment there exists a nonaccelerated reference system S' whose coordinate axes coincide with the coordinate axes of Σ at the moment in question (at a given time t' of S'). If the coordinates of a point event occurring at this time t' are ξ, η, ζ with respect to Σ, we will have

$$\left.\begin{aligned} x' &= \xi \\ y' &= \eta \\ z' &= \zeta \end{aligned}\right\},$$

because in accordance with what we said above, we are not to assume that acceleration affects the shape of the measuring instruments used for measuring ξ, η, ζ. We shall also imagine that the clocks of Σ are set at time t' of S' such that their readings at that moment equal t'. What about the rate of the clocks in the next time element τ?

First of all, we have to bear in mind that a specific effect of *acceleration* on the rate of the clocks of Σ need not be taken into account, since it would have to be of the order γ^2. Furthermore, since the effect of the velocity attained during τ on the rate of the clocks is negligible, and the distances traveled by the clocks during the time τ relative to those traveled by S' are also of the order τ^2, i.e., negligible, the readings of the clocks of Σ may be fully replaced by readings of the clocks of S' for the time element τ. [95]

From the foregoing it follows that, relative to Σ, light in vacuum is propagated during the time element τ with the universal velocity c if we define simultaneity in the system S' which is momentarily at rest relative to Σ, and if the clocks and measuring rods we use for measuring the time and length are identical with those used for the measurement of time and space in nonaccelerated systems. Thus the principle of constancy of the velocity of light can be used here too to define simultaneity if one restricts oneself to very short light paths.

We now imagine that the clocks of Σ are adjusted, in the way described, at that time $t = 0$ of S at which Σ is instantaneously at rest relative to S. The totality of readings of the clocks of Σ adjusted in
[96] this way is called the "local time" σ of the system Σ. It is immediately evident that the physical meaning of the local time σ is as follows. If one uses the local time σ for the temporal evaluation of processes occurring in the individual space elements of Σ, then the laws obeyed by these processes cannot depend on the position of these space elements, i.e., on their coordinates, if not only the clocks, but also the other measuring tools used in the
[97] various space elements are identical.

However, we must not simply refer to the local time σ as the "time" of Σ, because according to the definition given above, two point events occurring at different points of Σ are not simultaneous when their local times σ are equal. For if at time $t = 0$ two clocks of Σ are synchronous with respect to S and are subjected to the same motions, then they remain forever synchronous with respect to S. However, for this reason, in accordance with §4, they do not run synchronously with respect to a reference system S' instantaneously at rest relative to Σ but in motion relative to S, and hence according to our definition they do not run synchronously with respect to Σ either.

We now define the "time" τ of the system Σ as the totality of those readings of the clock situated at the coordinate origin of Σ which are, according to the above definition, simultaneous with the events which are to be temporally evaluated.[1]

We shall now determines the relation between the time τ and the local time σ of a point event. It follows from the first of equations (1) that

[1]Thus the symbol "τ" is used here in a different sense than above.

two events are simultaneous with respect to S', and thus also with respect to Σ, if

$$t_1 - \frac{v}{c^2} x_1 = t_2 - \frac{v}{c^2} x_2 ,$$

where the subscripts refer to the one or to the other point event, respectively. We shall first confine ourselves to the consideration of times that are so short[1] that all terms containing the second or higher power of τ or v can be omitted; taking (1) and (29) into account, we then have to put [98]

$$\begin{gathered} x_2 - x_1 = x_2' - x_2' = \xi_2 - \xi_1 \\ t_1 = \sigma_1 \qquad\qquad t_2 = \sigma_2 \\ v = \gamma t = \gamma\tau , \end{gathered}$$ [99]

so that we obtain from the above equation

$$\sigma_2 - \sigma_1 = \frac{\gamma\tau}{c^2} (\xi_2 - \xi_1) .$$

If we move the first point event to the coordinate origin, so that $\sigma_1 = \tau$ and $\xi_1 = 0$, we obtain, omitting the subscript for the second point event,

$$\sigma = \tau\left[1 + \frac{\gamma\xi}{c^2}\right] . \tag{30}$$

This equation holds first of all if τ and ξ lie below certain limits. It is obvious that it holds for arbitrarily large τ if the acceleration γ is constant with respect to Σ, because the relation between σ and τ must then be linear. Equation (30) does not hold for arbitrarily large ξ. From the fact that the choice of the coordinate origin must not affect the relation, one must conclude that, strictly speaking, equation (30) should be replaced by the equation

$$\sigma = \tau e^{\frac{\gamma\xi}{c^2}} .$$

Nevertheless, we shall maintain formula (30).

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[1]In accordance with (1), we thereby also assume a certain restriction with respect to the values of $\xi = x'$.

According to §17, equation (30) is also applicable to a coordinate system in which a homogeneous gravitational field is acting. In that case we have to put $\Phi = \gamma\xi$, where Φ is the gravitational potential, so that we obtain

$$\sigma = \tau\left[1 + \frac{\Phi}{c^2}\right] . \tag{30a}$$

We have defined two kinds of times for Σ. Which of the two definitions do we have to use in the various cases? Let us assume that at two locations of different gravitational potentials $(\gamma\xi)$ there exists one physical system each, and we want to compare their physical quantities. To do this, the most natural procedure might be as follows: First we take our measuring tools to the first physical system and carry out our measurements there; then we take our measuring tools to the second system to carry out the same measurement here. If the two sets of measurements give the same results, we shall denote the two physical systems as "equal." The measuring tools include a clock with which we measure local times σ. From this it follows that to define the physical quantities at some position of the gravitational field, it is natural to use the time σ.

However, if we deal with a phenomenon in which objects situated at positions with different gravitational potentials must be considered simultaneously, we have to use the time τ in those terms in which time occurs explicitly (i.e., not only in the definition of physical quantities), because otherwise the simultaneity of the events would not be expressed by the equality of the time values of the two events. Since in the definition of the time τ a clock situated in an arbitrarily chosen position is used, but not an arbitrarily chosen instant, when using time τ the laws of nature can vary with position but not with time.

§19. *The effect of the gravitational field on clocks*

If a clock showing local time is located in a point P of gravitational potential Φ, then, according to (30a), its reading will be $(1 + \frac{\Phi}{c^2})$ times greater than the time τ, i.e., it runs $(1 + \frac{\Phi}{c^2})$ times faster than an identical clock located at the coordinate origin. Suppose an observer located somewhere in space perceives the indications of the two clocks in a certain way, e.g., optically. As the time $\Delta\tau$ that elapses between the instants at which a clock indication occurs and at which this indication is perceived by the observer is independent of τ, for an observer situated somewhere in space the clock in point P runs $(1 + \frac{\Phi}{c^2})$ times faster than the clock at the coordinate origin. In this sense we may say that the process occurring in the clock, and, more generally, any physical process, proceeds faster the greater the gravitational potential at the position of the process taking place.

There exist "clocks" that are present at locations of different gravitational potentials and whose rates can be controlled with great precision; these are the producers of spectral lines. It can be concluded from the aforesaid[1] that the wave length of light coming from the sun's surface, which originates from such a producer, is larger by about one part in two millionth than that of light produced by the same substance on earth. [100]

§20. *The effect of gravitation on electromagnetic phenomena*

If we refer an electromagnetic process at some point of time to a non-accelerated reference system S' that is instantaneously at rest relative to the reference system Σ accelerated as above, then the following equations will hold according to (5) and (6):

$$\frac{1}{c}\left[\rho' u'_x + \frac{\partial X'}{\partial t'}\right] = \frac{\partial N'}{\partial y'} - \frac{\partial M'}{\partial z'}\ , \text{ etc.}$$

and

$$\frac{1}{c}\frac{\partial L'}{\partial t'} = \frac{\partial Y'}{\partial z'} - \frac{\partial Z'}{\partial y'}\ , \text{ etc.}$$

In accordance with the above, we may readily equate the S'-referred quantities ρ', u', X', L', x', etc., with the corresponding Σ-referred

[1]While assuming that equation (30a) holds for an inhomogeneous gravitational field as well.

quantities ρ, u, X, L, ξ, etc., if we limit ourselves to an infinitesimally short period[1] that is infinitesimally close to the time of relative rest of S' and Σ. Further, we have to replace t' by the local time σ. However, we must not simply put

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial \sigma} ,$$

because a point which is at rest relative to Σ, and to which equations transformed to Σ should refer, changes its velocity relative to S' during the time element $dt' = d\sigma$, to which change, according to equations (7a) and (7b), there corresponds a temporal change of the Σ-related field component. Hence we have to put

$$\frac{\partial X'}{\partial t'} = \frac{\partial X}{\partial \sigma} \qquad \frac{\partial L'}{\partial t'} = \frac{\partial L}{\partial \sigma}$$

$$\frac{\partial Y'}{\partial t'} = \frac{\partial Y}{\partial \sigma} + \frac{\gamma}{c} N \qquad \frac{\partial M'}{\partial t'} = \frac{\partial M}{\partial \sigma} - \frac{\gamma}{c} Z$$

$$\frac{\partial Z'}{\partial t'} = \frac{\partial Z}{\partial \sigma} - \frac{\gamma}{c} M \qquad \frac{\partial N'}{\partial t'} = \frac{\partial N}{\partial \sigma} + \frac{\gamma}{c} Y .$$

Hence the Σ-referred electromagnetic equations are

$$\frac{1}{c}\left[\rho u_{\xi} + \frac{\partial X}{\partial \sigma}\right] = \frac{\partial N}{\partial \eta} - \frac{\partial M}{\partial \zeta}$$

$$\frac{1}{c}\left[\rho u_{\eta} + \frac{\partial Y}{\partial \sigma} + \frac{\gamma}{c} N\right] = \frac{\partial L}{\partial \zeta} - \frac{\partial N}{\partial \xi}$$

$$\frac{1}{c}\left[\rho u_{\xi} + \frac{\partial Z}{\partial \sigma} - \frac{\gamma}{c} M\right] = \frac{\partial M}{\partial \xi} - \frac{\partial L}{\partial \eta}$$

$$\frac{1}{c} \frac{\partial L}{\partial \sigma} = \frac{\partial Y}{\partial \zeta} - \frac{\partial Z}{\partial \eta}$$

$$\frac{1}{c}\left[\frac{\partial M}{\partial \sigma} - \frac{\gamma}{c} Z\right] = \frac{\partial Z}{\partial \xi} - \frac{\partial X}{\partial \zeta}$$

$$\frac{1}{c}\left[\frac{\partial N}{\partial \sigma} + \frac{\gamma}{c} Y\right] = \frac{\partial X}{\partial \eta} - \frac{\partial Y}{\partial \xi} .$$

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[1]This restriction does not affect the range of validity of our results because inherently the laws to be derived cannot depend on the time.

We multiply these equations by $\left[1 + \frac{\gamma\xi}{c^2}\right]$ and put for the sake of brevity

$$X^* = X\left[1 + \frac{\gamma\xi}{c^2}\right], \qquad Y^* = Y\left[1 + \frac{\gamma\xi}{c^2}\right] \text{ , etc.}$$

$$\rho^* = \rho\left[1 + \frac{\gamma\xi}{c^2}\right] .$$

Neglecting terms of the second power in γ, we obtain the equations

$$\left.\begin{aligned} \frac{1}{c}\left[\rho^* u_\xi + \frac{\partial X^*}{\partial\sigma}\right] &= \frac{\partial N^*}{\partial\eta} - \frac{\partial M^*}{\partial\zeta} \\ \frac{1}{c}\left[\rho^* u_\eta + \frac{\partial Y^*}{\partial\sigma}\right] &= \frac{\partial L^*}{\partial\zeta} - \frac{\partial N^*}{\partial\xi} \\ \frac{1}{c}\left[\rho^* u_\zeta + \frac{\partial Z^*}{\partial\sigma}\right] &= \frac{\partial M^*}{\partial\xi} - \frac{\partial L^*}{\partial\eta} \end{aligned}\right\} \qquad (31a)$$

$$\left.\begin{aligned} \frac{1}{c}\frac{\partial L^*}{\partial\sigma} &= \frac{\partial Y^*}{\partial\zeta} - \frac{\partial Z^*}{\partial\eta} \\ \frac{1}{c}\frac{\partial M^*}{\partial\sigma} &= \frac{\partial Z^*}{\partial\xi} - \frac{\partial X^*}{\partial\zeta} \\ \frac{1}{c}\frac{\partial N^*}{\partial\sigma} &= \frac{\partial X^*}{\partial\eta} - \frac{\partial Y^*}{\partial\xi} . \end{aligned}\right\} \qquad (32a)$$

These equations show first of all how the gravitational field affects the static and stationary phenomena. The same laws hold as in the gravitation-free field, except that the field components X, etc. are replaced by $X\left[1 + \frac{\gamma\xi}{c^2}\right]$, etc., and ρ is replaced by $\rho\left[1 + \frac{\gamma\xi}{c^2}\right]$.

Furthermore, to follow the development of nonstationary states, we make use of the time τ in the terms differentiated with respect to time as well as in the definition of the velocity of electricity, i.e., we put according to (30)

$$\frac{\partial}{\partial\tau} = \left[1 + \frac{\gamma\xi}{c^2}\right] \frac{\partial}{\partial\tau} \qquad [101]$$

and

$$w_\xi = \left[1 + \frac{\gamma\xi}{c^2}\right] . \qquad [102]$$

We thus obtain

$$\frac{1}{c\left[1+\frac{\gamma\xi}{c^2}\right]}\left[\rho^* w_\xi + \frac{\partial X^*}{\partial \tau}\right] = \frac{\partial N^*}{\partial \eta} - \frac{\partial M^*}{\partial \zeta} \quad \text{etc.} \tag{31b}$$

and

[103]
$$\frac{1}{c\left[1+\frac{\gamma\xi}{c^2}\right]}\frac{\partial L^*}{\partial \tau} = \frac{\partial Y^*}{\partial \zeta} = \frac{\partial Z^*}{\partial \eta} \quad \text{etc.} \tag{32b}$$

These equations too have the same form as the corresponding equations of the nonaccelerated or gravitation-free space; however, c is here replaced by the value

$$c\left[1+\frac{\gamma\xi}{c^2}\right] = c\left[1+\frac{\Phi}{c^2}\right] .$$

From this it follows that those light rays that do not propagate along the ξ-axis are bent by the gravitational field; it can easily be seen that the change of direction amounts to $\frac{\gamma}{c^2}\sin\varphi$ per cm light path, where φ
[104] denotes the angle between the direction of gravity and that of the light ray.

With the help of these equations and the equations relating the field strength and the electric current of one point, which are known from the optics of bodies at rest, we can calculate the effect of the gravitational field on optical phenomena in bodies at rest. One has to bear in mind, however, that the above-mentioned equations from the optics of bodies at rest hold for the local time σ. Unfortunately, the effect of the terrestrial gravitational field is so small according to our theory (because of the smallness of $\frac{\gamma x}{c^2}$) that there is no prospect of a comparison of the results of
[105] the theory with experience.

If we successively multiply equations (31a) and (32a) by $\frac{X^*}{4\pi} \cdots\cdots \frac{N^*}{4\pi}$ and integrate over infinite space, we obtain, using our earlier notation,

[106]
$$\int\left[1+\frac{\gamma\xi}{c^2}\right]^2\frac{\rho}{4\pi}(uX+u_\eta Y+uZ)\,d\omega+\int\left[1+\frac{\gamma\xi}{c^2}\right]^2\cdot\frac{1}{8\pi}\frac{\partial}{\partial\sigma}(X^2+Y^2+\cdots+N^2)\,d\omega = 0 .$$

[107] $\frac{\rho}{4\pi}(uX + u_\eta Y + u_\xi Z)$ is the energy η_σ supplied to the matter per unit volume and unit local time σ if this energy is measured by measuring tools situated at the corresponding location. Hence, according to (30),

$\eta_\tau = \eta^\sigma\left[1 + \frac{\gamma\xi}{c^2}\right]$ is the (similarly measured) energy supplied to the matter per [108]
unit volume and unit local time τ; $\frac{1}{8\pi}(X^2 + Y^2 \cdots + N^2)$ is the electromagnetic energy ϵ per unit volume, measured the same way. If we take into account that according to (30) we have to set $\frac{\partial}{\partial\sigma} = \left[1 - \frac{\gamma\xi}{c^2}\right]\frac{\partial}{\partial\tau}$, we obtain

$$\int \left[1 + \frac{\gamma\xi}{c^2}\right]\eta_\tau d\omega + \frac{d}{d\tau}\left\{\int \left[1 + \frac{\gamma\xi}{c^2}\right]\epsilon \; d\omega\right\} = 0 \; .$$

This equation expresses the principle of conservation of energy and contains a very remarkable result. An energy, or energy input, that, measured locally, has the value $E = \epsilon d\omega$ or $E = \eta \; d\omega d\tau$, respectively, contributes to the energy integral, in addition to the value E that corresponds to its magnitude, also a value $\frac{E}{c^2}\gamma\xi = \frac{E}{c^2}\Phi$ that corresponds to its *position*. Thus, to each energy E in the gravitational field there corresponds an energy of position that equals the potential energy of a "ponderable" mass of magnitude $\frac{E}{c^2}$.

Thus the proposition derived in §11, that to an amount of energy E there corresponds a mass of magnitude $\frac{E}{c^2}$, holds not only for the *inertial* but also for the *gravitational* mass, if the assumption introduced in §17 is correct.

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Peter Havas, CONSULTANT