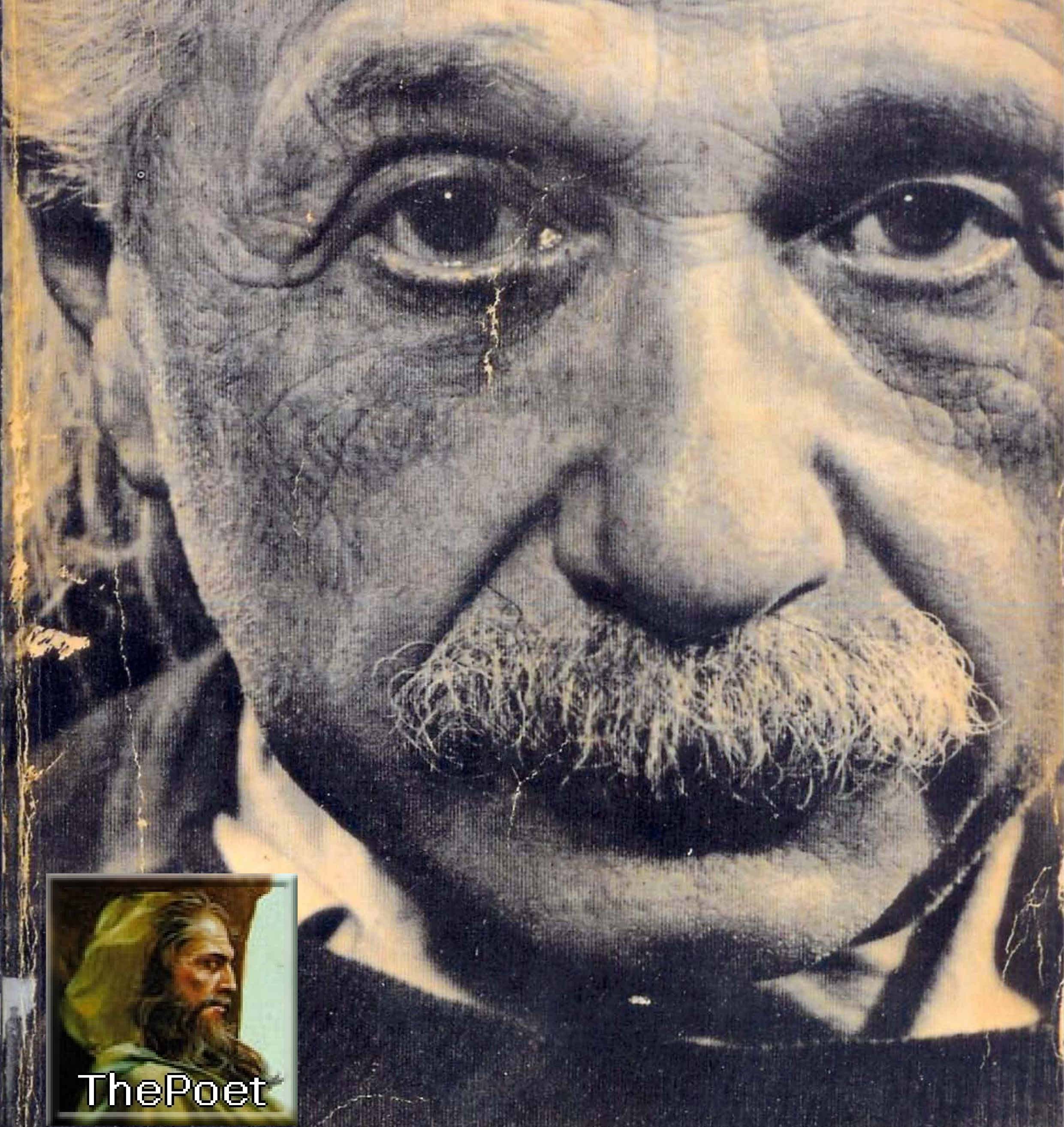

IDEAS AND OPINIONS

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ALBERT EINSTEIN

Ideas and Opinions

by

Albert Einstein

Based on MEIN WELTBILD,
edited by Carl Seelig,
and other sources

New translations and revisions
by Sonja Bargmann

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MESSAGE IN THE TIME-CAPSULE

World's Fair, 1939.

Our time is rich in inventive minds, the inventions of which could facilitate our lives considerably. We are crossing the seas by power and utilize power also in order to relieve humanity from all tiring muscular work. We have learned to fly and we are able to send messages and news without any difficulty over the entire world through electric waves.

However, the production and distribution of commodities is entirely unorganized so that everybody must live in fear of being eliminated from the economic cycle, in this way suffering for the want of everything. Furthermore, people living in different countries kill each other at irregular time intervals, so that also for this reason anyone who thinks about the future must live in fear and terror. This is due to the fact that the intelligence and character of the masses are incomparably lower than the intelligence and character of the few who produce something valuable for the community.

I trust that posterity will read these statements with a feeling of proud and justified superiority.

REMARKS ON BERTRAND RUSSELL'S THEORY OF KNOWLEDGE

From The Philosophy of Bertrand Russell, *Vol. V of "The Library of Living Philosophers," edited by Paul Arthur Schilpp, 1944. Translated from the original German by Paul Arthur Schilpp. Tudor Publishers.*

When the editor asked me to write something about Bertrand Russell, my admiration and respect for that author at once induced me to say yes. I owe innumerable happy hours to the reading of Russell's works, something which I cannot say of any other contemporary scientific writer, with the exception of Thorstein Veblen. Soon, however, I discovered that it is easier to give such a promise than to fulfill it. I had promised to say something about Russell as philosopher and epistemologist. After having in full confidence begun with it, I quickly recognized what a slippery field I had ventured upon, having, due to lack of experience, until now cautiously limited myself to the field of physics. The present difficulties of his science force the physicist to come to grips with philosophical problems to a greater degree than was the case with earlier generations. Although I shall not speak here of those difficulties, it was my concern with them, more than anything else, which led me to the position outlined in this essay.

In the evolution of philosophic thought through the centuries the following question has played a major rôle: what knowledge is pure thought able to supply independently of sense perception? Is there any such knowledge? If not, what precisely is the relation between our knowledge and the raw material furnished by sense impressions? An almost boundless chaos of philosophical opinions corresponds to these questions and to a few others intimately connected with them. Nevertheless there is visible in this process of relatively fruitless but heroic endeavors a systematic trend of development, namely, an increasing skepticism concerning every attempt by means of pure thought to learn something about the "objective world," about the world of "things" in contrast to the world of mere "concepts and ideas." Be it said parenthetically that, just as on the part of a real philosopher, quotation marks are used here to introduce an illegitimate concept, which the reader is asked to permit for the moment, although the concept is suspect in the eyes of the philosophical police.

During philosophy's childhood it was rather generally believed that it is possible to find everything which can be known by means of mere reflection. It was an illusion which anyone can easily understand if, for a moment, he dismisses what he has learned from later philosophy and from natural science; he will not be surprised to find that Plato ascribed a higher reality to "ideas" than to empirically experienceable things. Even in Spinoza and as late as in Hegel this prejudice was the vitalizing force which seems still to have played the major rôle. Someone, indeed, might even raise the question whether, without something of this illusion, anything really great can be achieved in the realm of philosophic thought—but we do not wish to ask this question.

This more aristocratic illusion concerning the unlimited penetrative power of thought has as its counterpart the more plebeian illusion of naïve realism, according to which things "are" as they are perceived by us through our senses. This illusion dominates the daily life of men and of animals; it is also the point of departure in all of the sciences, especially of the natural sciences.

These two illusions cannot be overcome independently. The overcoming of naïve realism has been relatively simple. In his introduction to his volume, *An Inquiry Into Meaning and Truth,* Russell has characterized this process in a marvelously concise fashion:

> We all start from "naïve realism," i.e., the doctrine that things are what they seem. We think that grass is green, that stones are hard, and that snow is cold. But physics assures us that the greenness of grass, the hardness of stones, and the coldness of snow are not the greenness, hardness, and coldness that we know in our own experience, but something very different. The observer, when he seems to himself to be observing a stone, is really, if physics is to be believed, observing the effects of the stone upon himself. Thus science seems to be at war with itself: when it most means to be objective, it finds itself plunged into subjectivity against its will. Naïve realism leads to physics, and physics, if true, shows that naïve realism is false. Therefore naïve realism, if true, is false; therefore it is false. (pp. 14–15)

Apart from their masterful formulation these lines say something which had never previously occurred to me. For, superficially considered, the mode of thought in Berkeley and Hume seems to stand in contrast to the mode of thought in the natural sciences. However, Russell's just cited remark uncovers a connection: if Berkeley relies upon the fact that we do not directly grasp the "things" of the external world through our senses, but that only events causally connected with the presence of "things" reach our sense organs, then this is a consideration which gets its persuasive character from our confidence in the physical mode of thought. For, if one doubts the physical mode of thought in even its most general features, there is no necessity to interpolate between the object and the act of vision anything which separates the object from the subject and makes the "existence of the object" problematical.

It was, however, the very same physical mode of thought and its practical successes which have shaken the confidence in the possibility of understanding things and their relations by means of purely speculative thought. Gradually the conviction gained recognition that all knowledge about things is exclusively a working-over of the raw material furnished by the senses. In this general (and intentionally somewhat vaguely stated) form this sentence is probably today commonly accepted. But *this conviction does not rest on the supposition that anyone has* actually proved the impossibility of gaining knowledge of reality by means of pure speculation, but rather upon the fact that the empirical (in the above-mentioned sense) procedure alone has shown its capacity to be the source of knowledge. Galileo and Hume first upheld this principle with full clarity and decisiveness.

Hume saw that concepts which we must regard as essential, such as, for example, causal connection, cannot be gained from material given to us by the senses. This insight led him to a skeptical attitude as concerns knowledge of any kind. If one reads Hume's books, one is amazed that many and sometimes even highly esteemed philosophers after him have been able to write so much obscure stuff and even find grateful readers for it. Hume has permanently influenced the development of the best of philosophers who came after him. One senses him in the reading of Russell's philosophical analyses, whose acumen and simplicity of expression have often reminded me of Hume.

Man has an intense desire for assured knowledge. That is why Hume's clear message seemed crushing: the sensory raw material, the only source of our knowledge, through habit may lead us to belief and expectation but not to the knowledge and still less to the understanding of lawful relations. Then Kant took the stage with an idea which, though certainly untenable in the form in which he put it, signified a step towards the solution of Hume's dilemma: whatever in knowledge is of empirical origin is never certain (Hume). If, therefore, we have definitely assured knowledge, it must be grounded in reason itself. This is held to be the case, for example, in the propositions of geometry and in the principle of causality. These and certain other types of knowledge are, so to speak, a part of the implements of thinking and therefore do not previously have to be gained from sense data (i.e., they are *a priori* knowledge). Today everyone knows, of course, that the mentioned concepts contain nothing of the certainty, of the inherent necessity, which Kant had attributed to them. The following, however, appears to me to be correct in Kant's statement of the problem: in thinking we use, with a certain "right," concepts to which there is no access from the materials of sensory experience, if the situation is viewed from the logical point of view.

As a matter of fact, I am convinced that even much more is to be asserted: the concepts which arise in our thought and in our linguistic expressions are all—when viewed logically—the free creations of thought which cannot inductively be gained from sense experiences. This is not so easily noticed only because we have the habit of combining certain concepts and conceptual relations (propositions) so definitely with certain sense experiences that we do not become conscious of the gulf—logically unbridgeable—which separates the world of sensory experiences from the world of concepts and propositions.

Thus, for example, the series of integers is obviously an invention of the human mind, a self-created tool which simplifies the ordering of certain sensory experiences. But there is no way in which this concept could be made to grow, as it were, directly out of sense experiences. It is deliberately that I choose here the concept of number, because it belongs to pre-scientific thinking and because, in spite of that fact, its constructive character is still easily recognizable. The more, however, we turn to the most primitive concepts of everyday life, the more difficult it becomes amidst the mass of inveterate habits to recognize the concept as an independent creation of thinking. It was thus that the fateful conception—fateful, that is to say, for an understanding of the here-existing conditions—could arise, according to which the concepts originate from experience by way of "abstraction," i.e., through omission of a part of its content. I want to indicate now why this conception appears to me to be so fateful.

As soon as one is at home in Hume's critique one is easily led to believe that all those concepts and propositions which cannot be deduced from the sensory raw material are, on account of their "metaphysical" character, to be removed from thinking. For all thought acquires material content only through its relationship with that sensory material. This latter proposition I take to be entirely true; but I hold the prescription for thinking which is grounded on this proposition to be false. For this claim—if only carried through consistently—absolutely excludes thinking of any kind as "metaphysical."

In order that thinking might not degenerate into "metaphysics," or into empty talk, it is only necessary that enough propositions of the conceptual system be firmly enough connected with sensory experiences and that the conceptional system, in view of its task of ordering and surveying sense experience, should show as much unity and parsimony as possible. Beyond that, however, the "system" is (as regards logic) a free play with symbols according to (logically) arbitrarily given rules of the game. All this applies as much (and in the same manner) to the thinking in daily life as to the more consciously and systematically constructed thinking in the sciences.

It will now be clear what is meant if I make the following statement: by his clear critique Hume did not only advance philosophy in a decisive way but also—though through no fault of his—created a danger for philosophy in that, following his critique, a fateful "fear of metaphysics" arose which has come to be a malady of contemporary empiricistic philosophizing; this malady is the counterpart to that earlier philosophizing in the clouds, which thought it could neglect and dispense with what was given by the senses.

No matter how much one may admire the acute analysis which Russell has given us in his latest book on *Meaning and Truth,* it still seems to me that even there the specter of the metaphysical fear has caused some damage. For this fear seems to me, for example, to be the cause for conceiving of the "thing" as a "bundle of qualities," such that the "qualities" are to be taken from the sensory raw material. Now the fact that two things are said to be one and the same thing, if they coincide in all qualities, forces one to consider the geometrical relations between things as belonging to their qualities. (Otherwise one is forced to look upon the Eiffel Tower in Paris and a New York skyscraper as "the same thing.")* However, I see no "metaphysical" danger in taking the thing (the object in the sense of physics) as an independent concept into the system together with the proper spatio-temporal structure.

In view of these endeavors I am particularly pleased to note that, in the last chapter of the book, it finally turns out that one can, after all, not get along without "metaphysics." The only thing to which I take exception there is the bad intellectual conscience which shines through between the lines.

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* Compare Russell's *An Inquiry Into Meaning and Truth,* 119–120, chapter on "Proper Names."

3. In a displacement of the spectral lines toward the red end of the spectrum in the case of light transmitted to us from stars of considerable magnitude (unconfirmed so far).*

The chief attraction of the theory lies in its logical completeness. If a single one of the conclusions drawn from it proves wrong, it must be given up; to modify it without destroying the whole structure seems to be impossible.

Let no one suppose, however, that the mighty work of Newton can really be superseded by this or any other theory. His great and lucid ideas will retain their unique significance for all time as the foundation of our whole modern conceptual structure in the sphere of natural philosophy.

Note: Some of the statements in your paper concerning my life and person owe their origin to the lively imagination of the writer. Here is yet another application of the principle of relativity for the delectation of the reader: today I am described in Germany as a "German savant," and in England as a "Swiss Jew." Should it ever be my fate to be represented as a *bête noire,* I should, on the contrary, become a "Swiss Jew" for the Germans and a "German savant" for the English.

GEOMETRY AND EXPERIENCE

Lecture before the Prussian Academy of Sciences, January 27, 1921. The last part appeared first in a reprint by Springer, Berlin, 1921.

One reason why mathematics enjoys special esteem, above all other sciences, is that its propositions are absolutely certain and indisputable, while those of all other sciences are to some extent debatable and in constant danger of being overthrown by newly discovered facts. In spite of this, the investigator in

* This criterion has since been confirmed.

another department of science would not need to envy the mathematician if the propositions of mathematics referred to objects of our mere imagination, and not to objects of reality. For it cannot occasion surprise that different persons should arrive at the same logical conclusions when they have already agreed upon the fundamental propositions (axioms), as well as the methods by which other propositions are to be deduced therefrom. But there is another reason for the high repute of mathematics, in that it is mathematics which affords the exact natural sciences a certain measure of certainty, to which without mathematics they could not attain.

At this point an enigma presents itself which in all ages has agitated inquiring minds. How can it be that mathematics, being after all a product of human thought which is independent of experience, is so admirably appropriate to the objects of reality? Is human reason, then, without experience, merely by taking thought, able to fathom the properties of real things?

In my opinion the answer to this question is, briefly, this: as far as the propositions of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality. It seems to me that complete clarity as to this state of things became common property only through that trend in mathematics which is known by the name of "axiomatics." The progress achieved by axiomatics consists in its having neatly separated the logical-formal from its objective or intuitive content; according to axiomatics the logical-formal alone forms the subject matter of mathematics, which is not concerned with the intuitive or other content associated with the logical-formal.

Let us for a moment consider from this point of view any axiom of geometry, for instance, the following: through two points in space there always passes one and only one straight line. How is this axiom to be interpreted in the older sense and in the more modern sense?

The older interpretation: everyone knows what a straight line is, and what a point is. Whether this knowledge springs from an ability of the human mind or from experience, from some cooperation of the two or from some other source, is not for the mathematician to decide. He leaves the question to the philosopher. Being based upon this knowledge, which precedes all mathematics, the axiom stated above is, like all other axioms, self-evident, that is, it is the expression of a part of this *a priori* knowledge.

The more modern interpretation: geometry treats of objects which are denoted by the words straight line, point, etc. No knowledge or intuition of these objects is assumed but only the validity of the axioms, such as the one stated above, which are to be taken in a purely formal sense, i.e., as void of all content of intuition or experience. These axioms are free creations of the human mind. All other propositions of geometry are logical inferences from the axioms (which are to be taken in the nominalistic sense only). The axioms *define* the objects of which geometry treats. Schlick in his book on epistemology has therefore characterized axioms very aptly as "implicit definitions."

This view of axioms, advocated by modern axiomatics, purges mathematics of all extraneous elements, and thus dispels the mystic obscurity which formerly surrounded the basis of mathematics. But such an expurgated exposition of mathematics makes it also evident that mathematics as such cannot predicate anything about objects of our intuition or real objects. In axiomatic geometry the words "point," "straight line," etc., stand only for empty conceptual schemata. That which gives them content is not relevant to mathematics.

Yet on the other hand it is certain that mathematics generally, and particularly geometry, owes its existence to the need which was felt of learning something about the behavior of real objects. The very word geometry, which, of course, means earth-measuring, proves this. For earth-measuring has to do with the possibilities of the disposition of certain natural objects with respect to one another, namely, with parts of the earth, measuring-lines, measuring-wands, etc. It is clear that the system of concepts of axiomatic geometry alone cannot make any assertions as to the behavior of real objects of this kind, which we will call practically-rigid bodies. To be able to make such assertions, geometry must be stripped of its merely logical-formal character by the coordination of real objects of experience with the empty conceptual schemata of axiomatic geometry. To accomplish this, we need only add the proposition: solid bodies are related, with respect to their possible dispositions, as are bodies in Euclidean geometry of three dimensions. Then the propositions of Euclid contain affirmations as to the behavior of practically-rigid bodies.

Geometry thus completed is evidently a natural science; we may in fact regard it as the most ancient branch of physics. Its affirmations rest essentially on induction from experience, but not on logical inferences only. We will call this completed geometry "practical geometry," and shall distinguish it in what follows from "purely axiomatic geometry." The question whether the practical geometry of the universe is Euclidean or not has a clear meaning, and its answer can only be furnished by experience. All length-measurements in physics constitute practical geometry in this sense, so, too, do geodetic and astronomical length measurements, if one utilizes the empirical law that light is propagated in a straight line, and indeed in a straight line in the sense of practical geometry.

I attach special importance to the view of geometry which I have just set forth, because without it I should have been unable to formulate the theory of relativity. Without it the following reflection would have been impossible: in a system of reference rotating relatively to an inertial system, the laws of disposition of rigid bodies do not correspond to the rules of Euclidean geometry on account of the Lorentz contraction; thus if we admit non-inertial systems on an equal footing, we must abandon Euclidean geometry. Without the above interpretation the decisive step in the transition to generally covariant equations would certainly not have been taken. If we reject the relation between the body of axiomatic Euclidean geometry and the practically-rigid body of reality, we readily arrive at the following view, which was entertained by that acute and profound thinker, H. Poincaré: Euclidean geometry is distinguished above all other conceivable axiomatic geometries by its simplicity. Now since axiomatic geometry by itself contains no assertions as to the reality which can be experienced, but can do so only in combination with physical laws, it should be possible and reasonable—whatever may be the nature of reality—to retain Euclidean geometry. For if contradictions between theory and experience manifest themselves, we should rather decide to change physical laws than to change axiomatic Euclidean geometry. If we reject the relation between the practically-rigid body and geometry, we shall indeed not easily free ourselves from the convention that Euclidean geometry is to be retained as the simplest.

Why is the equivalence of the practically-rigid body and the body of geometry—which suggests itself so readily—rejected by Poincaré and other investigators? Simply because under closer inspection the real solid bodies in nature are not rigid, because their geometrical behavior, that is, their possibilities of relative disposition, depend upon temperature, external forces, etc. Thus the original, immediate relation between geometry and physical reality appears destroyed, and we feel impelled toward the following more general view, which characterizes Poincaré's standpoint. Geometry (G) predicates nothing about the behavior of real things, but only geometry together with the totality (P) of physical laws can do so. Using symbols, we may say that only the sum of (G) + (P) is subject to experimental verification. Thus (G) may be chosen arbitrarily, and also parts of (P); all these laws are conventions. All that is necessary to avoid contradictions is to choose the remainder of (P) so that (G) and the whole of (P) are together in accord with experience. Envisaged in this way, axiomatic geometry and the part of natural law which has been given a conventional status appear as epistemologically equivalent.

Sub specie aeterni Poincaré, in my opinion, is right. The idea of the measuring-rod and the idea of the clock coordinated with it in the theory of relativity do not find their exact correspondence in the real world. It is also clear that the solid body and the clock do not in the conceptual edifice of physics play the part of irreducible elements, but that of composite structures, which must not play any independent part in theoretical physics. But it is my conviction that in the present stage of development of theoretical physics these concepts must still be employed as independent concepts; for we are still far from possessing such certain knowledge of the theoretical principles of atomic structure as to be able to construct solid bodies and clocks theoretically from elementary concepts.

Further, as to the objection that there are no really rigid bodies in nature, and that therefore the properties predicated of rigid bodies do not apply to physical reality—this objection is by no means so radical as might appear from a hasty examination. For it is not a difficult task to determine the physical state of a measuring-body so accurately that its behavior relative to other measuring-bodies shall be sufficiently free from ambiguity to allow it to be substituted for the "rigid" body. It is to measuring-bodies of this kind that statements about rigid bodies must be referred.

All practical geometry is based upon a principle which is accessible to experience, and which we will now try to realize. Suppose two marks have been put upon a practically-rigid body. A pair of two such marks we shall call a tract. We imagine two practically-rigid bodies, each with a tract marked out on it. These two tracts are said to be "equal to one another" if the marks of the one tract can be brought to coincide permanently with the marks of the other. We now assume that:

If two tracts are found to be equal once and anywhere, they are equal always and everywhere.

Not only the practical geometry of Euclid, but also its nearest generalization, the practical geometry of Riemann, and therewith the general theory of relativity, rest upon this assumption. Of the experimental reasons which warrant this assumption I will mention only one. The phenomenon of the propagation of light in empty space assigns a tract, namely, the appropriate path of light, to each interval of local time, and conversely. Thence it follows that the above assumption for tracts must also hold good for intervals of clock-time in the theory of relativity. Consequently it may be formulated as follows: if two ideal clocks are going at the same rate at any time and at any place (being then in immediate proximity to each other), they will always go at the same rate, no matter where and when they are again compared with each other at one place. If this law were not valid for natural clocks, the proper frequencies for the separate atoms of the same chemical element would not be in such exact agreement as experience demonstrates. The existence of sharp spectral lines is a convincing experimental proof of the above-mentioned principle of practical geometry. This, in the last analysis, is the reason which enables us to speak meaningfully of a Riemannian metric of the four-dimensional space-time continuum.

According to the view advocated here, the question whether this continuum has a Euclidean, Riemannian, or any other structure is a question of physics proper which must be answered by experience, and not a question of a convention to be chosen on grounds of mere expediency. Riemann's geometry will hold if the laws of disposition of practically-rigid bodies approach those of Euclidean geometry the more closely the smaller the dimensions of the region of space-time under consideration.

It is true that this proposed physical interpretation of geometry breaks down when applied immediately to spaces of submolecular order of magnitude. But nevertheless, even in questions as to the constitution of elementary particles, it retains part of its significance. For even when it is a question of describing the electrical elementary particles constituting matter, the attempt may still be made to ascribe physical meaning to those field concepts which have been physically defined for the purpose of describing the geometrical behavior of bodies which are large as compared with the molecule. Success alone can decide as to the justification of such an attempt, which postulates physical reality for the fundamental principles of Riemann's geometry outside of the domain of their physical definitions. It might possibly turn out that this extrapolation has no better warrant than the extrapolation of the concept of temperature to parts of a body of molecular order of magnitude.

It appears less problematical to extend the concepts of practical geometry to spaces of cosmic order of magnitude. It might, of course, be objected that a construction composed of solid rods departs the more from ideal rigidity the greater its spatial extent. But it will hardly be possible, I think, to assign fundamental significance to this objection. Therefore the question whether the universe is spatially finite or not seems to me an entirely meaningful question in the sense of practical geometry. I do not even consider it impossible that this question will be answered before long by astronomy. Let us call to mind what the general theory of relativity teaches in this respect. It offers two possibilities:

1. The universe is spatially infinite. This is possible only if in the universe the average spatial density of matter, concentrated in the stars, vanishes, i.e., if the ratio of the total mass of the stars to the volume of the space through which they are scattered indefinitely approaches zero as greater and greater volumes are considered.

2. The universe is spatially finite. This must be so, if there exists an average density of the ponderable matter in the universe which is different from zero. The smaller that average density, the greater is the volume of the universe.

I must not fail to mention that a theoretical argument can be adduced in favor of the hypothesis of a finite universe. The general theory of relativity teaches that the inertia of a given body is greater as there are more ponderable masses in proximity to it; thus it seems very natural to reduce the total inertia of a body to interaction between it and the other bodies in the universe, as indeed, ever since Newton's time, gravity has been completely reduced to interaction between bodies. From the equations of the general theory of relativity it can be deduced that this total reduction of inertia to interaction between masses —as demanded by E. Mach, for example—is possible only if the universe is spatially finite.

Many physicists and astronomers are not impressed by this argument. In the last analysis, experience alone can decide which of the two possibilities is realized in nature. How can experience furnish an answer? At first it might seem possible to determine the average density of matter by observation of that part of the universe which is accessible to our observation. This hope is illusory. The distribution of the visible stars is extremely irregular, so that we on no account may venture to set the average density of star-matter in the universe equal to, let us say, the average density in the Galaxy. In any case, however great the space examined may be, we could not feel convinced that there were any more stars beyond that space. So it seems impossible to estimate the average density.

But there is another road, which seems to me more practicable, although it also presents great difficulties. For if we inquire into the deviations of the consequences of the general theory of relativity which are accessible to experience, from the consequences of the Newtonian theory, we first of all find a deviation which manifests itself in close proximity to gravitating mass, and has been confirmed in the case of the planet Mercury. But if the universe is spatially finite, there is a second deviation from the Newtonian theory, which, in the language of the Newtonian theory, may be expressed thus: the gravitational field is such as if it were produced, not only by the ponderable masses, but in addition by a mass-density of negative sign, distributed uniformly throughout space. Since this fictitious mass-density would have to be extremely small, it would be noticeable only in very extensive gravitating systems.

Assuming that we know, let us say, the statistical distribution and the masses of the stars in the Galaxy, then by Newton's law we can calculate the gravitational field and the average velocities which the stars must have, so that the Galaxy should not collapse under the mutual attraction of its stars, but should maintain its actual extent. Now if the actual velocities of the stars—which can be measured—were smaller than the calculated velocities, we should have a proof that the actual attractions at great distances are smaller than by Newton's law. From such a deviation it could be proved indirectly that the universe is finite. It would even be possible to estimate its spatial dimensions.

Can we visualize a three-dimensional universe which is finite, yet unbounded?

The usual answer to this question is "No," but that is not the right answer. The purpose of the following remarks is to show that the answer should be "Yes." I want to show that without any extraordinary difficulty we can illustrate the theory of a finite universe by means of a mental picture to which, with some practice, we shall soon grow accustomed.

First of all, an observation of epistemological nature. A geometrical-physical theory as such is incapable of being directly pictured, being merely a system of concepts. But these concepts serve the purpose of bringing a multiplicity of real or imaginary sensory experiences into connection in the mind. To "visualize" a theory therefore means to bring to mind that abundance of sensible experiences for which the theory supplies the schematic arrangement. In the present case we have to ask ourselves how we can represent that behavior of solid bodies with respect to their mutual disposition (contact) which corresponds to the theory of a finite universe. There is really nothing new in what I have to say about this; but innumerable questions addressed to me prove that the curiosity of those who are interested in these matters has not yet been completely satisfied. So, will the initiated please pardon me, in that part of what I shall say has long been known?

What do we wish to express when we say that our space is infinite? Nothing more than that we might lay any number of bodies of equal sizes side by side without ever filling space. Suppose that we are provided with a great many cubic boxes all of the same size. In accordance with Euclidean geometry we can place them above, beside, and behind one another so as to fill an arbitrarily large part of space; but this construction would never be finished; we could go on adding more and more cubes without ever finding that there was no more room. That is what we wish to express when we say that space is infinite. It would be better to say that space is infinite in relation to practically-rigid bodies, assuming that the laws of disposition for these bodies are given by Euclidean geometry.

Another example of an infinite continuum is the plane. On a plane surface we may lay squares of cardboard so that each side of any square has the side of another square adjacent to it. The construction is never finished; we can always go on laying squares—if their laws of disposition correspond to those of plane figures of Euclidean geometry. The plane is therefore infinite in relation to the cardboard squares. Accordingly we say that the plane is an infinite continuum of two dimensions, and space an infinite continuum of three dimensions. What is here meant by the number of dimensions, I think I may assume to be known.

Now we take an example of a two-dimensional continuum which is finite, but unbounded. We imagine the surface of a large globe and a quantity of small paper discs, all of the same size. We place one of the discs anywhere on the surface of the globe. If we move the disc about, anywhere we like, on the surface of the globe, we do not come upon a boundary anywhere on the journey. Therefore we say that the spherical surface of the globe is an unbounded continuum. Moreover, the spherical surface is a finite continuum. For if we stick the paper discs on the globe, so that no disc overlaps another, the surface of the globe will finally become so full that there is no room for another disc. This means exactly that the spherical surface of the globe is finite in relation to the paper discs. Further, the spherical surface is a non-Euclidean continuum of two dimensions, that is to say, the laws of disposition for the rigid figures lying in it do not agree with those of the Euclidean plane. This can be shown in the following way. Take a disc and surround it in a circle by six more discs, each of which is to be surrounded in turn by six discs, and so on. If this construction is made on a plane surface, we obtain an uninterrupted arrangement in which there are six discs touching every disc except those which lie on the outside. On the spherical surface the construction also

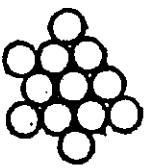

FIG. 1

seems to promise success at the outset, and the smaller the radius of the disc in proportion to that of the sphere, the more promising it seems. But as the construction progresses it becomes more and more patent that the arrangement of the discs in the manner indicated, without interruption, is not possible, as it should be possible by the Euclidean geometry of the plane. In this way creatures which cannot leave the spherical surface, and cannot even peep out from the spherical surface into three-dimensional space, might discover, merely by experimenting with discs, that their two-dimensional "space" is not Euclidean, but spherical space.

From the latest results of the theory of relativity it is probable that our three-dimensional space is also approximately spherical, that is, that the laws of disposition of rigid bodies in it are not given by Euclidean geometry, but approximately by spherical geometry, if only we consider parts of space which are sufficiently extended. Now this is the place where the reader's imagination boggles. "Nobody can imagine this thing," he cries indignantly. "It can be said, but cannot be thought. I can imagine a spherical surface well enough, but nothing analogous to it in three dimensions."

We must try to surmount this barrier in the mind, and the patient reader will see that it is by no means a particularly difficult task. For this purpose we will first give our attention once more to the geometry of two-dimensional spherical surfaces. In the adjoining figure let *K* be the spherical surface, touched at *S* by a plane, *E*, which, for facility of presentation, is shown in the drawing as a bounded surface. Let *L* be a disc on the spherical surface. Now let us imagine that at the point *N* of the

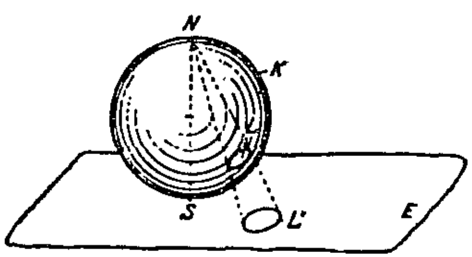

FIG. 2

spherical surface, diametrically opposite to S, there is a luminous point, throwing a shadow L' of the disc L upon the plane E. Every point on the sphere has its shadow on the plane. If the disc on the sphere K is moved, its shadow L' on the plane E also moves. When the disc L is at S, it almost exactly coincides with its shadow. If it moves on the spherical surface away from S upwards, the disc shadow L' on the plane also moves away from S on the plane outwards, growing bigger and bigger. As the disc L approaches the luminous point N, the shadow moves off to infinity, and becomes infinitely great.

Now we put the question: what are the laws of disposition of the disc-shadows L' on the plane E? Evidently they are exactly the same as the laws of disposition of the discs L on the spherical surface. For to each original figure on K there is a corresponding shadow figure on E. If two discs on K are touching, their shadows on E also touch. The shadow-geometry on the plane agrees with the disc-geometry on the sphere. If we call the disc-shadows rigid figures, then spherical geometry holds good on the plane E with respect to these rigid figures. In particular, the plane is finite with respect to the disc-shadows, since only a finite number of the shadows can find room on the plane.

At this point somebody will say, "That is nonsense. The disc-shadows are *not* rigid figures. We have only to move a two-foot rule about on the plane E to convince ourselves that the shadows constantly increase in size as they move away from S on the plane toward infinity." But what if the two-foot rule were to behave on the plane E in the same way as the disc-shadows L'? It would then be impossible to show that the shadows increase in size as they move away from S; such an assertion would then no longer have any meaning whatever. In fact the only objective assertion that can be made about the disc-shadows is just this, that they are related in exactly the same way as are the rigid discs on the spherical surface in the sense of Euclidean geometry.

We must carefully bear in mind that our statement as to the growth of the disc-shadows, as they move away from S toward infinity, has in itself no objective meaning, as long as we are unable to compare the disc-shadows with Euclidean rigid bodies which can be moved about on the plane E. In respect of the laws of disposition of the shadows L', the point S has no special privileges on the plane any more than on the spherical surface.

The representation given above of spherical geometry on the plane is important for us, because it readily allows itself to be transferred to the three-dimensional case.

Let us imagine a point S of our space, and a great number of small spheres, L', which can all be brought to coincide with one another. But these spheres are not to be rigid in the sense of Euclidean geometry; their radius is to increase (in the sense of Euclidean geometry) when they are moved away from S toward infinity; it is to increase according to the same law as the radii of the disc-shadows L' on the plane.

After having gained a vivid mental image of the geometrical behavior of our L' spheres, let us assume that in our space there are no rigid bodies at all in the sense of Euclidean geometry, but only bodies having the behavior of our L' spheres. Then we shall have a clear picture of three-dimensional spherical space, or, rather of three-dimensional spherical geometry. Here our spheres must be called "rigid" spheres. Their increase in size as they depart from S is not to be detected by measuring with measuring-rods, any more than in the case of the disc-shadows on E, because the standards of measurement behave in the same way as the spheres. Space is homogeneous, that is to say, the same spherical configurations are possible in the neighborhood of every point.* Our space is finite, because, in consequence of the "growth" of the spheres, only a finite number of them can find room in space.

In this way, by using as a crutch the practice in thinking and visualization which Euclidean geometry gives us, we have acquired a mental picture of spherical geometry. We may without difficulty impart more depth and vigor to these ideas by carrying out special imaginary constructions. Nor would it be difficult to represent the case of what is called elliptical geometry in

* This is intelligible without calculation—but only for the two-dimensional case—if we revert once more to the case of the disc on the surface of the sphere.

since, according to that theory, the physical properties of space are affected by ponderable matter. In my opinion the general theory of relativity can solve this problem satisfactorily only if it regards the world as spatially closed. The mathematical results of the theory force one to this view, if one believes that the mean density of ponderable matter in the world possesses some finite value, however small.

THE CAUSE OF THE FORMATION OF MEANDERS IN THE COURSES OF RIVERS AND OF THE SO-CALLED BAER'S LAW

Read before the Prussian Academy, January 7, 1926. Published in the German periodical, Die Naturwissenschaften, *Vol. 14, 1926.*

It is common knowledge that streams tend to curve in serpentine shapes instead of following the line of the maximum declivity of the ground. It is also well known to geographers that the rivers of the northern hemisphere tend to erode chiefly on the right side. The rivers of the southern hemisphere behave in the opposite manner (Baer's law). Many attempts have been made to explain this phenomenon, and I am not sure whether anything I say in the following pages will be new to the expert; some of my considerations are certainly known. Nevertheless, having found nobody who was thoroughly familiar with the causal relations involved, I think it is appropriate to give a short qualitative exposition of them.

First of all, it is clear that the erosion must be stronger the greater the velocity of the current where it touches the bank in question, or rather the more steeply it falls to zero at any particular point of the confining wall. This is equally true under all circumstances, whether the erosion depends on mechanical or on physico-chemical factors (decomposition of the ground). We must then concentrate our attention on the circumstances which affect the steepness of the velocity gradient at the wall.

the *probabilities* of the occurrence of a physical reality that we have in view. Dirac, to whom, in my opinion, we owe the most perfect exposition, logically, of this theory, rightly points out that it would probably be difficult, for example, to give a theoretical description of a photon such as would give enough information to enable one to decide whether it will pass a polarizer placed (obliquely) in its way or not.

I am still inclined to the view that physicists will not in the long run content themselves with that sort of indirect description of the real, even if the theory can eventually be adapted to the postulate of general relativity in a satisfactory manner. We shall then, I feel sure, have to return to the attempt to carry out the program which may be described properly as the Maxwellian—namely, the description of physical reality in terms of fields which satisfy partial differential equations without singularities.

ON THE METHOD OF THEORETICAL PHYSICS

The Herbert Spencer lecture, delivered at Oxford, June 10, 1933. Published in Mein Weltbild, *Amsterdam: Querido Verlag, 1934.*

If you want to find out anything from the theoretical physicists about the methods they use, I advise you to stick closely to one principle: don't listen to their words, fix your attention on their deeds. To him who is a discoverer in this field, the products of his imagination appear so necessary and natural that he regards them, and would like to have them regarded by others, not as creations of thought but as given realities.

These words sound like an invitation to you to walk out of this lecture. You will say to yourselves, the fellow's a working physicist himself and ought therefore to leave all questions of the structure of theoretical science to the epistemologists.

Against such criticism I can defend myself from the personal point of view by assuring you that it is not at my own instance but at the kind invitation of others that I have mounted this rostrum, which serves to commemorate a man who fought hard all his life for the unity of knowledge. Objectively, however, my enterprise can be justified on the ground that it may, after all, be of interest to know how one who has spent a lifetime in striving with all his might to clear up and rectify its fundamentals looks upon his own branch of science. The way in which he regards its past and present may depend too much on what he hopes for the future and aims at in the present; but that is the inevitable fate of anybody who has occupied himself intensively with a world of ideas. The same thing happens to him as to the historian, who in the same way, even though perhaps unconsciously, groups actual events round ideals which he has formed for himself on the subject of human society.

Let us now cast an eye over the development of the theoretical system, paying special attention to the relations between the content of the theory and the totality of empirical fact. We are concerned with the eternal antithesis between the two inseparable components of our knowledge, the empirical and the rational, in our department.

We reverence ancient Greece as the cradle of western science Here for the first time the world witnessed the miracle of a logical system which proceeded from step to step with such precision that every single one of its propositions was absolutely indubitable—I refer to Euclid's geometry. This admirable triumph of reasoning gave the human intellect the necessary confidence in itself for its subsequent achievements. If Euclid failed to kindle your youthful enthusiasm, then you were not born to be a scientific thinker.

But before mankind could be ripe for a science which takes in the whole of reality, a second fundamental truth was needed, which only became common property among philosophers with the advent of Kepler and Galileo. Pure logical thinking cannot yield us any knowledge of the empirical world; all knowledge of reality starts from experience and ends in it. Propositions arrived at by purely logical means are completely empty as regards reality. Because Galileo saw this, and particularly because he drummed it into the scientific world, he is the father of modern physics—indeed, of modern science altogether.

If, then, experience is the alpha and the omega of all our knowledge of reality, what is the function of pure reason in science?

A complete system of theoretical physics is made up of concepts, fundamental laws which are supposed to be valid for those concepts and conclusions to be reached by logical deduction. It is these conclusions which must correspond with our separate experiences; in any theoretical treatise their logical deduction occupies almost the whole book.

This is exactly what happens in Euclid's geometry, except that there the fundamental laws are called axioms and there is no question of the conclusions having to correspond to any sort of experience. If, however, one regards Euclidean geometry as the science of the possible mutual relations of practically rigid bodies in space, that is to say, treats it as a physical science, without abstracting from its original empirical content, the logical homogeneity of geometry and theoretical physics becomes complete.

We have thus assigned to pure reason and experience their places in a theoretical system of physics. The structure of the system is the work of reason; the empirical contents and their mutual relations must find their representation in the conclusions of the theory. In the possibility of such a representation lie the sole value and justification of the whole system, and especially of the concepts and fundamental principles which underlie it. Apart from that, these latter are free inventions of the human intellect, which cannot be justified either by the nature of that intellect or in any other fashion *a priori*.

These fundamental concepts and postulates, which cannot be further reduced logically, form the essential part of a theory, which reason cannot touch. It is the grand object of all theory to make these irreducible elements as simple and as few in number as possible, without having to renounce the adequate representation of any empirical content whatever.

The view I have just outlined of the purely fictitious character of the fundamentals of scientific theory was by no means the prevailing one in the eighteenth and nineteenth centuries. But it is steadily gaining ground from the fact that the distance in thought between the fundamental concepts and laws on one side and, on the other, the conclusions which have to be brought into relation with our experience grows larger and larger, the simpler the logical structure becomes—that is to say, the smaller the number of logically independent conceptual elements which are found necessary to support the structure.

Newton, the first creator of a comprehensive, workable system of theoretical physics, still believed that the basic concepts and laws of his system could be derived from experience. This is no doubt the meaning of his saying, *hypotheses non fingo.*

Actually the concepts of time and space appeared at that time to present no difficulties. The concepts of mass, inertia, and force, and the laws connecting them, seemed to be drawn directly from experience. Once this basis is accepted, the expression for the force of gravitation appears derivable from experience, and it was reasonable to expect the same in regard to other forces.

We can indeed see from Newton's formulation of it that the concept of absolute space, which comprised that of absolute rest, made him feel uncomfortable; he realized that there seemed to be nothing in experience corresponding to this last concept. He was also not quite comfortable about the introduction of forces operating at a distance. But the tremendous practical success of his doctrines may well have prevented him and the physicists of the eighteenth and nineteenth centuries from recognizing the fictitious character of the foundations of his system.

The natural philosophers of those days were, on the contrary, most of them possessed with the idea that the fundamental concepts and postulates of physics were not in the logical sense free inventions of the human mind but could be deduced from experience by "abstraction"—that is to say, by logical means. A clear recognition of the erroneousness of this notion really only came with the general theory of relativity, which showed that one could take account of a wider range of empirical facts, and that, too, in a more satisfactory and complete manner, on a foundation quite different from the Newtonian. But quite apart from the question of the superiority of one or the other, the fictitious character of fundamental principles is perfectly evident from the fact that we can point to two essentially different principles, both of which correspond with experience to a large extent; this proves at the same time that every attempt at a logical deduction of the basic concepts and postulates of mechanics from elementary experiences is doomed to failure.

If, then, it is true that the axiomatic basis of theoretical physics cannot be extracted from experience but must be freely invented, can we ever hope to find the right way? Nay, more, has this right way any existence outside our illusions? Can we hope to be guided safely by experience at all when there exist theories (such as classical mechanics) which to a large extent do justice to experience, without getting to the root of the matter? I answer without hesitation that there is, in my opinion, a right way, and that we are capable of finding it. Our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of purely mathematical constructions the concepts and the laws connecting them with each other, which furnish the key to the understanding of natural phenomena. Experience may suggest the appropriate mathematical concepts, but they most certainly cannot be deduced from it. Experience remains, of course, the sole criterion of the physical utility of a mathematical construction. But the creative principle resides in mathematics. In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed.

In order to justify this confidence, I am compelled to make use of a mathematical concept. The physical world is represented as a four-dimensional continuum. If I assume a Riemannian metric in it and ask what are the simplest laws which such a metric can satisfy, I arrive at the relativistic theory of gravitation in empty space. If in that space I assume a vector-field or an anti-symmetrical tensor-field which can be derived from it, and ask what are the simplest laws which such a field can satisfy, I arrive at Maxwell's equations for empty space.

At this point we still lack a theory for those parts of space in which electrical charge density does not disappear. De Broglie conjectured the existence of a wave field, which served to explain certain quantum properties of matter. Dirac found in the spinors field-magnitudes of a new sort, whose simplest equations enable one to a large extent to deduce the properties of the electron. Subsequently I discovered, in conjunction with my colleague, Dr. Walter Mayer, that these spinors form a special case of a new sort of field, mathematically connected with the four-dimensional system, which we called "semivectors." The simplest equations which such semivectors can satisfy furnish a key to the understanding of the existence of two sorts of elementary particles, of different ponderable mass and equal but opposite electrical charge. These semivectors are, after ordinary vectors, the simplest mathematical fields that are possible in a metrical continuum of four dimensions, and it looks as if they described, in a natural way, certain essential properties of electrical particles.

The important point for us to observe is that all these constructions and the laws connecting them can be arrived at by the principle of looking for the mathematically simplest concepts and the link between them. In the limited number of the mathematically existent simple field types, and the simple equations possible between them, lies the theorist's hope of grasping the real in all its depth.

Meanwhile the great stumbling-block for a field-theory of this kind lies in the conception of the atomic structure of matter and energy. For the theory is fundamentally non-atomic in so far as it operates exclusively with continuous functions of space, in contrast to classical mechanics, whose most important element, the material point, in itself does justice to the atomic structure of matter.

The modern quantum theory in the form associated with the names of de Broglie, Schrödinger, and Dirac, which operates with continuous functions, has overcome these difficulties by a bold piece of interpretation which was first given a clear form by Max Born. According to this, the spatial functions which appear in the equations make no claim to be a mathematical model of the atomic structure. Those functions are only supposed to determine the mathematical probabilities to find such structures, if measurements are taken, at a particular spot or in a certain state of motion. This notion is logically unobjectionable and has important successes to its credit. Unfortunately, however, it compels one to use a continuum the number of whose dimensions is not that ascribed to space by physics hitherto (four) but rises indefinitely with the number of the particles constituting the system under consideration. I cannot but confess that I attach only a transitory importance to this interpretation. I still believe in the possibility of a model of reality—that is to say, of a theory which represents things themselves and not merely the probability of their occurrence.

On the other hand, it seems to me certain that we must give up the idea of a complete localization of the particles in a theoretical model. This seems to me to be the permanent upshot of Heisenberg's principle of uncertainty. But an atomic theory in the true sense of the word (not merely on the basis of an interpretation) without localization of particles in a mathematical model is perfectly thinkable. For instance, to account for the atomic character of electricity, the field equations need only lead to the following conclusions: A region of three-dimensional space at whose boundary electrical density vanishes everywhere always contains a total electrical charge whose size is represented by a whole number. In a continuum-theory atomic characteristics would be satisfactorily expressed by integral laws without localization of the entities which constitute the atomic structure.

Not until the atomic structure has been successfully represented in such a manner would I consider the quantum-riddle solved.

THE PROBLEM OF SPACE, ETHER, AND THE FIELD IN PHYSICS

Mein Weltbild, *Amsterdam: Querido Verlag, 1934.*

Scientific thought is a development of pre-scientific thought. As the concept of space was already fundamental in the latter, we must begin with the concept of space in pre-scientific thought. There are two ways of regarding concepts, both of which are indispensable to understanding. The first is that of logical analysis. It answers the question, How do concepts and judgments depend on each other? In answering it we are on comparatively safe ground. It is the certainty by which we are so much impressed in mathematics. But this certainty is purthe light—only those who have experienced it can understand that.

PHYSICS AND REALITY

From The Journal of the Franklin Institute, *Vol. 221, No. 3. March, 1936.*

I. General Consideration Concerning the Method of Science

It has often been said, and certainly not without justification, that the man of science is a poor philosopher. Why, then, should it not be the right thing for the physicist to let the philosopher do the philosophizing? Such might indeed be the right thing at a time when the physicist believes he has at his disposal a rigid system of fundamental concepts and fundamental laws which are so well established that waves of doubt cannot reach them; but, it cannot be right at a time when the very foundations of physics itself have become problematic as they are now. At a time like the present, when experience forces us to seek a newer and more solid foundation, the physicist cannot simply surrender to the philosopher the critical contemplation of the theoretical foundations; for, he himself knows best, and feels more surely where the shoe pinches. In looking for a new foundation, he must try to make clear in his own mind just how far the concepts which he uses are justified, and are necessities.

The whole of science is nothing more than a refinement of everyday thinking. It is for this reason that the critical thinking of the physicist cannot possibly be restricted to the examination of the concepts of his own specific field. He cannot proceed without considering critically a much more difficult problem, the problem of analyzing the nature of everyday thinking.

Our psychological experience contains, in colorful succession, sense experiences, memory pictures of them, images, and feelings. In contrast to psychology, physics treats directly only of sense experiences and of the "understanding" of their connection. But even the concept of the "real external world" of everyday thinking rests exclusively on sense impressions.

Now we must first remark that the differentiation between sense impressions and images is not possible; or, at least it is not possible with absolute certainty. With the discussion of this problem, which affects also the notion of reality, we will not concern ourselves but we shall take the existence of sense experiences as given, that is to say, as psychic experiences of a special kind.

I believe that the first step in the setting of a "real external world" is the formation of the concept of bodily objects and of bodily objects of various kinds. Out of the multitude of our sense experiences we take, mentally and arbitrarily, certain repeatedly occurring complexes of sense impressions (partly in conjunction with sense impressions which are interpreted as signs for sense experiences of others), and we correlate to them a concept—the concept of the bodily object. Considered logically this concept is not identical with the totality of sense impressions referred to; but it is a free creation of the human (or animal) mind. On the other hand, this concept owes its meaning and its justification exclusively to the totality of the sense impressions which we associate with it.

The second step is to be found in the fact that, in our thinking (which determines our expectation), we attribute to this concept of the bodily object a significance, which is to a high degree independent of the sense impressions which originally give rise to it. This is what we mean when we attribute to the bodily object "a real existence." The justification of such a setting rests exclusively on the fact that, by means of such concepts and mental relations between them, we are able to orient ourselves in the labyrinth of sense impressions. These notions and relations, although free mental creations, appear to us as stronger and more unalterable than the individual sense experience itself, the character of which as anything other than the result of an illusion or hallucination is never completely guaranteed. On the other hand, these concepts and relations, and indeed the postulation of real objects and, generally speaking, of the existence of "the real world," have justification only in so far as they are connected with sense impressions between which they form a mental connection.

The very fact that the totality of our sense experiences is such that by means of thinking (operations with concepts, and the creation and use of definite functional relations between them, and the coordination of sense experiences to these concepts) it can be put in order, this fact is one which leaves us in awe, but which we shall never understand. One may say "the eternal mystery of the world is its comprehensibility." It is *one of the great realizations of* Immanuel Kant *that the* postulation of a real external world would be senseless without this comprehensibility.

In speaking here of "comprehensibility," the expression is used in its most modest sense. It implies: the production of some sort of order among sense impressions, this order being produced by the creation of general concepts, relations between these concepts, and by definite relations of some kind between the concepts and sense experience. It is in this sense that the *world of our sense experiences is comprehensible.* The *fact that* it is comprehensible is a miracle.

In my opinion, nothing can be said *a priori* concerning the manner in which the concepts are to be formed and connected, and how we are to coordinate them to sense experiences. In guiding us in the creation of such an order of sense experiences, success alone is the determining factor. All that is necessary is to fix a set of rules, since without such rules the acquisition of knowledge in the desired sense would be impossible. One may compare these rules with the rules of a game in which, while the rules themselves are arbitrary, it is their rigidity alone which makes the game possible. However, the fixation will never be final. It will have validity only for a special field of application (i.e., there are no final categories in the sense of Kant).

The connection of the elementary concepts of everyday thinking with complexes of sense experiences can only be comprehended intuitively and it is unadaptable to scientifically logical fixation. The totality of these connections—none of which is expressible in conceptual terms—is the only thing *which differentiates the great building which is science from* a logical but empty scheme of concepts. By means of these connections, the purely conceptual propositions of science become general statements about complexes of sense experiences.

We shall call "primary concepts" such concepts as are directly and intuitively connected with typical complexes of sense experiences. All other notions are—from the physical point of view—possessed of meaning only in so far as they are connected, by propositions, with the primary notions. These propositions are partially definitions of the concepts (and of the statements derived logically from them) and partially propositions not derivable from the definitions, which express at least indirect relations between the "primary concepts," and in this way between sense experiences. Propositions of the latter kind are "statements about reality" or laws of nature, i.e., propositions which have to show their validity when applied to sense experiences covered by primary concepts. The question as to which of the propositions shall be considered as definitions and which as natural laws will depend largely upon the chosen representation. It really becomes absolutely necessary to make this differentiation only when one examines the degree to which the whole system of concepts considered is not empty from the physical point of view.

STRATIFICATION OF THE SCIENTIFIC SYSTEM

The aim of science is, on the one hand, a comprehension, as *complete* as possible, of the connection between the sense experiences in their totality, and, on the other hand, the accomplishment of this aim *by the use of a minimum of primary concepts and relations.* (Seeking, as far as possible, logical unity in the world picture, i.e., paucity in logical elements.)

Science uses the totality of the primary concepts, i.e., concepts directly connected with sense experiences, and propositions connecting them. In its first stage of development, science does not contain anything else. Our everyday thinking is satisfied on the whole with this level. Such a state of affairs cannot, however, satisfy a spirit which is really scientifically minded; because the totality of concepts and relations obtained in this manner is utterly lacking in logical unity. In order to sup-

*plement this deficiency, one invents a system poorer in con*cepts and relations, a system retaining the primary concepts and relations of the "first layer" as logically derived concepts and relations. This new "secondary system" pays for its higher *logical unity by having elementary concepts (concepts of the* second layer), which are no longer directly connected with complexes of sense experiences. Further striving for logical unity brings us to a tertiary system, still poorer in concepts and relations, for the deduction of the concepts and relations of the secondary (and so indirectly of the primary) layer. Thus the story goes on until we have arrived at a system of the greatest conceivable unity, and of the greatest poverty of concepts of the logical foundations, which is still compatible with the observations made by our senses. We do not know whether or not this ambition will ever result in a definitive system. If one is asked for his opinion, he is inclined to answer no. While wrestling with the problems, however, one will never give up the hope that this greatest of all aims can really be attained to a very high degree.

An adherent to the theory of abstraction or induction might call our layers "degrees of abstraction"; but I do not consider it justifiable to veil the logical independence of the concept from the sense experiences. The relation is not analogous to that of soup to beef but rather of check number to overcoat.

The layers are furthermore not clearly separated. It is not even absolutely clear which concepts belong to the primary layer. As a matter of fact, we are dealing with freely formed concepts, which, with a certainty sufficient for practical use, are intuitively connected with complexes of sense experiences in such a manner that, in any given case of experience, there is no uncertainty as to the validity of an assertion. The essential thing is the aim to represent the multitude of concepts and propositions, close to experience, as propositions, logically deduced from a basis, as narrow as possible, of fundamental concepts and fundamental relations which themselves can be chosen freely (axioms). The liberty of choice, however, is of a special kind; it is not in any way similar to the liberty of a writer of fiction. Rather, it is similar to that of a man engaged in solving a well-designed word puzzle. He may, it is true, propose any word as the solution; but, there is only *one* word which really solves the puzzle in all its parts. It is a matter of faith that nature —as she is perceptible to our five senses—takes the character of such a well-formulated puzzle. The successes reaped up to now by science do, it is true, give a certain encouragement for this faith.

The multitude of layers discussed above corresponds to the several stages of progress which have resulted from the struggle for unity in the course of development. As regards the final aim, intermediary layers are only of temporary nature. They must eventually disappear as irrelevant. We have to deal, however, with the science of today, in which these strata represent problematic partial successes which support one another but which also threaten one another, because today's system of concepts contains deep-seated incongruities which we shall meet later on.

It will be the aim of the following lines to demonstrate what paths the constructive human mind has entered, in order to arrive at a basis of physics which is logically as uniform as possible.

II. Mechanics and the Attempts to Base All Physics upon It

An important property of our sense experiences, and, more generally, of all of our experiences, is their temporal order. This kind of order leads to the mental conception of a subjective time, an ordering scheme for our experience. The subjective time leads then via the concept of the bodily object and of space to the concept of objective time, as we shall see later on.

Ahead of the notion of objective time there is, however, the concept of space; and ahead of the latter we find the concept of the bodily object. The latter is directly connected with complexes of sense experiences. It has been pointed out that one property which is characteristic of the notion "bodily object" is the property which provides that we coordinate to it an existence, independent of (subjective) time, and independent of the fact that it is perceived by our senses. We do this in spite of the fact that we perceive temporal alterations in it. Poincaré has justly emphasized the fact that we distinguish two kinds of alterations of the bodily object, "changes of state" and "changes of position." The latter, he remarked, are alterations which we can reverse by voluntary motions of our bodies.

That there are bodily objects to which we have to ascribe, within a certain sphere of perception, no alteration of state, but only alterations of position, is a fact of fundamental importance for the formation of the concept of space (in a certain degree even for the justification of the notion of the bodily object itself). Let us call such an object "practically rigid."

If, as the object of our perception, we consider simultaneously (i.e., as a single unit) two practically rigid bodies, then there exist for this ensemble such alterations as can *not* possibly be considered as changes of position of the whole, notwithstanding the fact that this is the case for each one of the two constituents. This leads to the notion of "change of relative position" of the two objects; and, in this way, also to the notion of "relative position" of the two objects. It is found moreover that among the relative positions, there is one of a specific kind which we designate as "contact." * Permanent contact of two bodies in three or more "points" means that they are united to a quasi-rigid compound body. It is permissible to say that the second body forms then a (quasi-rigid) continuation of the first body and may, in its turn, be continued quasi-rigidly. The possibility of the quasi-rigid continuation of a body is unlimited. The totality of all conceivable quasi-rigid continuations of a body B_0 is the infinite "space" determined by it.

In my opinion, the fact that every bodily object situated in any arbitrary manner can be put into contact with the quasi-rigid continuation of some given body B_0 (body of reference), this fact is the empirical basis of our conception of space. In pre-scientific thinking, the solid earth's crust plays the role of B_0 and its continuation. The very name geometry indicates

* It is in the nature of things that we are able to talk about these objects only by means of concepts of our own creation, concepts which themselves are not subject to definition. It is essential, however, that we make use only of such concepts concerning whose coordination to our experience we feel no doubt.

that the concept of space is psychologically connected with the earth as an ever present body of reference.

The bold notion of "space" which preceded all scientific geometry transformed our mental concept of the relations of positions of bodily objects into the notion of the position of these bodily objects in "space." This, of itself, represents a great formal simplification. Through this concept of space one reaches, moreover, an attitude in which any description of position is implicitly a description of contact; the statement that a point of a bodily object is located at a point P of space means that the object touches the point P of the standard body of reference B_0 (supposed appropriately continued) at the point considered.

In the geometry of the Greeks, space plays only a qualitative role, since the position of bodies in relation to space is considered as given, it is true, but is not described by means of numbers. Descartes was the first to introduce this method. In his language, the whole content of Euclidean geometry can axiomatically be founded upon the following statements: (1) Two specified points of a rigid body determine a segment. (2) We may associate triples of numbers X_1, X_2, X_3, to points of space in such a manner that for every segment $P' - P''$ under consideration, the coordinates of whose end points are X_1', X_2', X_3'; X_1'', X_2'', X_3'', the expression

$$s^2 = (X_1'' - X_1')^2 + (X_2'' - X_2')^2 + (X_3'' - X_3')^2$$

is independent of the position of the body, and of the positions of any and all other bodies.

The (positive) number s is called the length of the segment, or the distance between the two points P' and P'' of space (which are coincident with the points P' and P'' of the segment).

The formulation is chosen, intentionally, in such a way that it expresses clearly, not only the logical and axiomatic, but also the empirical content of Euclidean geometry. The purely logical (axiomatic) representation of Euclidean geometry has, it is true, the advantage of greater simplicity and clarity. It pays for this, however, by renouncing a representation of the connection between the conceptual construction and the sense experiences upon which connection, alone, the significance of geometry for physics rests. The fatal error that logical necessity, preceding all experience, was the basis of Euclidean geometry and the concept of space belonging to it, this fatal error arose from the fact that the empirical basis, on which the axiomatic construction of Euclidean geometry rests, had fallen into oblivion.

In so far as one can speak of the existence of rigid bodies in nature, Euclidean geometry is a physical science, which must be confirmed by sense experiences. It concerns the totality of laws which must hold for the relative positions of rigid bodies independently of time. As one may see, the physical notion of space also, as originally used in physics, is tied to the existence of rigid bodies.

From the physicist's point of view, the central importance of Euclidean geometry rests in the fact that its laws are independent of the specific nature of the bodies whose relative positions it discusses. Its formal simplicity is characterized by the properties of homogeneity and isotropy (and the existence of similar entities).

The concept of space is, it is true, useful, but not indispensable for geometry proper, i.e., for the formulation of rules about the relative positions of rigid bodies. By contrast, the concept of objective time, without which the formulation of the fundamentals of classical mechanics is impossible, is linked with the concept of the spatial continuum.

The introduction of objective time involves two postulates which are independent of each other.

1. The introduction of the objective local time by connecting the temporal sequence of experiences with the readings of a "clock," i.e., of a periodically recurring closed system.

2. The introduction of the notion of objective time for the events in the whole space, by which notion alone the idea of local time is extended to the idea of time in physics.

Note concerning 1. As I see it, it does not mean a "petitio principii" if one puts the concept of periodical recurrence ahead of the concept of time, while one is concerned with the clarification of the origin and of the empirical content of the concept of time. Such a conception corresponds exactly to the precedence of the concept of the rigid (or quasi-rigid) body in the interpretation of the concept of space.

Further discussion of 2. The illusion which prevailed prior to the enunciation of the theory of relativity—that, from the point of view of experience the meaning of simultaneity in relation to spatially distant events and, consequently, that the meaning of physical time is *a priori* clear—this illusion had its origin in the fact that in our everyday experience we can neglect the time of propagation of light. We are accustomed on this account to fail to differentiate between "simultaneously seen" and "simultaneously happening"; and, as a result, the difference between time and local time is blurred.

The lack of definiteness which, from the point of view of its empirical significance, adheres to the notion of time in classical mechanics was veiled by the axiomatic representation of space and time as given independently of our sense experiences. Such a use of notions—independent of the empirical basis to which they owe their existence—does not necessarily damage science. One may, however, easily be led into the error of believing that these notions, whose origin is forgotten, are logically necessary and therefore unalterable, and this error may constitute a serious danger to the progress of science.

It was fortunate for the development of mechanics and hence also for the development of physics in general, that the lack of definiteness in the concept of objective time remained hidden from the earlier philosophers as regards its empirical interpretation. Full of confidence in the real meaning of the space-time construction, they developed the foundations of mechanics which we shall characterize, schematically, as follows:

(*a*) Concept of a material point: a bodily object which—as regards its position and motion—can be described with sufficient accuracy as a point with coordinates X_1, X_2, X_3. Description of its motion (in relation to the "space" B_0) by giving X_1, X_2, X_3, as functions of the time.

(*b*) Law of inertia: the disappearance of the components of acceleration for a material point which is sufficiently far away from all other points.

(*c*) Law of motion (for the material point): Force $=$ mass $\times$ acceleration.

(*d*) Laws of force (interactions between material points).

In this, (*b*) is merely an important special case of (*c*). A real theory exists only when the laws of force are given. The forces must in the first place only obey the law of equality of action and reaction in order that a system of points—permanently connected to each other by forces—may behave like *one* material point.

These fundamental laws, together with Newton's law for the gravitational force, form the basis of the mechanics of celestial bodies. In this mechanics of Newton, and in contrast to the above conceptions of space derived from rigid bodies, the space B_0 enters in a form which contains a new idea; it is not for every B_0 that validity is asserted (for a given law of force) for (*b*) and (*c*), but only for a B_0 in an appropriate state of motion (inertial system). On account of this fact, the coordinate space acquired an independent physical property which is not contained in the purely geometrical notion of space, a circumstance which gave Newton considerable food for thought (pail-experiment).*

Classical mechanics is only a general scheme; it becomes a theory only by explicit indication of the force laws (*d*) as was done so very successfully by Newton for celestial mechanics. From the point of view of the aim of the greatest logical simplicity of the foundations, this theoretical method is deficient in so far as the laws of force cannot be obtained by logical and formal considerations, so that their choice is *a priori* to a large extent arbitrary. Also Newton's law of gravitation is distinguished from other conceivable laws of force exclusively by its *success*.

In spite of the fact that, today, we know positively that classical mechanics fails as a foundation dominating all physics, it still occupies the center of all of our thinking in physics. The reason for this lies in the fact that, regardless of important

* This defect of the theory could only be eliminated by such a formulation of mechanics as would claim validity for all B_0. This is one of the steps which led to the general theory of relativity. A second defect, also eliminated only by the introduction of the general theory of relativity, lies in the fact that there is no reason given by mechanics itself for the equality of the gravitational and inertial mass of the material point.

progress reached since the time of Newton, we have not yet arrived at a new foundation of physics concerning which we may be certain that the manifold of all investigated phenomena, and of successful partial theoretical systems, could be deduced logically from it. In the following lines I shall try to describe briefly how the matter stands.

First we try to get clearly in our minds how far the system of classical mechanics has shown itself adequate to serve as a basis for the whole of physics. Since we are dealing here only with the foundations of physics and with its development, we need not concern ourselves with the purely *formal* progresses of mechanics (equations of Lagrange, canonical equations, etc.). *One* remark, however, appears indispensable. The notion "material point" is fundamental for mechanics. If now we seek to develop the mechanics of a bodily object which itself can *not* be treated as a material point—and strictly speaking every object "perceptible to our senses" is of this category—then the question arises: How shall we imagine the object to be built up out of material points, and what forces must we assume as acting between them? The formulation of this question is indispensable, if mechanics is to pretend to describe the object *completely*.

It is in line with the natural tendency of mechanics to assume these material points, and the laws of forces acting between them, as invariable, since temporal changes would lie outside of the scope of mechanical explanation. From this we can see that classical mechanics must lead us to an atomistic construction of matter. We now realize, with special clarity, how much in error are those theorists who believe that theory comes inductively from experience. Even the great Newton could not free himself from this error (*"Hypotheses non fingo"**).

In order to save itself from becoming hopelessly lost in this line of thought (atomism), science proceeded first in the following manner. The mechanics of a system is determined if its potential energy is given as a function of its configuration. Now, if the acting forces are of such a kind as to guarantee the

* "I make no hypotheses."

maintenance of certain structural properties of the system's configuration, then the configuration may be described with sufficient accuracy by a relatively small number of configuration variables q_r; the potential energy is considered only in so far as it is dependent upon *these* variables (for instance, description of the configuration of a practically rigid body by six variables).

A second method of application of mechanics, which avoids the consideration of a subdivision of matter down to "real" material points, is the mechanics of so-called continuous media. This mechanics is characterized by the fiction that the density and the velocity of matter depend continuously upon coordinates and time, and that the part of the interactions not explicitly given can be considered as surface forces (pressure forces) which again are continuous functions of position. Herein we find the hydrodynamic theory, and the theory of elasticity of solid bodies. These theories avoid the explicit introduction of material points by fictions which, in the light of the foundation of classical mechanics, can only have an approximate significance.

In addition to their great *practical* significance, these categories of science have—by developing new mathematical concepts—created those formal tools (partial differential equations) which have been necessary for the subsequent attempts at a new foundation of all of physics.

These two modes of application of mechanics belong to the so-called "phenomenological" physics. It is characteristic of this kind of physics that it makes as much use as possible of concepts which are close to experience but, for this reason, has to give up, to a large extent, unity in the foundations. Heat, electricity, and light are described by separate variables of state and material constants other than the mechanical quantities; and to determine all of these variables in their mutual and temporal dependence was a task which, in the main, could only be solved empirically. Many contemporaries of Maxwell saw in such a manner of presentation the ultimate aim of physics, which they thought could be obtained purely inductively from experience on account of the relative closeness of the concepts used to experience. From the point of view of theories of knowledge St. Mill and E. Mach took their stand approximately on this ground.

In my view, the greatest achievement of Newton's mechanics lies in the fact that its consistent application has led beyond this phenomenological point of view, particularly in the field of heat phenomena. This occurred in the kinetic theory of gases and in statistical mechanics in general. The former connected the equation of state of the ideal gases, viscosity, diffusion, and heat conductivity of gases and radiometric phenomena of gases, and gave the logical connection of phenomena which, from the point of view of direct experience, had nothing whatever to do with one another. The latter gave a mechanical interpretation of the thermodynamic ideas and laws and led to the discovery of the limit of applicability of the notions and laws of the classical theory of heat. This kinetic theory, which by far surpassed phenomenological physics as regards the logical unity of its foundations, produced, moreover, definite values for the true magnitudes of atoms and molecules which resulted from several independent methods and were thus placed beyond the realm of reasonable doubt. These decisive progresses were paid for by the coordination of atomistic entities to the material points, the constructively speculative character of these entities being obvious. Nobody could hope ever to "perceive directly" an atom. Laws concerning variables connected more directly with experimental facts (for example: temperature, pressure, speed) were deduced from the fundamental ideas by means of complicated calculations. In this manner physics (at least part of it), originally more phenomenologically constructed, was reduced, by being founded upon Newton's mechanics for atoms and molecules, to a basis further removed from direct experiment, but more uniform in character.

III. The Field Concept

In explaining optical and electrical phenomena, Newton's mechanics has been far less successful than it had been in the fields cited above. It is true that Newton tried to reduce light to the motion of material points in his corpuscular theory of light. Later on, however, as the phenomena of polarization, diffraction, and interference of light forced upon this theory more and more unnatural modifications, Huygens' undulatory theory of light prevailed. Probably this theory owes its origin essentially to the phenomena of crystal optics and to the theory of sound, which was then already elaborated to a certain degree. It must be admitted that Huygens' theory also was based in the first instance upon classical mechanics; the all-penetrating ether had to be assumed as the carrier of the waves, but no known phenomenon suggested the way in which the ether was built up from material points. One could never get a clear picture of the internal forces governing the ether, nor of the forces acting between the ether and "ponderable" matter. The foundations of this theory remained, therefore, eternally in the dark. The true basis was a partial differential equation, the reduction of which to mechanical elements remained always problematic.

For the theoretical conception of electric and magnetic phenomena one introduced, again, masses of a special kind, and between these masses one assumed the existence of forces acting at a distance, similar to Newton's gravitational forces. This special kind of matter, however, appeared to be lacking in the fundamental property of inertia; and the forces acting between these masses and the ponderable matter remained obscure. To these difficulties there had to be added the polar character of these kinds of matter which did not fit into the scheme of classical mechanics. The basis of the theory became still more unsatisfactory when electrodynamic phenomena became known, notwithstanding the fact that these phenomena brought the physicist to the explanation of magnetic phenomena through electrodynamic phenomena and, in this way, made the assumption of magnetic masses superfluous. This progress had, indeed, to be paid for by increasing the complexity of the forces of interaction which had to be assumed as existing between electrical masses in motion.

The escape from this unsatisfactory situation by the electric field theory of Faraday and Maxwell represents probably the most profound transformation of the foundations of physics since Newton's time. Again, it has been a step in the direction of constructive speculation which has increased the distance between the foundation of the theory and sense experiences. The existence of the field manifests itself, indeed, only when electrically charged bodies are introduced into it. The differential equations of Maxwell connect the spatial and temporal differential coefficients of the electric and magnetic fields. The electric masses are nothing more than places of non-vanishing divergence of the electric field. Light waves appear as undulatory electromagnetic field processes in space.

To be sure, Maxwell still tried to interpret his field theory mechanically by means of mechanical ether models. But these attempts receded gradually to the background following the representation of the theory—purged of any unnecessary trimmings—by Heinrich Hertz, so that in this theory the field finally took the fundamental position which had been occupied in Newton's mechanics by the material points. Primarily, however, this applied only for electromagnetic fields in empty space.

In its initial stage the theory was yet quite unsatisfactory for the interior of matter, because there, two electric vectors had to be introduced, which were connected by relations dependent on the nature of the medium, these relations being inaccessible to any theoretical analysis. An analogous situation arose in connection with the magnetic field, as well as in the relation between electric current density and the field.

Here H. A. Lorentz found a way out which showed, at the same time, the way to an electrodynamic theory of bodies in motion, a theory which was more or less free from arbitrary assumptions. His theory was built on the following fundamental hypotheses:

Everywhere (including the interior of ponderable bodies) the seat of the field is the empty space. The participation of matter in electromagnetic phenomena has its origin only in the fact that the elementary particles of matter carry unalterable electric charges, and, on this account, are subject on the one hand to the actions of ponderomotive forces and on the other hand possess the property of generating a field. The elementary particles obey Newton's law of motion for material points.

This is the basis on which H. A. Lorentz obtained his synthesis of Newton's mechanics and Maxwell's field theory. The weakness of this theory lies in the fact that it tried to determine the phenomena by a combination of partial differential equations (Maxwell's field equations for empty space) and total differential equations (equations of motion of points), which procedure was obviously unnatural. The inadequacy of this point of view manifested itself in the necessity of assuming finite dimensions for the particles in order to prevent the electromagnetic field existing at their surfaces from becoming infinitely large. The theory failed, moreover, to give any explanation concerning the tremendous forces which hold the electric charges on the individual particles. H. A. Lorentz accepted these weaknesses of his theory, which were well known to him, in order to explain the phenomena correctly at least in general outline.

Furthermore, there was one consideration which pointed beyond the frame of Lorentz's theory. In the environment of an electrically charged body there is a magnetic field which furnishes an (apparent) contribution to its inertia. Should it not be possible to explain the *total* inertia of the particles electromagnetically? It is clear that this problem could be worked out satisfactorily only if the particles could be interpreted as regular solutions of the electromagnetic partial differential equations. The Maxwell equations in their original form do not, however, allow such a description of particles, because their corresponding solutions contain a singularity. Theoretical physicists have tried for a long time, therefore, to reach the goal by a modification of Maxwell's equations. These attempts have, however, not been crowned with success. Thus it happened that the goal of erecting a pure electromagnetic field theory of matter remained unattained for the time being, although in principle no objection could be raised against the possibility of reaching such a goal. The lack of any systematic method leading to a solution discouraged further attempts in this direction. What appears certain to me, however, is that, in the foundations of any consistent field theory, the particle concept must not appear in addition to the field concept. The whole theory must be based solely on partial differential equations and their singularity-free solutions.

IV. THE THEORY OF RELATIVITY

There is no inductive method which could lead to the fundamental concepts of physics. Failure to understand this fact constituted the basic philosophical error of so many investigators of the nineteenth century. It was probably the reason why the molecular theory and Maxwell's theory were able to establish themselves only at a relatively late date. Logical thinking is necessarily deductive; it is based upon hypothetical concepts and axioms. How can we expect to choose the latter so that we might hope for a confirmation of the consequences derived from them?

The most satisfactory situation is evidently to be found in cases where the new fundamental hypotheses are suggested by the world of experience itself. The hypothesis of the non-existence of perpetual motion as a basis for thermodynamics affords such an example of a fundamental hypothesis suggested by experience; the same holds for Galileo's principle of inertia. In the same category, moreover, we find the fundamental hypotheses of the theory of relativity, which theory has led to an unexpected expansion and broadening of the field theory, and to the superseding of the foundations of classical mechanics.

The success of the Maxwell-Lorentz theory has given great confidence in the validity of the electromagnetic equations for empty space, and hence, in particular, in the assertion that light travels "in space" with a certain constant speed c. Is this assertion of the constancy of light velocity valid for every inertial system? If it were not, then one specific inertial system or, more accurately, one specific state of motion (of a body of reference) would be distinguished from all others. This, however, appeared to contradict all mechanical and electromagnetic-optical experimental facts.

For these reasons it was necessary to raise to the rank of a principle the validity of the law of constancy of light velocity for all inertial systems. From this, it follows that the spatial coordinates X_1, X_2, X_3, and the time X_4, must be transformed according to the "Lorentz-transformation" which is characterized by the invariance of the expression

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2$$

(if the unit of time is chosen in such a manner that the speed of light $c = 1$).

By this procedure time lost its absolute character, and was adjoined to the "spatial" coordinates as of algebraically (nearly) similar character. The absolute character of time and particularly of simultaneity was destroyed, and the four-dimensional description was introduced as the only adequate one.

In order to account, also, for the equivalence of all inertial systems with regard to all the phenomena of nature, it is necessary to postulate invariance of all systems of physical equations which express general laws with respect to Lorentz transformations. The elaboration of this requirement forms the content of the special theory of relativity.

This theory is compatible with the equations of Maxwell; but it is incompatible with the basis of classical mechanics. It is true that the equations of motion of the material point can be modified (and with them the expressions for momentum and kinetic energy of the material point) in such a manner as to satisfy the theory; but, the concept of the force of interaction, and with it the concept of potential energy of a system, lose their basis, because these concepts rest upon the idea of absolute simultaneity. The field, as determined by differential equations, takes the place of the force.

Since the foregoing theory allows interaction only by fields, it requires a field theory of gravitation. Indeed, it is not difficult to formulate such a theory in which, as in Newton's theory, the gravitational fields can be reduced to a scalar which is the solution of a partial differential equation. However, the experimental facts expressed in Newton's theory of gravitation lead in another direction, that of the general theory of relativity.

It is an unsatisfactory feature of classical mechanics that in its fundamental laws the same mass constant appears in two different rôles, namely as "inertial mass" in the law of motion, and as "gravitational mass" in the law of gravitation. As a result, the acceleration of a body in a pure gravitational field is independent of its material; or, in a uniformly accelerated coordinate system (accelerated in relation to an "inertial system") the motions take place as they would in a homogeneous gravitational field (in relation to a "motionless" system of coordinates). If one assumes that the equivalence of these two cases is complete, then one attains an adaptation of our theoretical thinking to the fact that the gravitational and inertial masses are equal.

From this it follows that there is no longer any reason for favoring, as a matter of principle, the "inertial systems"; and, we must admit on an equal footing also *non-linear* transformations of the coordinates (x_1, x_2, x_3, x_4). If we make such a transformation of a system of coordinates of the special theory of relativity, then the metric

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2$$

goes over into a general (Riemannian) metric of the form

$$ds^2 = g_{\mu\nu}\, dx_\mu\, dx_\nu \quad \text{(summed over } \mu \text{ and } \nu)$$

where the $g_{\mu\nu}$, symmetrical in μ and ν, are certain functions of $x_1 \ldots x_4$ which describe both the metric properties, and the gravitational field in relation to the new system of coordinates.

The foregoing improvement in the interpretation of the mechanical basis must, however, be paid for in that—as becomes evident on closer scrutiny—the new coordinates can no longer be interpreted as results of measurements on rigid bodies and clocks, as they could in the original system (an inertial system with vanishing gravitational field).

The passage to the general theory of relativity is realized by the assumption that such a representation of the field properties of space already mentioned, by functions $g_{\mu\nu}$ (that is to say, by a Riemann metric), is also justified in the *general* case in which there is no system of coordinates in relation to which the metric takes the simple quasi-Euclidean form of the special theory of relativity.

Now the coordinates, by themselves, no longer express metric relations, but only the "closeness" of objects whose coordinates differ but little from one another. All transformations of the coordinates have to be admitted so long as these transformations are free from singularities. Only such equations as are covariant in relation to arbitrary transformations in this sense have meaning as expressions of general laws of nature (postulate of general covariance).

The first aim of the general theory of relativity was a preliminary version which, while not meeting the requirements for constituting a closed system, could be connected in as simple a manner as possible with "directly observable facts." If the theory were restricted to pure gravitational mechanics, Newton's gravitational theory could serve as a model. This preliminary version may be characterized as follows:

1. The concept of the material point and of its mass is retained. A law of motion is given for it, this law of motion being the translation of the law of inertia into the language of the general theory of relativity. This law is a system of total differential equations, the system characteristic of the geodesic line.

2. Newton's law of interaction by gravitation is replaced by the system of the simplest generally covariant differential equations which can be set up for the $g_{\mu\nu}$-tensor. It is formed by equating to zero the once contracted Riemannian curvature tensor ($R_{\mu\nu} = 0$).

This formulation permits the treatment of the problem of the planets. More accurately speaking, it allows the treatment of the problem of motion of material points of practically negligible mass in the (centrally symmetric) gravitational field produced by a material point supposed to be "at rest." It does not take into account the reaction of the "moving" material points on the gravitational field, nor does it consider how the central mass produces this gravitational field.

Analogy with classical mechanics shows that the following is a way to complete the theory. One sets up as field equations

$$R_{ik} - \tfrac{1}{2} g_{ik} R = - T_{ik}$$

where R represents the scalar of Riemannian curvature, T_{ik} the energy tensor of the matter in a phenomenological representation. The left side of the equation is chosen in such a manner that its divergence disappears identically. The resulting disappearance of the divergence of the right side produces the "equations of motion" of matter, in the form of partial differential equations for the case where T_{ik} introduces, for the description of the matter, only *four* further independent functions (for instance, density, pressure, and velocity components, where there is between the latter an identity, and between pressure and density an equation of condition).

By this formulation one reduces the whole mechanics of gravitation to the solution of a single system of covariant partial differential equations. The theory avoids all the shortcomings which we have charged against the basis of classical mechanics. It is sufficient—as far as we know—for the representation of the observed facts of celestial mechanics. But it is similar to a building, one wing of which is made of fine marble (left part of the equation), but the other wing of which is built of low-grade wood (right side of equation). The phenomenological representation of matter is, in fact, only a crude substitute for a representation which would do justice to all known properties of matter.

There is no difficulty in connecting Maxwell's theory of the electromagnetic field with the theory of the gravitational field so long as one restricts himself to space free of ponderable matter and free of electric density. All that is necessary is to put on the right-hand side of the above equation for T_{ik} the energy tensor of the electromagnetic field in empty space and to adjoin to the so modified system of equations the Maxwell field equation for empty space, written in general covariant form. Under these conditions there will exist, between all these equations, a sufficient number of differential identities to guarantee their consistency. We may add that this necessary formal property of the total system of equations leaves arbitrary the choice of the sign of the member T_{ik}, a fact which later turned out to be important.

The desire to have, for the foundations of the theory, the greatest possible unity has resulted in several attempts to include the gravitational field and the electromagnetic field in one unified formal picture. Here we must mention particularly the five-dimensional theory of Kaluza and Klein. Having considered this possibility very carefully, I feel that it is more desirable to accept the lack of internal uniformity of the original theory, because I do not think that the totality of the hypotheses at the basis of the five-dimensional theory contains less arbitrary features than does the original theory. The same statement may be made for the projective version of the theory, which has been elaborated with great care, in particular, by v. Dantzig and by Pauli.

The foregoing considerations concern, exclusively, the theory of the field, free of matter. How are we to proceed from this point in order to obtain a complete theory of atomically constituted matter? In such a theory, singularities must certainly be excluded, since without such exclusion the differential equations do not completely determine the total field. Here, in the field theory of general relativity, we meet the same problem of a field-theoretical representation of matter as was met originally in connection with the pure Maxwell theory.

Here again the attempt of a field-theoretical construction of particles leads apparently to singularities. Here also the endeavor has been made to overcome this defect by the introduction of new field variables and by elaborating and extending the system of field equations. Recently, however, I discovered, in collaboration with Dr. Rosen, that the above-mentioned simplest combination of the field equations of gravitation and electricity produces centrally symmetrical solutions which can be represented as free of singularity (the well-known centrally symmetrical solutions of Schwarzschild for the pure gravitational field, and those of Reissner for the electric field with consideration of its gravitational action). We shall refer to this shortly in the paragraph next but one. In this way it seems possible to get for matter and its interactions a pure field theory free of additional hypotheses, one moreover whose test by submission to facts of experience does not lead to difficulties other than purely mathematical ones, which difficulties, however, are very serious.

V. Quantum Theory and the Fundamentals of Physics

The theoretical physicists of our generation are expecting the erection of a new theoretical basis for physics which would make use of fundamental concepts greatly different from those of the field theory considered up to now. The reason is that it has been found necessary to use—for the mathematical representation of the so-called quantum phenomena—entirely new methods.

While the failure of classical mechanics, as revealed by the theory of relativity, is connected with the finite speed of light (its not being ∞), it was discovered at the beginning of our century that there were other kinds of inconsistencies between deductions from mechanics and experimental facts, which inconsistencies are connected with the finite magnitude (its not being zero) of Planck's constant h. In particular, while molecular mechanics requires that both heat content and (monochromatic) radiation density of solid bodies should decrease *in proportion* to the decreasing absolute temperature, experience has shown that they decrease much more rapidly than the absolute temperature. For a theoretical explanation of this behavior it was necessary to assume that the energy of a mechanical system cannot assume arbitrary values, but only certain discrete values whose mathematical expressions were always dependent upon Planck's constant h. Moreover, this conception was essential for the theory of the atom (Bohr's theory). For the transitions of these states into one another—with or without emission or absorption of radiation—no causal laws could be given, but only statistical ones; and a similar conclusion holds for the radioactive decay of atoms, which was carefully investigated about the same time. For more than two decades physicists tried vainly to find a uniform interpretation of this "quantum character" of systems and phenomena. Such an attempt was successful about ten years ago, through the agency of two entirely different theoretical methods of attack. We owe one of these to Heisenberg and Dirac, and the other to de Broglie and Schrödinger. The mathematical equivalence of the two methods was soon recognized by Schrödinger. I shall try here to sketch the line of thought of de Broglie and Schrödinger, which lies closer to the physicist's method of thinking, and shall accompany the description with certain general considerations.

The question is first: How can one assign a discrete succession of energy values H_σ to a system specified in the sense of classical mechanics (the energy function is a given function of the coordinates q_r and the corresponding momenta p_r)? Planck's constant h relates the frequency H_σ/h to the energy values H_σ. It is therefore sufficient to assign to the system a succession of discrete *frequency* values. This reminds us of the fact that in acoustics a series of discrete frequency values is coordinated to a linear partial differential equation (for given boundary conditions) namely, the sinusoidal periodic solutions. In corresponding manner, Schrödinger set himself the task of coordinating a partial differential equation for a scalar function ψ to the given energy function $\mathcal{E}(q_r, p_r)$, where the q_r and the time t are independent variables. In this he succeeded (for a complex function ψ) in such a manner that the theoretical values of the energy H_σ, as required by the statistical theory, actually resulted in a satisfactory manner from the periodic solutions of the equation.

To be sure, it did not happen to be possible to associate a definite movement, in the sense of mechanics of material points, with a definite solution $\psi(q_r, t)$ of the Schrödinger equation. This means that the ψ function does not determine, at any rate *exactly*, the story of the q_r as functions of the time t. According to Born, however, an interpretation of the physical meaning of the ψ functions was shown to be possible in the following manner: $\psi\bar{\psi}$ (the square of the absolute value of the complex function ψ) is the probability density at the point under consideration in the configuration-space of the q_r, at the time t. It is therefore possible to characterize the content of the Schrödinger equation in a manner, easy to be understood, but not quite accurate, as follows: it determines how the probability density of a statistical ensemble of systems varies in the configuration-space with the time. Briefly: the Schrödinger equation determines the change of the function ψ of the q_r with time.

It must be mentioned that the results of this theory contain —as limiting values—the results of particle mechanics if the wave-lengths encountered in the solution of the Schrödinger problem are everywhere so small that the potential energy varies by a practically infinitely small amount for a distance of one wave-length in the configuration-space. Under these conditions the following can in fact be shown: We choose a region G_0 in the configuration-space which, although large (in every direction) in relation to the wave-length, is small in relation to the relevant dimensions of the configuration-space. Under these conditions it is possible to choose a function ψ for an initial time t_0 in such a manner that it vanishes outside the region G_0, and behaves, according to the Schrödinger equation, in such a manner that it retains this property—approximately at least—also for a later time, but with the region G_0 having passed at that time t into another region G. In this manner one can, with a certain degree of approximation, speak of the motion of the region G as a whole, and one can approximate this motion by the motion of a point in the configuration-space. This motion then coincides with the motion which is required by the equations of classical mechanics.

Experiments on interference made with particle rays have given a brilliant proof that the wave character of the phenomena of motion as assumed by the theory does, really, correspond to the facts. In addition to this, the theory succeeded, easily, in demonstrating the statistical laws of the transition of a system from one quantum state to another under the action of external forces, which, from the standpoint of classical mechanics, appears as a miracle. The external forces were here represented by small time dependent additions to the potential energy. Now, while in classical mechanics, such additions can produce only correspondingly small changes of the system, in the quantum mechanics they produce changes of any magnitude however large, but with correspondingly small probability, a consequence in perfect harmony with experience. Even an understanding of the laws of radioactive decay, at least in broad outline, was provided by the theory.

Probably never before has a theory been evolved which has given a key to the interpretation and calculation of such a heterogeneous group of phenomena of experience as has quantum theory. In spite of this, however, I believe that the theory is apt to beguile us into error in our search for a uniform basis for physics, because, in my belief, it is an *incomplete* representation of real things, although it is the only one which can be built out of the fundamental concepts of force and material points (quantum corrections to classical mechanics). The incompleteness of the representation leads necessarily to the statistical nature (incompleteness) of the laws. I will now give my reasons for this opinion.

I ask first: How far does the ψ function describe a real state of a mechanical system? Let us assume the ψ_r to be the periodic solutions (put in the order of increasing energy values) of the Schrödinger equation. I shall leave open, for the time being, the question as to how far the individual ψ_r are *complete* descriptions of physical states. A system is first in the state ψ_1 of lowest energy $\mathcal{E}_1$. Then during a finite time a small disturbing force acts upon the system. At a later instant one obtains then from the Schrödinger equation a ψ function of the form

$$\psi = \Sigma\, c_r \psi_r$$

where the c_r are (complex) constants. If the ψ_r are "normalized," then $|c_1|$ is nearly equal to 1, $|c_2|$ etc. is small compared with 1. One may now ask: Does ψ describe a real state of the system? If the answer is yes, then we can hardly do otherwise than ascribe * to this state a definite energy $\mathcal{E}$, and, in particular, an energy which exceeds $\mathcal{E}_1$ by a small amount (in any case $\mathcal{E}_1 < \mathcal{E} < \mathcal{E}_2$). Such an assumption is, however, at variance with the experiments on electron impact such as have been made by J. Franck and G. Hertz, if one takes into account Millikan's demonstration of the discrete nature of electricity. As a matter of fact, these experiments lead to the conclusion that energy values lying between the quantum values do not exist. From this it follows that our function ψ does not in any way describe a homogeneous state of the system, but represents rather a statistical description in which the c_r represent probabilities of the individual energy values. It seems to be clear, therefore, that Born's statistical interpretation of quantum theory is the only possible one. The ψ function does not in any way describe a state which could be that of a single system; it relates rather to many systems, to "an en-

* Because, according to a well-established consequence of the relativity theory, the energy of a complete system (at rest) is equal to its inertia (as a whole). This, however, must have a well-defined value.

semble of systems" in the sense of statistical mechanics. If, except for certain special cases, the ψ function furnishes only *statistical* data concerning measurable magnitudes, the reason lies not only in the fact that the *operation of measuring* introduces unknown elements, which can be grasped only statistically, but because of the very fact that the ψ function does not, in any sense, describe the state of *one* single system. The Schrödinger equation determines the time variations which are experienced by the ensemble of systems which may exist with or without external action on the single system.

Such an interpretation eliminates also the paradox recently demonstrated by myself and two collaborators, and which relates to the following problem.

Consider a mechanical system consisting of two partial systems *A* and *B* which interact with each other only during a limited time. Let the ψ function before their interaction be given. Then the Schrödinger equation will furnish the ψ function after the interaction has taken place. Let us now determine the physical state of the partial system *A* as completely as possible by measurements. Then quantum mechanics allows us to determine the ψ function of the partial system *B* from the measurements made, and from the ψ function of the total system. This determination, however, gives a result which depends upon *which* of the physical quantities (observables) of *A* have been measured (for instance, coordinates *or* momenta). Since there can be only *one* physical state of *B* after the interaction which cannot reasonably be considered to depend on the particular measurement we perform on the system *A* separated from *B* it may be concluded that the ψ function is *not* unambiguously coordinated to the physical state. This coordination of several ψ functions to the same physical state of system *B* shows again that the ψ function cannot be interpreted as a (complete) description of a physical state of a single system. Here also the coordination of the ψ function to an ensemble of systems eliminates every difficulty.*

* A measurement on *A*, for example, thus involves a transition to a narrower ensemble of systems. The latter (hence also its ψ function) depends upon the point of view according to which this reduction of the ensemble of systems is carried out.

The fact that quantum mechanics affords, in such a simple manner, statements concerning (apparently) discontinuous transitions from one state to another without actually giving a description of the specific process—this fact is connected with another, namely, the fact that the theory, in reality, does not operate with the single system, but with a totality of systems. The coefficients c_r of our first example are really altered very *little* under the action of the external force. With this interpretation of quantum mechanics one can understand why this theory can easily account for the fact that weak disturbing forces are able to produce changes of any magnitude in the physical state of a system. Such disturbing forces produce, indeed, only correspondingly small changes of the *statistical density* in the ensemble of systems, and hence only infinitely weak changes of the ψ functions, the mathematical description of which offers far less difficulty than would be involved in the mathematical description of finite changes experienced by part of the single systems. What happens to the single system remains, it is true, *entirely unclarified by this mode of consideration; this enigmatic event is entirely eliminated from the description by the statistical approach.*

But now I ask: Is there really any physicist who believes that we shall never get any insight into these important changes in the single systems, in their structure and their causal connections, regardless of the fact that these single events have been brought so close to us, thanks to the marvelous inventions of the Wilson chamber and the Geiger counter? To believe this is logically possible without contradiction; but, it is so very contrary to my scientific instinct that I cannot forego the search for a more complete conception.

To these considerations we should add those of another kind which also appear to indicate that the methods introduced by quantum mechanics are not likely to give a useful basis for the whole of physics. In the Schrödinger equation, absolute time, and also the potential energy, play a decisive rôle, while these two concepts have been recognized by the theory of relativity as inadmissible in principle. If one wishes to escape from this difficulty, he must found the theory upon field and field laws instead of upon forces of interaction. This leads us to apply the statistical methods of quantum mechanics to fields, that is, to systems of infinitely many degrees of freedom. Although the attempts so far made are restricted to linear equations, which, as we know from the results of the general theory of relativity, are insufficient, the complications met up to now by the very ingenious attempts are already terrifying. They certainly will multiply if one wishes to obey the requirements of the general theory of relativity, the justification of which in principle nobody doubts.

To be sure, it has been pointed out that the introduction of a space-time continuum may be considered as contrary to nature in view of the molecular structure of everything which happens on a small scale. It is maintained that perhaps the success of the Heisenberg method points to a purely algebraical method of description of nature, that is, to the elimination of continuous functions from physics. Then, however, we must also give up, on principle, the space-time continuum. It is conceivable that human ingenuity will some day find methods which will make it possible to proceed along such a path. At the present time, however, such a program looks like an attempt to breathe in empty space.

There is no doubt that quantum mechanics has seized hold of a good deal of truth, and that it will be a touchstone for any future theoretical basis, in that it must be deducible as a limiting case from that basis, just as electrostatics is deducible from the Maxwell equations of the electromagnetic field or as thermodynamics is deducible from classical mechanics. However, I do not believe that quantum mechanics can serve as a *starting point* in the search for this basis, just as, vice versa, one could not find from thermodynamics (resp. statistical mechanics) the foundations of mechanics.

In view of this situation, it seems to be entirely justifiable seriously to consider the question as to whether the basis of field physics cannot by *any* means be put into harmony with quantum phenomena. Is this not the only basis which, with the presently available mathematical tools, can be adapted to the requirements of the general theory of relativity? The belief, prevailing among the physicists of today, that such an attempt would be hopeless, may have its root in the unwarranted assumption that such a theory must lead, in first approximation, to the equations of classical mechanics for the motion of corpuscles, or at least to total differential equations. As a matter of fact, up to now we have never succeeded in a field-theoretical description of corpuscles free of singularities, and we can, *a priori*, say nothing about the behavior of such entities. *One thing*, however, is certain: if a field theory results in a representation of corpuscles free of singularities, then the behavior of these corpuscles in time is determined solely by the differential equations of the field.

VI. Relativity Theory and Corpuscles

I shall now show that, according to the general theory of relativity, there exist singularity-free solutions of field equations which can be interpreted as representing corpuscles. I restrict myself here to neutral particles because, in another recent publication in collaboration with Dr. Rosen, I have treated this question in detail, and because the essentials of the problem can be completely exhibited in this case.

The gravitational field is entirely described by the tensor $g_{\mu\nu}$. In the three-index symbols $\Gamma^{\sigma}_{\mu\nu}$, there appear also the contravariant $g^{\mu\nu}$ which are defined as the minors of the $g_{\mu\nu}$ divided by the determinant $g(=|g_{\alpha\beta}|)$. In order that the R_{ik} shall be defined and finite, it is not sufficient that there shall be, in the neighborhood of every point of the continuum, a system of coordinates in which the $g_{\mu\nu}$ and their first differential quotients are continuous and differentiable, but it is also necessary that the determinant g shall nowhere vanish. This last restriction disappears, however, if one replaces the differential equations $R_{ik}=0$ by $g^2R_{ik}=0$, the left-hand sides of which are *whole* rational functions of the g_{ik} and of their derivatives.

These equations have the centrally symmetrical solution given by Schwarzschild

$$ds^2 = -\frac{1}{1-2m/r}\,dr^2 - r^2(d\theta^2+\sin^2\theta d\varphi^2) + \left(1-\frac{2m}{r}\right)dt^2$$

This solution has a singularity at $r=2m$, since the coefficient of dr^2 (i.e., g_{11}), becomes infinite on this hypersurface. If, however, we replace the variable r by ρ defined by the equation

$$\rho^2 = r - 2m$$

we obtain

$$ds^2 = -4(2m + \rho^2)d\rho^2 - (2m + \rho^2)^2(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{\rho^2}{2m + \rho^2}\, dt^2$$

This solution behaves regularly for all values of ρ. The vanishing of the coefficient of dt^2 (i.e., g_{44}) for $\rho = 0$ results, it is true, in the consequence that the determinant g vanishes for this value; but, with the methods of writing the field equations actually adopted, this does not constitute a singularity.

If ρ varies from $-\infty$ to $+\infty$, then r varies from $+\infty$ to $r = 2m$ and then back to $+\infty$, while for such values of r as correspond to $r < 2m$ there are no corresponding real values of ρ. Hence the Schwarzschild solution becomes a regular solution by representing the physical space as consisting of two identical "sheets" in contact along the hypersurface $\rho = 0$ (i.e., $r = 2m$), on which the determinant g vanishes. Let us call such a connection between the two (identical) sheets a "bridge." Hence the existence of such a bridge between the two sheets in the finite realm corresponds to the existence of a material neutral particle which is described in a manner free from singularities.

The solution of the problem of the motion of neutral particles evidently amounts to the discovery of such solutions of the gravitational equations (written free of denominators), as contain several bridges.

The conception sketched above corresponds, *a priori*, to the atomistic structure of matter in so far as the "bridge" is by its nature a discrete element. Moreover, we see that the mass constant m of the neutral particles must necessarily be positive, since no solution free of singularities can correspond to the Schwarzschild solution for a negative value of m. Only the examination of the several-bridge-problem can show whether or not this theoretical method furnishes an explanation of the empirically demonstrated equality of the masses of the particles found in nature, and whether it takes into account the facts which the quantum mechanics has so wonderfully comprehended.

In an analogous manner, it is possible to demonstrate that the combined equations of gravitation and electricity (with appropriate choice of the sign of the electrical member in the gravitational equations) produce a singularity-free bridge-representation of the electric corpuscle. The simplest solution of this kind is that for an electrical particle without gravitational mass.

So long as the considerable mathematical difficulties concerned with the solution of the several-bridge-problem are not overcome, nothing can be said concerning the usefulness of the theory from the physicist's point of view. However, it constitutes, as a matter of fact, the first attempt toward the consistent elaboration of a field theory which presents a possibility of explaining the properties of matter. In favor of this attempt one should also add that it is based on the simplest possible relativistic field equations known today.

Summary

Physics constitutes a logical system of thought which is in a state of evolution, whose basis cannot be distilled, as it were, from *experience by an inductive method*, but *can only* be arrived at by free invention. The justification (truth content) of the system rests in the verification of the derived propositions by sense experiences, whereby the relations of the latter to the former can only be comprehended intuitively. Evolution is proceeding in the direction of increasing simplicity of the logical basis. In order further to approach this goal, we must resign to *the fact that the logical basis departs more and more from the* facts of experience, and that the path of our thought from the fundamental basis to those derived propositions, which correlate with sense experiences, becomes continually harder and longer.

Our aim has been to sketch, as briefly as possible, the development of the fundamental concepts in their dependence upon the facts of experience and upon the endeavor to achieve *internal perfection of the system. These considerations* were intended to illuminate the present state of affairs, as it appears to me. (It is unavoidable that a schematic historic exposition is subjectively colored.)

I try to demonstrate how the concepts of bodily objects, space, subjective and objective time, are connected with one another and with the nature of our experience. In classical mechanics the concepts of space and time become independent. The concept of the bodily object is replaced in the foundations by the concept of the material point, by which means mechanics becomes fundamentally atomistic. Light and electricity produce insurmountable difficulties when one attempts to make mechanics the basis of all physics. We are thus led to the field theory of electricity, and, later on to the attempt to base physics entirely upon the concept of the field (after an attempted compromise with classical mechanics). This attempt leads to the theory of relativity (evolution of the notion of space and time into that of the continuum with metric structure).

I try to demonstrate, furthermore, why in my opinion quantum theory does not seem capable to furnish an adequate foundation for physics: one becomes involved in contradictions if one tries to consider the theoretical quantum description as a *complete* description of the individual physical system or event.

On the other hand, the field theory is as yet unable to explain the molecular structure of matter and of quantum phenomena. It is shown, however, that the conviction of the inability of field theory to solve these problems by its methods rests upon prejudice.

THE FUNDAMENTS OF THEORETICAL PHYSICS

From Science, *Washington, D. C. May 24, 1940.*

Science is the attempt to make the chaotic diversity of our sense-experience correspond to a logically uniform system of thought. In this system single experiences must be correlated with the theoretic structure in such a way that the resulting coordination is unique and convincing.

The sense-experiences are the given subject-matter. But the theory that shall interpret them is man-made. It is the result of an extremely laborious process of adaptation: hypothetical, forgotten about his responsibility and dignity? My answer is: while it is true that an inherently free and scrupulous person may be destroyed, such an individual can never be enslaved or used as a blind tool.

If the man of science of our own days could find the time and the courage to think over honestly and critically his situation and the tasks before him and if he would act accordingly, the possibilities for a sensible and satisfactory solution of the present dangerous international situation would be considerably improved.

MESSAGE ON THE 410TH ANNIVERSARY OF THE DEATH OF COPERNICUS

On the occasion of the commemoration evening held at Columbia University, New York, in December, 1953.

We are honoring today, with joy and gratitude, the memory of a man who, more than almost anyone else, contributed to the liberation of the mind from the chains of clerical and scientific dominance in the Occident.

It is true that some scholars in the classic Greek period had become convinced that the earth is not the natural center of the world. But this comprehension of the universe could not gain real recognition in antiquity. Aristotle and the Greek school of astronomers continued to adhere to the geocentric conception, and hardly anyone had any doubt about it.

A rare independence of thought and intuition as well as a mastery of the astronomical facts, not easily accessible in those days, were necessary to expound the superiority of the heliocentric conception convincingly. This great accomplishment of Copernicus not only paved the way to modern astronomy; it also helped to bring about a decisive change in man's attitude toward the cosmos. Once it was recognized that the earth was not the center of the world, but only one of the smaller planets, the illusion of the central significance of man himself became untenable. Hence, Copernicus, through his work and the greatness of his personality, taught man to be modest.

No nation should find pride in the fact that such a man developed in its midst. For national pride is quite a petty weakness which is hardly justifiable in face of a man of such inner independence as Copernicus.

RELATIVITY AND THE PROBLEM OF SPACE

From the revised edition of Relativity, the Special and the General Theory: A Popular Exposition. *Translated by Robert W. Lawson. London: Methuen, 1954.*

It is characteristic of Newtonian physics that it has to ascribe independent and real existence to space and time as well as to matter, for in Newton's law of motion the concept of acceleration appears. But in this theory, acceleration can only denote "acceleration with respect to space." Newton's space must thus be thought of as "at rest," or at least as "unaccelerated," in order that one can consider the acceleration, which appears in the law of motion, as being a magnitude with any meaning. Much the same holds with time, which of course likewise enters into the concept of acceleration. Newton himself and his most critical contemporaries felt it to be disturbing that one had to ascribe physical reality both to space itself as well as to its state of motion; but there was at that time no other alternative, if one wished to ascribe to mechanics a clear meaning.

It is indeed an exacting requirement to have at all to ascribe physical reality to space, and especially to empty space. Time and again since remotest times philosophers have resisted such a presumption. Descartes argued somewhat on these lines: space is identical with extension, but extension is connected with bodies; thus there is no space without bodies and hence no empty space. The weakness of this argument lies primarily in what follows. It is certainly true that the concept of extension owes its origin to our experiences of laying out or bringing into contact solid bodies. But from this it cannot be concluded that the concept of extension may not be justified in cases which have not themselves given rise to the formation of this concept. Such an enlargement of concepts can be justified indirectly by its value for the comprehension of empirical results. The assertion that extension is confined to bodies is therefore of itself certainly unfounded. We shall see later, however, that the general theory of relativity confirms Descartes' conception in a roundabout way. What brought Descartes to his seemingly odd view was certainly the feeling that, without compelling necessity, one ought not to ascribe reality to a thing like space, which is not capable of being "directly experienced." *

The psychological origin of the idea of space, or of the necessity for it, is far from being so obvious as it may appear to be on the basis of our customary habit of thought. The old geometers deal with conceptual objects (straight line, point, surface), but not really with space as such, as was done later in analytical geometry. The idea of space, however, is suggested by certain primitive experiences. Suppose that a box has been constructed. Objects can be arranged in a certain way inside the box, so that it becomes full. The possibility of such arrangements is a property of the material object "box," something that is given with the box, the "space enclosed" by the box. This is something which is different for different boxes, something that is thought quite naturally as being independent of whether or not, at any moment, there are any objects at all in the box. When there are no objects in the box, its space appears to be "empty."

So far, our concept of space has been associated with the box. It turns out, however, that the storage possibilities that make up the box-space are independent of the thickness of the walls of the box. Cannot this thickness be reduced to zero, without the "space" being lost as a result? The naturalness of such a limiting process is obvious, and now there remains for our thought the space without the box, a self-evident thing, yet it appears to be so unreal if we forget the origin of this concept. One can understand that it was repugnant to Descartes to consider space as independent of material objects, a thing that might exist without matter.† (At the same time, this does not prevent him from treating space as a fundamental concept in his analytical geometry.) The drawing of attention to the vacuum in a mer-

* This expression is to be taken *cum grano salis.*

† Kant's attempt to remove the embarrassment by denial of the objectivity of space can, however, hardly be taken seriously. The possibilities of packing inherent in the inside space of a box are objective in the same sense as the box itself, and as the objects which can be packed inside it.

cury barometer has certainly disarmed the last of the Cartesians. But it is not to be denied that, even at this primitive stage, something unsatisfactory clings to the concept of space, or to space thought of as an independent real thing.

The ways in which bodies can be packed into space (box) are the subject of three-dimensional Euclidean geometry, whose axiomatic structure readily deceives us into forgetting that it refers to realizable situations.

If now the concept of space is formed in the manner outlined above, and following on from experience about the "filling" of the box, then this space is primarily a *bounded* space. This limitation does not appear to be essential, however, for apparently a larger box can always be introduced to enclose the smaller one. In this way space appears as something unbounded.

I shall not consider here how the concepts of the three-dimensional and the Euclidean nature of space can be traced back to relatively primitive experiences. Rather, I shall consider first of all from other points of view the rôle of the concept of space in the development of physical thought.

When a smaller box *s* is situated, relatively at rest, inside the hollow space of a larger box *S*, then the hollow space of *s* is a part of the hollow space of *S*, and the same "space," which contains both of them, belongs to each of the boxes. When *s* is in motion with respect to *S*, however, the concept is less simple. One is then inclined to think that *s* encloses always the same space, but a variable part of the space *S*. It then becomes necessary to apportion to each box its particular space, not thought of as bounded, and to assume that these two spaces are in motion with respect to each other.

Before one has become aware of this complication, space appears as an unbounded medium or container in which material objects swim around. But it must now be remembered that there is an infinite number of spaces, which are in motion with respect to each other. The concept of space as something existing objectively and independent of things belongs to prescientific thought, but not so the idea of the existence of an infinite number of spaces in motion relatively to each other. This latter idea is indeed logically unavoidable, but is far from having played a considerable rôle even in scientific thought.

But what about the psychological origin of the concept of time? This concept is undoubtedly associated with the fact of "calling to mind," as well as with the differentiation between sense experiences and the recollection of these. Of itself it is doubtful whether the differentiation between sense experience and recollection (or a mere mental image) is something psychologically directly given to us. Everyone has experienced that he has been in doubt whether he has actually experienced something with his senses or has simply dreamed about it. Probably the ability to discriminate between these alternatives first comes about as the result of an activity of the mind creating order.

An experience is associated with a "recollection," and it is considered as being "earlier" in comparison with "present experiences." This is a conceptual ordering principle for recollected experiences, and the possibility of its accomplishment gives rise to the subjective concept of time, i.e., that concept of time which refers to the arrangement of the experiences of the individual.

What do we mean by rendering objective the concept of time? Let us consider an example. A person *A* ("I") has the experience "it is lightning." At the same time the person *A* also experiences such a behavior of the person *B* as brings the behavior of *B* into relation with his own experience "it is lightning." Thus it comes about that *A* associates with *B* the experience "it is lightning." For the person *A* the idea arises that other persons also participate in the experience "it is lightning." "It is lightning" is now no longer interpreted as an exclusively personal experience, but as an experience of other persons (or eventually only as a "potential experience"). In this way arises the interpretation that "it is lightning," which originally entered into the consciousness as an "experience," is now also interpreted as an (objective) "event." It is just the sum total of all events that we mean when we speak of the "real external world."

We have seen that we feel ourselves impelled to ascribe a temporal arrangement to our experiences, somewhat as follows. If β is later than α and γ later than β, then γ is also later than α

("sequence of experiences"). Now what is the position in this respect with the "events" which we have associated with the experiences? At first sight it seems obvious to assume that a temporal arrangement of events exists which agrees with the temporal arrangement of the experiences. In general, and unconsciously this was done, until skeptical doubts made themselves felt.* In order to arrive at the idea of an objective world, an additional constructive concept still is necessary: the event is localized not only in time, but also in space.

In the previous paragraphs we have attempted to describe how the concepts space, time, and event can be put psychologically into relation with experiences. Considered logically, they are free creations of the human intelligence, tools of thought, which are to serve the purpose of bringing experiences into relation with each other, so that in this way they can be better surveyed. The attempt to become conscious of the empirical sources of these fundamental concepts should show to what extent we are actually bound to these concepts. In this way we become aware of our freedom, of which, in case of necessity, it is always a difficult matter to make sensible use.

We still have something essential to add to this sketch concerning the psychological origin of the concepts space-time-event (we will call them more briefly "space-like," in contrast to concepts from the psychological sphere). We have linked up the concept of space with experiences using boxes and the arrangement of material objects in them. Thus this formation of concepts already presupposes the concept of material objects (e.g., "boxes"). In the same way persons, who had to be introduced for the formation of an objective concept of time, also play the rôle of material objects in this connection. It appears to me, therefore, that the formation of the concept of the material object must precede our concepts of time and space.

All these space-like concepts already belong to pre-scientific thought, along with concepts like pain, goal, purpose, etc., from the field of psychology. Now it is characteristic of thought in

* For example, the order of experiences in time obtained by acoustical means can differ from the temporal order gained visually, so that one cannot simply identify the time sequence of events with the time sequence of experiences.

physics, as of thought in natural science generally, that it endeavors in principle to make do with "space-like" concepts *alone*, and strives to express with their aid all relations having the form of laws. The physicist seeks to reduce colors and tones to vibrations; the physiologist, thought and pain to nerve processes, in such a way that the psychical element as such is eliminated from the causal nexus of existence, and thus nowhere occurs as an independent link in the causal associations. It is no doubt this attitude, which considers the comprehension of all relations by the exclusive use of only "space-like" concepts as being possible in principle, that is at the present time understood by the term "materialism" (since "matter" has lost its rôle as a fundamental concept).

Why is it necessary to drag down from the Olympian fields of Plato the fundamental ideas of thought in natural science, and to attempt to reveal their earthly lineage? Answer: In order to free these ideas from the taboo attached to them, and thus to achieve greater freedom in the formation of ideas or concepts. It is to the immortal credit of D. Hume and E. Mach that they, above all others, introduced this critical conception.

Science has taken over from pre-scientific thought the concepts space, time, and material object (with the important special case "solid body"), and has modified them and rendered them more precise. Its first significant accomplishment was the development of Euclidean geometry, whose axiomatic formulation must not be allowed to blind us to its empirical origin (the possibilities of laying out or juxtaposing solid bodies). In particular, the three-dimensional nature of space as well as its Euclidean character are of empirical origin (it can be wholly filled by like constituted "cubes").

The subtlety of the concept of space was enhanced by the discovery that there exist no completely rigid bodies. All bodies are elastically deformable and alter in volume with change in temperature. The structures, whose possible configurations are to be described by Euclidean geometry, cannot therefore be characterized without reference to the content of physics. But since physics after all must make use of geometry in the establishment of its concepts, the empirical content of geometry can be stated and tested only in the framework of the whole of physics.

In this connection atomistics must also be borne in mind, and its conception of finite divisibility; for spaces of sub-atomic extension cannot be measured up. Atomistics also compels us to give up, in principle, the idea of sharply and statically defined bounding surfaces of solid bodies. Strictly speaking, there are no *precise* laws, even in the macro-region, for the possible configurations of solid bodies touching each other.

In spite of this, no one thought of giving up the concept of space, for it appeared indispensable in the eminently satisfactory whole system of natural science. Mach, in the nineteenth century, was the only one who thought seriously of an elimination of the concept of space, in that he sought to replace it by the notion of the totality of the instantaneous distances between all material points. (He made this attempt in order to arrive at a satisfactory understanding of inertia.)

The Field

In Newtonian mechanics, space and time play a dual rôle. First, they play the part of carrier or frame for things that happen in physics, in reference to which events are described by the space coordinates and the time. In principle, matter is thought of as consisting of "material points," the motions of which constitute physical happening. When matter is thought of as being continuous, this is done, as it were, provisionally in those cases where one does not wish to or cannot describe the discrete structure. In this case small parts (elements of volume) of the matter are treated similarly to material points, at least in so far as we are concerned merely with motions and not with occurrences which, at the moment, it is not possible or serves no useful purpose to attribute to motions (e.g., temperature changes, chemical processes). The second rôle of space and time was that of being an "inertial system." Inertial systems were considered to be distinguished among all conceivable systems of reference in that, with respect to them, the law of inertia claimed validity.

In this, the essential thing is that "physical reality," thought of as being independent of the subjects experiencing it, was conceived as consisting, at least in principle, of space and time on one hand, and of permanently existing material points, moving with respect to space and time, on the other. The idea of the independent existence of space and time can be expressed drastically in this way: if matter were to disappear, space and time alone would remain behind (as a kind of stage for physical happening).

This standpoint was overcome in the course of a development which, in the first place, appeared to have nothing to do with the problem of space-time, namely, the appearance of the *concept of field* and its final claim to replace, in principle, the idea of a particle (material point). In the framework of classical physics, the concept of field appeared as an auxiliary concept, in cases in which matter was treated as a continuum. For example, in the consideration of the heat conduction in a solid body, the state of the body is described by giving the temperature at every point of the body for every definite time. Mathematically, this means that the temperature T is represented as a mathematical expression (function) of the space coordinates and the time t (temperature field). The law of heat conduction is represented as a local relation (differential equation), which embraces all special cases of the conduction of heat. The temperature is here a simple example of the concept of field. This is a quantity (or a complex of quantities), which is a function of the coordinates and the time. Another example is the description of the motion of a liquid. At every point there exists at any time a velocity, which is quantitatively described by its three "components" with respect to the axes of a coordinate system (vector). The components of the velocity at a point (field components), here also are functions of the coordinates (x, y, z) and the time (t).

It is characteristic of the fields mentioned that they occur only within a ponderable mass; they serve only to describe a state of this matter. In accordance with the historical development of the field concept, where no matter was available there could also exist no field. But in the first quarter of the nineteenth century it was shown that the phenomena of the interference and the diffraction of light could be explained with astonishing accuracy when light was regarded as a wave-field, completely analogous to the mechanical vibration field in an elastic solid body. It was thus felt necessary to introduce a field, that could also exist in "empty space" in the absence of ponderable matter.

This state of affairs created a paradoxical situation, because, in accordance with its origin, the field concept appeared to be restricted to the description of states in the inside of a ponderable body. This seemed to be all the more certain, inasmuch as the conviction was held that every field is to be regarded as a state capable of mechanical interpretation, and this presupposed the presence of matter. One thus felt compelled, even in the space which had hitherto been regarded as empty, to assume everywhere the existence of a form of matter, which was called "ether."

The emancipation of the field concept from the assumption of its association with a mechanical carrier finds a place among the psychologically most interesting events in the development of physical thought. During the second half of the nineteenth century, in connection with the researches of Faraday and Maxwell, it became more and more clear that the description of electromagnetic processes in terms of field was vastly superior to a treatment on the basis of the mechanical concepts of material points. By the introduction of the field concept in electrodynamics, Maxwell succeeded in predicting the existence of electromagnetic waves, the essential identity of which with light waves could not be doubted, if only because of the equality of their velocity of propagation. As a result of this, optics was, in principle, absorbed by electrodynamics. *One* psychological effect of this immense success was that the field concept gradually won greater independence from the mechanistic framework of classical physics.

Nevertheless, it was at first taken for granted that electromagnetic fields had to be interpreted as states of the ether, and it was zealously sought to explain these states as mechanical ones. But as these efforts always met with frustration, science gradually became accustomed to the idea of renouncing such a mechanical interpretation. Nevertheless, the conviction still remained that electromagnetic fields must be states of the ether, and this was the position at the turn of the century.

The ether-theory brought with it the question: how does the ether behave from the mechanical point of view with respect to ponderable bodies? Does it take part in the motions of the bodies, or do its parts remain at rest relatively to each other? Many ingenious experiments were undertaken to decide this question. The following important facts should be mentioned in this connection: the "aberration" of the fixed stars in consequence of the annual motion of the earth, and the "Doppler effect," i.e., the influence of the relative motion of the fixed stars on the frequency of the light reaching us from them, for known frequencies of emission. The results of all these facts and experiments, except for one, the Michelson-Morley experiment, were explained by H. A. Lorentz on the assumption that the ether does not take part in the motions of ponderable bodies, and that the parts of the ether have no relative motions at all with respect to each other. Thus the ether appeared, as it were, as the embodiment of a space absolutely at rest. But the investigation of Lorentz accomplished still more. It explained all the electromagnetic and optical processes within ponderable bodies known at that time, on the assumption that the influence of ponderable matter on the electric field—and conversely—is due solely to the fact that the constituent particles of matter carry electrical charges, which share the motion of the particles. Concerning the experiment of Michelson and Morley, H. A. Lorentz showed that the result obtained at least does not contradict the theory of an ether at rest.

In spite of all these beautiful successes the state of the theory was not yet wholly satisfactory, and for the following reasons. Classical mechanics, of which it could not be doubted that it holds with a close degree of approximation, teaches the equivalence of all inertial systems or inertial "spaces" for the formulation of natural laws, i.e., the invariance of natural laws with respect to the transition from one inertial system to another. Electromagnetic and optical *experiments* taught the same thing with considerable accuracy. But the foundation of electromagnetic *theory* taught that a particular inertial system must be given preference, namely, that of the luminiferous ether at rest. This view of the theoretical foundation was much too unsatisfactory. Was there no modification that, like classical mechanics, would uphold the equivalence of inertial systems (special principle of relativity)?

The answer to this question is the special theory of relativity. This takes over from the theory of Maxwell-Lorentz the assumption of the constancy of the velocity of light in empty space. In order to bring this into harmony with the equivalence of inertial systems (special principle of relativity), the idea of the absolute character of simultaneity must be given up; in addition, the Lorentz transformations for the time and the space coordinates follow for the transition from one inertial system to another. The whole content of the special theory of relativity is included in the postulate: the laws of nature are invariant with respect to the Lorentz transformations. The importance of this requirement lies in the fact that it limits the possible natural laws in a definite manner.

What is the position of the special theory of relativity in regard to the problem of space? In the first place we must guard against the opinion that the four-dimensionality of reality has been newly introduced for the first time by this theory. Even in classical physics the event is localized by four numbers, three spatial coordinates and a time coordinate; the totality of physical "events" is thus thought of as being embedded in a four-dimensional continuous manifold. But on the basis of classical mechanics this four-dimensional continuum breaks up objectively into the one-dimensional time and into three-dimensional spatial sections, the latter of which contain only simultaneous events. This resolution is the same for all inertial systems. The simultaneity of two definite events with reference to one inertial system involves the simultaneity of these events in reference to all inertial systems. This is what is meant when we say that the time of classical mechanics is absolute. According to the special theory of relativity it is otherwise. The sum total of events which are simultaneous with a selected event exist, it is true, in relation to a particular inertial system, but no longer independently of the choice of the inertial system. The four-dimensional continuum is now no longer resolvable objectively into sections, which contain all simultaneous events; "now" loses for the spatially extended world its objective meaning. It is because of this that space and time must be regarded as a four-dimensional continuum that is objectively unresolvable, if it is desired to express the purport of objective relations without unnecessary conventional arbitrariness.

Since the special theory of relativity revealed the physical equivalence of all inertial systems, it proved the untenability of the hypothesis of an ether at rest. It was therefore necessary to renounce the idea that the electromagnetic field is to be regarded as a state of a material carrier. The field thus becomes an irreducible element of physical description, irreducible in the same sense as the concept of matter in the theory of Newton.

Up to now we have directed our attention to finding in what respect the concepts of space and time were *modified* by the special theory of relativity. Let us now focus our attention on those elements which this theory has taken over from classical mechanics. Here also, natural laws claim validity only when an inertial system is taken as the basis of space-time description. The principle of inertia and the principle of the constancy of the velocity of light are valid only with respect to an *inertial system*. The field-laws also can claim to have meaning and validity only in regard to inertial systems. Thus, as in classical mechanics, space is here also an independent component in the representation of physical reality. If we imagine matter and field to be removed, inertial space or, more accurately, this space together with the associated time remains behind. The four-dimensional structure (Minkowski-space) is thought of as being the carrier of matter and of the field. Inertial spaces, with their associated times, are only privileged four-dimensional coordinate systems that are linked together by the linear Lorentz transformations. Since there exist in this four-dimensional structure no longer any sections which represent "now" objectively, the concepts of happening and becoming are indeed not completely suspended, but yet complicated. It appears therefore more natural to think of physical reality as a four-dimensional existence, instead of, as hitherto, the *evolution* of a three-dimensional existence.

This rigid four-dimensional space of the special theory of relativity is to some extent a four-dimensional analogue of H. A. Lorentz's rigid three-dimensional ether. For this theory also the following statement is valid: the description of physical states postulates space as being initially given and as existing independently. Thus even this theory does not dispel Descartes' uneasiness concerning the independent, or indeed, the *a priori* existence of "empty space." The real aim of the elementary discussion given here is to show to what extent these doubts are overcome by the general theory of relativity.

The Concept of Space in the General Theory of Relativity

This theory arose primarily from the endeavor to understand the equality of inertial and gravitational mass. We start out from an inertial system S_1, whose space is, from the physical point of view, empty. In other words, there exists in the part of space contemplated neither matter (in the usual sense) nor a field (in the sense of the special theory of relativity). With reference to S_1 let there be a second system of reference S_2 in uniform acceleration. Then S_2 is thus not an inertial system. With respect to S_2 every test mass would move with an acceleration, which is independent of its physical and chemical nature. Relative to S_2, therefore, there exists a state which, at least to a first approximation, cannot be distinguished from a gravitational field. The following concept is thus compatible with the observable facts: S_2 is also equivalent to an "inertial system"; but with respect to S_2 a (homogeneous) gravitational field is present (about the origin of which one does not worry in this connection). Thus when the gravitational field is included in the framework of the consideration, the inertial system loses its objective significance, assuming that this "principle of equivalence" can be extended to any relative motion whatsoever of the systems of reference. If it is possible to base a consistent theory on these fundamental ideas, it will satisfy of itself the fact of the equality of inertial and gravitational mass, which is strongly confirmed empirically.

Considered four-dimensionally, a non-linear transformation of the four coordinates corresponds to the transition from S_1 to S_2. *The question now arises: what kind of non-linear transformations are to be permitted, or, how is the Lorentz transformation to be generalized?* In order to answer this question, the following consideration is decisive.

We ascribe to the inertial system of the earlier theory this property: differences in coordinates are measured by stationary "rigid" measuring rods, and differences in time by clocks at rest. The first assumption is supplemented by another, namely, that for the relative laying out and fitting together of measuring rods at rest, the theorems on "lengths" in Euclidean geometry hold. From the results of the special theory of relativity it is then concluded, by elementary considerations, that this direct physical interpretation of the coordinates is lost for systems of reference (S_2) accelerated relatively to inertial systems (S_1). But if this is the case, the coordinates now express only the order or rank of the "contiguity" and hence also the number of dimensions of the space, but do not express any of its metrical properties. We are thus led to extend the transformations to arbitrary continuous transformations.* This implies the general principle of relativity: Natural laws must be covariant with respect to arbitrary continuous transformations of the coordinates. This requirement (combined with that of the greatest possible logical simplicity of the laws) limits the natural laws concerned incomparably more strongly than the special principle of relativity.

This train of ideas is based essentially on the field as an independent concept. For the conditions prevailing with respect to S_2 are interpreted as a gravitational field, without the question of the existence of masses which produce this field being raised. By virtue of this train of ideas it can also be grasped why the laws of the pure gravitational field are more directly linked with the idea of general relativity than the laws

* This inexact mode of expression will perhaps suffice here.

for fields of a general kind (when, for instance, an electromagnetic field is present). We have, namely, good ground for the assumption that the "field-free" Minkowski-space represents a special case possible in natural law, in fact, the simplest conceivable special case. With respect to its metrical character, such a space is characterized by the fact that $dx_1^2 + dx_2^2 + dx_3^2$ is the square of the spatial separation, measured with a unit gauge, of two infinitesimally neighboring points of a three-dimensional "space-like" cross section (Pythagorean theorem), whereas dx_4 is the temporal separation, measured with a suitable time gauge, of two events with common (x_1, x_2, x_3). All this simply means that an objective metrical significance is attached to the quantity

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 - dx_4^2 \quad . \quad (1)$$

as is readily shown with the aid of the Lorentz transformations. Mathematically, this fact corresponds to the condition that ds^2 is invariant with respect to Lorentz transformations.

If now, in the sense of the general principle of relativity, this space (cf. eq. (1)) is subjected to an arbitrary continuous transformation of the coordinates, then the objectively significant quantity ds is expressed in the new system of coordinates by the relation

$$ds^2 = g_{ik}dx_i dx_k \quad . \quad . \quad . \quad (1a)$$

which has to be summed up over the indices i and k for all combinations 11, 12, . . . up to 44. The terms g_{ik} now are not constants, but functions of the coordinates, which are determined by the arbitrarily chosen transformation. Nevertheless, the terms g_{ik} are not arbitrary functions of the new coordinates, but just functions of such a kind that the form (1*a*) can be transformed back again into the form (1) by a continuous transformation of the four coordinates. In order that this may be possible, the functions g_{ik} must satisfy certain general covariant equations of condition, which were derived by B. Riemann more than half a century before the formulation of the general theory of relativity ("Riemann condition"). According to the principle of equivalence, (1*a*) describes in general covariant form a gravitational field of a special kind, when the functions g_{ik} satisfy the Riemann condition.

It follows that the law for the pure gravitational field of a general kind must be satisfied when the Riemann condition is satisfied; but it must be weaker or less restricting than the Riemann condition. In this way the field law of pure gravitation is practically completely determined, a result which will not be justified in greater detail here.

We are now in a position to see how far the transition to the general theory of relativity modifies the concept of space. In accordance with classical mechanics and according to the special theory of relativity, space (space-time) has an existence independent of matter or field. In order to be able to describe at all that which fills up space and is dependent on the coordinates, space-time or the inertial system with its metrical properties must be thought of as existing to start with, for otherwise the *description of "that which fills up space" would have no meaning.** On the basis of the general theory of relativity, on the other hand, space as opposed to "what fills space," which is dependent on the coordinates, has no separate existence. Thus a pure gravitational field might have been described in terms of the g_{ik} (as functions of the coordinates), by solution of the gravitational equations. If we imagine the gravitational field, i.e., *the functions g_{ik}, to be removed, there does not remain* a space of the type (1), but absolutely *nothing*, and also no "topological space." For the functions g_{ik} describe not only the field, but at the same time also the topological and metrical structural properties of the manifold. A space of the type (1), judged from the standpoint of the general theory of relativity, is not a space without field, but a special case of the g_{ik} field, for which—for the coordinate system used, which in itself has no objective significance—the functions g_{ik} have values that do not depend on the coordinates. There is no such thing as an empty space, i.e., a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field.

Thus Descartes was not so far from the truth when he be-

* If we consider that which fills space (e.g., the field) to be removed, there still remains the metric space in accordance with (1), which would also determine the inertial behavior of a test body introduced into it.

lieved he must exclude the existence of an empty space. The notion indeed appears absurd, as long as physical reality is seen exclusively in ponderable bodies. It requires the idea of the field as the representative of reality, in combination with the general principle of relativity, to show the true kernel of Descartes' idea; there exists no space "empty of field."

Generalized Theory of Gravitation

The theory of the pure gravitational field on the basis of the general theory of relativity is therefore readily obtainable, because we may be confident that the "field-free" Minkowski-space with its metric in conformity with (1) must satisfy the general laws of field. From this special case the law of gravitation follows by a generalization which is practically free from arbitrariness. The further development of the theory is not so unequivocally determined by the general principle of relativity; it has been attempted in various directions during the last few decades. It is common to all these attempts, to conceive physical reality as a field, and moreover, one which is a generalization of the gravitational field, and in which the field law is a generalization of the law for the pure gravitational field. After long probing I believe that I have now found * the most natural form for this generalization, but I have not yet been able to find out whether this generalized law can stand up against the facts of experience.

The question of the particular field law is secondary in the preceding general considerations. At the present time, the main question is whether a field theory of the kind here contemplated can lead to the goal at all. By this is meant a theory which describes exhaustively physical reality, including four-dimensional space, by a field. The present-day generation of physicists is inclined to answer this question in the negative. In

* The generalization can be characterized in the following way. In accordance with its derivation from empty "Minkowski space," the pure gravitational field of the functions g_{ik} has the property of symmetry given by $g_{ik} = g_{ki}$ ($g_{12} = g_{21}$, etc.). The generalized field is of the same kind, but without this property of symmetry. The derivation of the field law is completely analogous to that of the special case of pure gravitation.

conformity with the present form of the quantum theory, it believes that the state of a system cannot be specified directly, but only in an indirect way by a statement of the statistics of the results of measurements attainable on the system. The conviction prevails that the experimentally assured duality (corpuscular and wave structure) can be realized only by such a weakening of the concept of reality. I think that such a far-reaching theoretical renunciation is not for the present justified by our actual knowledge, and that one should not desist from pursuing to the end the path of the relativistic field theory.