Attempt at a Theory of Electrical and Optical Phenomena in Moving Bodies

Hendrik Lorentz

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of

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by

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Introduction

§ 1. The question as to whether the aether shares the motion of ponderable bodies or not, has still found no answer that satisfies all physicists. For the decision, primarily the aberration of light and related phenomena could be used, but so far none of the two contested theories, neither that of FRESNEL, nor that of STOKES, were fully confirmed with respect to all observations, so concerning the choice between the two views we can only weigh against each other the remaining problems for both of them. By that I was long ago led to believe that with FRESNEL's view, *i.e.* with the assumption of a stationary aether, we are on the right way. While against the view of STOKES there is hardly more than one objection, *i.e.* the doubt that his assumptions regarding the aether-motion in the vicinity of Earth are contradictory[1], but this objection is of great weight, and I can't see at all how it could be eliminated.

The difficulties for FRESNEL's theory stem from the known interference experiment of MICHELSON[2] and, as some think, from the experiments, by which DES COUDRES in vain sought to find an influence of Earth's motion on the induction of two circuits[3]. The results of the American scientist, however, allow of an interpretation by an auxiliary hypotheses, and the findings of DES COUDRES can easily be explained without such one.

Concerning the observations of FIZEAU[4] on the rotation of polarization in glass columns, the matter is as follows. At first glance, the result is decidedly against STOKES' view. Yet when I tried to improve FRESNEL's theory, the explanation of FIZEAU's experiments was not quite successful, so I gradually suspected that this result had been obtained by observational error, or at least it had not met the theoretical considerations which formed the basis of the experiments. And FIZEAU was so friendly to tell my colleague VAN DE SANDE BAKHUIJZEN after his request, that at present he himself doesn't see his observations as crucial.

In the further course of this work, I will come back in more detail to some of the issues raised at this place. Here I was concerned only with the preliminary justification of the standpoint I have taken.

In favor of FRESNEL's theory several well-known reasons can be cited. Especially the impossibility of locking the aether between solid or liquid walls. As far as we know, a space devoid of air behaves (in the mechanical sense) like a real vacuum, when ponderable bodies are in motion. When we see how the mercury of a barometer rises to the top when the tube is inclined, or how easily a closed metal shell can be compressed, one can not avoid the idea, that solid and liquid bodies let the aether pass through without hindrance.

One hardly will assume, that this medium could suffer a compression, without giving resistance to it.

That *transparent* bodies can move, without communicating their full velocity to the contained aether, was proven by FIZEAU's famous interference experiment with streaming water[5]. This experiment, that later was repeated by MICHELSON and MORLEY[6] on a larger scale, could impossibly have had the observed success, when *everything* within the tube would possess a common velocity. By that, only the behavior of nontransparent substances and very extended bodies remains questionable.

It should be noted, moreover, that we can imagine the permeability of a body in two ways. First, this property might not be present in individual atoms, yet when the atoms were very small compared to the gaps between them, it might be present in matter of greater extension; but secondly, it may be assumed - and this hypothesis I will use in the following - that ponderable matter is *absolutely* permeable, namely that at the location of an atom, also the aether exists at the same time, which would be understandable if we were allowed to see the atoms as local modifications of the aether.

It is not my intention to enter into such speculations more closely, or to express assumptions about the nature of the aether. I only wish to keep myself as free as possible from preconceived opinions about that substance, and I won't, for example, attribute to it the properties of ordinary liquids and gases. If it is the case, that a representation of the phenomena would succeed best under the condition of absolute permeability, then one should admit of such an assumption for the time being, and leave it to the subsequent research, to give us a deeper understanding.

That we cannot speak about an *absolute* rest of the aether, is self-evident; this expression would not even make sense. When I say for the sake of brevity, that the aether would be at rest, then this only means that one part of this medium does not move against the other one and that all perceptible motions are relative motions of the celestial bodies in relation to the aether.

§ 2. Since MAXWELL's views became more and more accepted, the question of the properties of the aether became highly important also for the theory of elasticity. Strictly speaking, not a single experiment in which a charged body or a current conductor moves, can be handled carefully, if the state of motion of the aether is not considered at the same time. In any phenomenon of electricity, the question arises whether an influence of the earth's motion is to be expected; and regarding the consequences of the latter for optical phenomena, we have to demand from the electro-magnetic theory of light that it can account for the already established facts. Namely, the aberration theory isn't one of those parts of optics, for which treatment the general principles of wave theory are sufficient. Once a telescope comes into play, one can not help but to apply FRESNEL's dragging coefficient to the lenses, yet its value can only be derived from special assumptions about the nature of light vibrations.

The fact that the electro-magnetic theory of light really leads to that coefficient assumed by FRESNEL, was shown by me two years ago[7]. Since then I have greatly simplified the theory and extended it also to the processes involved in reflection and refraction, as well as birefringent bodies[8]. It may be permitted for me, to come back to this matter.

To come to the basic equations for the phenomena of electricity in moving bodies, I joined an opinion that has been represented in recent years by several physicists; I have indeed assumed that small electrically charged molecules exist in all bodies, and that all electric processes are based on the location and motion of these "ions". As regards the electrolytes, this view is widely recognized as the only possible one, and GIESE[9], SCHUSTER[10], ARRHENIUS[11], ELSTER and GEITEL[12] have defended the view, that also as regards the electricity conduction in gases, we are dealing with a convection by ions. It seems to me, that nothing prevents us to believe that the molecules of ponderable dielectric bodies contain such particles, which are connected to certain equilibrium positions and are moved only by external electric forces thereof; just herein the "dielectric polarization" of such bodies would consist.

The periodically changing polarization, which forms a light ray according to MAXWELL's theory, become vibrations of the ions in this conception. It is well known that many researchers, who stood on the basis of the older theory of light, considered the resonance of ponderable matter as the cause of color dispersion, and this explanation can in the main also included into the electro-magnetic theory of light, for which it is only necessary to ascribe to the ions a certain mass. This I have shown in a previous paper[13], in which I admittedly have derived the equations of motion from actions at a distance, and not, what I now consider to be much easier, from MAXWELL's expressions. Later, VON HELMHOLTZ[14] in his electromagnetic theory of color dispersion started from the same point of view[15].

GIESE[16] has applied to various cases the hypothesis, that electricity is connected to ions in metallic conductors as well; but the picture which he gives of the processes in these bodies is at *one* point substantially different from the idea that we have on the conduction in electrolytes. While the particles of dissolved salt, however often they may be stopped by the water molecules, eventually might travel over large distances, the ions in a copper wire will hardly have such a great mobility. We can however be satisfied with forward and backward motion at molecular distances, if we only assume that one ion often transfers its charge to another, or that two oppositely charged ions, if they meet, or after they were "connected" with one another, exchange their charges against each other. In any case, such processes must take place at the boundary of two bodies, when a current flows from one to the other. If for example n positively charged copper atoms are separated at a copper plate, and we also want for the latter all the electricity be connected to ions, then we have to assume that the charges are transferred to n atoms in the plate, or that $\frac{1}{2}n$ of the deposited particles exchange their charges with $\frac{1}{2}n$ negatively charged copper atoms, which were already in the electrode.

Thus, if the adoption of this transition or exchange of the ionic charges - one of course still very dark process - is the essential complement to any theory that requires an entrainment of electricity by ions, then a persistent electric current never consists of a convection *alone*, at least not when the centers of two touching or interconnected particles are in some distance l from each other. Then the electricity motion happens without convection over a distance of order l, and only if this is very small in proportion to the distance over which a convection takes place, we on the whole are dealing almost exclusively with this latter phenomenon.

GIESE is of the opinion that in metals a real convection was not at all in play. But since it does not seem possible to include the "jumping" of the charges into the theory, then one would excuse, that for my part I totally disregard such a process, and that I interpret a current in a metal wire simply as a motion of charged particles.

Further research will have to decide whether the results of the theory remains at a different view.

§ 3. The theory of ions was very suitable for my purpose, because it makes it possible to introduce the permeability of the aether in a rather satisfactory way in the equations. Of course, these were decomposed into two groups. First, we have to express as to how the state of the aether by charge, position and motion of the ions is determined; then, secondly, we have to indicate by which forces the aether is acting on the charged particles. In my paper already cited[17] I have derived the formulas by means of D'ALEMBERT's principle from certain assumptions and therefore selected a path, that has much resemblance with MAXWELL's application of LAGRANGE's equations. Now I prefer for the sake of brevity, to introduce the basic equations themselves as hypotheses.

The formulas for the aether are in agreement, regarding the space between the ions, with the known equations of MAXWELL's theory, and generally express that any change that was caused by an ion in the aether, propagates with the velocity of light. But we regard the force exerted by the aether on a charged particle, as a function of the state of that medium at the point where the particle is located. The adopted fundamental law differs in a major point from the laws, that were introduced by Weber and Clausius. The influence that was suffered by a particle B due to the vicinity of a second one A, indeed depends on the motion of the latter, but not on its *instantaneous* motion. Much more relevant is the motion of A some time earlier, and the adopted law corresponds to the requirement for the theory of electrodynamics, that was presented by Gauss in 1845 in his known letter to Weber[18]

In general, the assumptions that I introduce represent in a certain sense a return to the earlier theories of electricity. The core of Maxwell's views is therefore not lost, but it cannot be denied that with the adoption of ions we are not far away from the electric particles, which were used earlier. In some simple cases, this occurs particularly clear. Since the essence of electric charge is seen by us in the accumulation of positive or negative charged particles, and since the basic formulas for stationary ions give Coulomb's law, therefore, for example, the entire electrostatics can be brought into the earlier form.

1. ↑ *Lorentz*. De l’influence du mouvement de la terre sur les phénomènes lumineux. Arch. néerl., T. 21, p. 103, 1887; Lodge. Aberration problems. London Phil. Trans., Vol. 184. A, p. 727, 1893; Lorentz. De aberratietheorie van Stokes. Zittingsverslagen der Akad. v. Wet. te Amsterdam, 1892—93, p. 97.
2. ↑ Michelson. American Journal of Science, 3d. Ser., Vol. 22, p. 120; Vol. 34, p. 333, 1887; Phil. Mag., 5th.

Ser., Vol. 24, p. 449, 1887.

3. ↑ Des Coudres. Wied. Ann., Bd. 38, p. 71, 1889.
4. ↑ Fizeau. Ann. de chim. et de phys., 3e sér., T. 58, p. 129, 1860; Pogg. Ann., Bd. 114, p. 554, 1861.
5. ↑ Fizeau. Ann. de chim. et de phys., 3e sér. T. 57, p. 385, 1859; Pogg. Ann., Erg. 3, p. 457, 1853.
6. ↑ Michelson and Morley. American Journal of Science, 3d. ser., Vol. 31, p. 377, 1886.
7. ↑ Lorentz. La théorie électromagnétique de Maxwell et son application aux corps mouvants. Leide, E. J. Brill, 1892. (Also published in Arch, néerl., T. 25).
8. ↑ A preliminary report about that was published in Zittingsverslagen der Akad. v. Wet. te Amsterdam, 1892—93, pp. 28 and 149.
9. ↑ Giese. Wied. Ann., Bd. 17, p. 538, 1882.
10. ↑ Schuster. Proc. Roy. Soc., Vol. 37, p. 317, 1884.
11. ↑ Arrhenius. Wied. Ann., Bd. 32, p. 565, 1887; Bd. 33, p. 638, 1888.
12. ↑ Elster and Geitel. Wiener Sitz.-Ber., Bd. 97, Abth. 2, p. 1255, 1888.
13. ↑ Lorentz. Over net vorband tusschen de voortplantingssnelheid van het licht en de dichtheid en samenstelling der middenstoffen. Verhandelingen der Akad. van Wet. te Amsterdam, Deel 18, 1878; Wied. Ann., Bd. 9, p. 641, 1880.
14. ↑ v. Helmholtz. Wied. Ann., Bd. 48, p. 389, 1893.
15. ↑ Also Koláček (Wied. Ann., Bd. 32, pp. 224 and 429, 1887) attempted to explain (albeit in a different manner) dispersion by electrical vibrations in the

molecules.

Also the theory of Goldhammer (Wied. Ann., Bd. 47, p. 93, 1892) has to be mentioned.

16. ↑ Giese. Wied. Ann., Bd. 37, p. 576, 1889.
17. ↑ Lorentz. La théorie électromagnétique de Maxwell et son application aux corps mouvants.
18. ↑ Gauss. Werke, Bd. 5, p. 629.

Some definitions and mathematical relations

§ 4. *a*. We want to say, that a rotation in a plane *corresponds* to a certain direction of the perpendicular, and namely it shall be the direction into that side, at which an observer must be located, so that for him the rotation is counter-clockwise.

b. The mutually perpendicular coordinate axes OX, OY, OZ are chosen by us, so that the direction of OZ corresponds to a rotation around a right angle of OX to OY.

c. A space, a surface and a line we denote by the letters τ, σ and s throughout, and infinitely small parts by $d\,\tau$, $d\,\sigma$ and $d\,s$.

The perpendicular to a surface will by sketched by *n*, and is always drawn into a certain side, the "positive" one. As regards the line, a certain direction will be called "positive", and namely we note, when we are dealing with the border line *s* of a surface σ, the following rule: If *P* is a fixed point of σ, very near to *s*, and if a second point *Q* traverses the nearest part of *s* in positive direction, then the rotation of

PQ shall correspond to the direction of the perpendicular to σ.

As regards a closed surface, the *outer side* shall be positive.

d. Usually we denote vectors by German letters; these sometimes also serve to denote the magnitude only. By $\mathfrak{A}_l$ we understand the component of the vector $\mathfrak{A}$ into the direction *l*; by $\mathfrak{A}_x, \mathfrak{A}_y, \mathfrak{A}_z$ therefore the components into the axis-directions.

For a vector with components X, Y, Z we sometimes also write (X, Y, Z).

e. If ϕ is a scalar magnitude, then we understand by $\dot{\phi}$ the derivative with respect to time *t*. The letter $\dot{\mathfrak{A}}$ denotes a vector with components: $\dot{\mathfrak{A}}_x, \dot{\mathfrak{A}}_y, \dot{\mathfrak{A}}_z$, or $\frac{\partial \mathfrak{A}_x}{\partial t}$ etc.

f. The expression

$$\int \mathfrak{A}_n \, d\,\sigma$$

we call the "integral of vector $\mathfrak{A}$ over the surface σ", and the magnitude

$$\int \mathfrak{A}_s \, d\,s$$

the "line integral of line *s*".

g. If a vector $\mathfrak{A}$ in any point of space is given, then

$$\frac{\partial \mathfrak{A}_x}{\partial x} + \frac{\partial \mathfrak{A}_y}{\partial y} + \frac{\partial \mathfrak{A}_z}{\partial z}$$

has everywhere a certain value, independent of the choice of coordinate system. We call this magnitude "divergence" of vector $\mathfrak{A}$ and denote it by

$$Div\,\mathfrak{A}.$$

For any space limited by a surface σ, the relation is given

$$\int Div\,\mathfrak{A}\,d\,\tau = \int \mathfrak{A}_n\,d\,\sigma,$$

when, as already mentioned, the perpendicular *n* will be drawn into the outside.

h. The magnitudes

$$\frac{\partial \mathfrak{A}_z}{\partial y} - \frac{\partial \mathfrak{A}_y}{\partial z}, \frac{\partial \mathfrak{A}_x}{\partial z} - \frac{\partial \mathfrak{A}_z}{\partial x}, \frac{\partial \mathfrak{A}_y}{\partial x} - \frac{\partial \mathfrak{A}_x}{\partial y}$$

can be interpreted as the components of vector $\mathfrak{B}$, which (independent from the choses coordinate system) is defined by the distribution of $\mathfrak{A}$. We call this vector the *rotation* of $\mathfrak{A}$ and denote it by

$$\boldsymbol{Rot}\,\mathfrak{A},$$

and its components occasionally by

$$[\boldsymbol{Rot}\,\mathfrak{A}]_l.$$

If *s* is the border line of surface σ, then we have

$$\int \mathfrak{A}_s\, d\, s = \int \mathfrak{B}_n\, d\, \sigma. \tag{1}$$

Furthermore we will easily find

$$\boldsymbol{Div}\,\boldsymbol{Rot}\,\mathfrak{A} = 0,$$

and for the components of vector $\boldsymbol{Rot}\,\boldsymbol{Rot}\,\mathfrak{A}$

$$\frac{\partial}{\partial x}\boldsymbol{Div}\,\mathfrak{A} - \triangle\mathfrak{A}_x, \text{ etc.}$$

Here, the letter Δ has, like in all our formulas, the meaning

$$\triangle = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

i. If *m* and *n* are scalar magnitudes, then we attribute to the expressions

$$-\mathfrak{A},\ m\,\mathfrak{A},\ m\,\mathfrak{A} \pm n\,\mathfrak{B}$$

the known meanings.

j. By $[\mathfrak{A}.\,\mathfrak{B}]$ we understand the so-called "vector product", namely a vector whose magnitude is given by the area of the parallelogram drawn over $\mathfrak{A}$ and $\mathfrak{B}$, and whose direction is perpendicular to the plane that is laid trough $\mathfrak{A}$ and $\mathfrak{B}$, and namely in a way, by that the direction of $\mathfrak{A}$ is transformed into the direction of $\mathfrak{B}$.

As regards the components it can be written $[\mathfrak{A}.\,\mathfrak{B}]_l$; the components into the axis-directions are:

$$\mathfrak{A}_y\mathfrak{B}_z - \mathfrak{A}_z\mathfrak{B}_y,\ \mathfrak{A}_z\mathfrak{B}_x - \mathfrak{A}_x\mathfrak{B}_z,\ \mathfrak{A}_x\mathfrak{B}_y - \mathfrak{A}_y\mathfrak{B}_x,$$

and

$$[\mathfrak{B}.\,\mathfrak{A}] = -[\mathfrak{A}.\,\mathfrak{B}].$$

k. The advantage of the previously introduced expressions mainly consists in the fact, that now three equations like

$$\mathfrak{A}_x = \mathfrak{B}_x,\ \mathfrak{A}_y = \mathfrak{B}_y,\ \mathfrak{A}_z = \mathfrak{B}_z$$

can be summarized in one formula

$$\mathfrak{A} = \mathfrak{B}$$

However, in the course of the investigation we will often use the three individual equations. If they have the same form, so that they transform into one another by cyclic permutation of the letters, then we can restrict ourselves to only writing down the first equation, and to sketch the two others by "etc.".

l. We will often have to consider bodies with molecular structure. Then functions arise, whose value *quickly* changes in the individual molecules and in the interspaces, and namely in a highly irregular way, as the molecules themselves are not always structured and oriented regularly. In those cases it is recommended, to calculate with *averages*, which we define as follows:

We describe around center-point P a sphere of area I, and calculate for it, when ϕ is the magnitude to be considered, the integral $\int \phi \, d\,\tau$. Then we call

$$\frac{1}{I} \int \phi \, d\,\tau, \tag{2}$$

for which we want to write $\bar{\phi}$, the "average of ϕ at point P".

If we give to the sphere, where ever P may lie, always the same magnitude, then $\bar{\phi}$ can obviously only depend on t and the coordinates x, y, z of point P. It is clear that also $\bar{\phi}$

will show "rapid" changes from point to point, as long the sphere encloses only a few molecules, yet by a continuing increase the changes will step back more and more. We think for once and for all time a certain *R* as chosen, which is just as great that — with respect to the degree of exactitude that can be reached by the observations — we can neglect the rapid changes in $\bar{\phi}$. Then only the slow changes from point to point remain, that are accessible to our senses, and in all real cases they proceed so slow, that they hardly appear in spaces which are considerably greater as the sphere *I*. In these cases, $\bar{\phi}$ will be given only by expression (2), when we don't apply it to the mentioned sphere, but to an arbitrary formed larger space.

Of course $\bar{\phi} = \phi$ everywhere, as soon as ϕ doesn't show rapid changes.

Furthermore we easily find

$$\frac{\partial\bar{\phi}}{\partial t} = \overline{\frac{\partial\phi}{\partial t}},\ \frac{\partial\bar{\phi}}{\partial x} = \overline{\frac{\partial\phi}{\partial x}},\ \mathbf{u.\ s.\ w.}$$

m. By the average of a vector $\mathfrak{A}$ we understand a vector — it may be called $\bar{\mathfrak{A}}$ —, whose components are the averages of $\mathfrak{A}_x, \mathfrak{A}_y, \mathfrak{A}_z$. Consequently we have

$$\dot{\bar{\mathfrak{A}}} = \bar{\dot{\mathfrak{A}}},\ Div\,\bar{\mathfrak{A}} = \overline{Div}\,\bar{\mathfrak{A}},\ Rot\,\bar{\mathfrak{A}} = \overline{Rot}\,\bar{\mathfrak{A}}.$$

The fundamental equations for a system of ions located in the aether.

The equations for the aether.

§ 5. When forming the equations of motion we will express all magnitudes in electromagnetic measure, and preliminarily use a coordinate system that is at rest in the aether. Now according to MAXWELL, two kinds of deviations from the equilibrium state can exist in this medium. The deviation of *first* kind, which (among others) can be found in the vicinity of any charged body, we call the *dielectric displacement*; it is a vector quantity and may get the designation $\mathfrak{d}$[1]. It is *solenoidally* distributed in "pure" aether, *i.e.* in the spaces between the ions, and we have

$$Div\,\mathfrak{d} = 0. \tag{3}$$

We now want to assume, that aether exists in the space where an ion is located, and that a dielectric displacement can happen at this place, *i.e.* that the dielectric displacement caused by a *single* ion is extended over the interior of the other ions.

The charge of an ion we see as distributed over a certain *space*; the spatial density may be called ρ, and we want to assume, that this function *steadily* goes over to 0 when passing from the interior of the particle into the pure aether. In this assumption, that gives us the advantage that no discontinuities must be considered, there is no essential restriction. Because the charge distribution over a surface and a discontinuity of ρ can be treated as limiting cases of states to which that assumption are applicable.

In the cases to be considered, ρ is different from zero only in the interior of a very great number of small spaces which are completely mutually separated. Yet we can start with the general case, that an electric density exists in arbitrary great spaces. Since we think of the electric charges as always connected to ponderable matter, then this would correspond to a continuous distribution of matter.

Ponderable matter, which is *not* charged, has only to be considered by us, when it exerts molecular forces on the ions. Concerning the electric phenomena, it has no influence at all and everything happens, as if the space where it is located would only contain the aether.

Where ρ is different form zero, equation (3) is not applicable anymore. According to a known theorem from MAXWELL's theory, we have for any closed surface σ, when E represents the entire charge in the interior,

$$\int \mathfrak{d}_n \, d\,\sigma = E = \int \rho \, d\,\tau,$$

or

$$\int Div\, \mathfrak{d} \, d\,\tau = \int \rho \, d\,\tau,$$

so everywhere it must be

$$Div\,\mathfrak{d} = \rho \tag{I}$$

If the ponderable matter is moving, then — since it carries the charge along with it — at a certain point of space there always exists another ρ, and soon it is (when we are dealing with mutually separated ions) different from zero here and there. Yet the condition of the aether has constantly to obey equation (I).

§ 6. The change of $\mathfrak{d}$, that happens with time at a certain point of space, constitutes an electric current (Maxwell's *displacement current*) that can be represented by $\dot{\mathfrak{d}}$. We assume that it exists also in the interior of charged matter. Yet additionally we find a *convection current* $\mathfrak{C}$ there. This is considered by me, when $\mathfrak{v}$ is the velocity of ponderable matter, as given in magnitude and direction by

$$\mathfrak{C} = \rho\mathfrak{v}$$

and I put for the total current

$$\mathfrak{S} = \mathfrak{C} + \dot{\mathfrak{d}} = \rho\mathfrak{v} + \dot{\mathfrak{d}}. \tag{4}$$

In charged matter, $\mathfrak{v}$ shall continuously vary from point to point[2]. Additionally the charge of every mass element shall stay unchanged during the motion. Thus $\rho\omega$ must be constant, when ω is the — maybe variable — volume of the element.

From this assumption we derive the property of solenoid distribution for the total current, which will be expressed by

$$Div\,\mathfrak{S} = 0. \tag{5}$$

§ 7. The *second* deviation of the equilibrium state of the aether will be determined by the *magnetic force* $\mathfrak{H}$. It depends on the instantaneous current distribution, and satisfies the requirements

$$Div\,\mathfrak{H} = 0, \tag{II}$$

$$Rot\,\mathfrak{H} = 4\pi\mathfrak{S}, \tag{III}$$

whose applicability we also presuppose for the interior of ponderable matter[3].

Eventually we also assume the relation, for the interior of the ions[4] as well as for the interspaces, by which in MAXWELL's theory the dielectric displacement is connected with the temporal variation of the magnetic force. The relation reads

$$-4\pi V^2\,Rot\,\mathfrak{d} = \dot{\mathfrak{H}}, \tag{IV}$$

if we denote by *V* the ratio of the electromagnetic and electrostatic units of electricity, or the velocity of light in the aether.

Now we have written down all equations for the aether. If $\mathfrak{d}$ and $\mathfrak{H}$ for $t = 0$ are given everywhere, we know for all subsequent instants the motion of charged bodies, and if we also add the requirement, that $\mathfrak{d}$ and $\mathfrak{H}$ vanish in infinite distance, then these vectors are definitely specified.

Where $\rho = 0$, the equations go over into the formulas for pure aether, from which it is knowingly given, that the variations represented by $\mathfrak{d}$ and $\mathfrak{H}$ propagate with the velocity of light.

Since the equations are linear, various solutions can be composed to a more general one by addition. For example, the motion of *n* ions shall be given, and *n* value systems of $\mathfrak{d}$ and $\mathfrak{H}$ shall be found that determine the state of the aether for the case in which only one ion exists, and the others were neglected. Then we obtain by superposition the state of the aether, being in agreement with the motions of all *n* ions. In this sense we may say, that any ion influences the state of aether in exactly such way, as if the others wouldn't exist.

§ 8. If the ponderable matter is at rest and $\mathfrak{d}$ is independent of time, then $\mathfrak{S}$ and $\mathfrak{H}$ vanish, while $\mathfrak{d}$ will be determined by

$$Div\,\mathfrak{d} = \rho \tag{I}$$

and

$$Rot\,\mathfrak{d} = 0.$$

This last equation says, that $\mathfrak{d}_x$, $\mathfrak{d}_y$, $\mathfrak{d}_z$ can be considered as partial derivatives of a single function, which we want to call $-\frac{\omega}{4\pi}$. We thus put

$$\mathfrak{d}_x = -\frac{1}{4\pi}\frac{\partial\omega}{\partial x}, \text{ etc.} \tag{6}$$

and derive from (I)

$$\triangle\omega = -4\pi\rho. \tag{7}$$

After we determined ω from that, $\mathfrak{d}_x$, $\mathfrak{d}_y$, $\mathfrak{d}_z$ can be calculated from (6).

The first part of the force acting on ponderable matter.

§ 9. According to the older electrostatics, whose conclusions are in agreement with experience, we obtain the force components that act on the volume element in the case previously considered, when we at first determine the "potential function" by means of POISSON's equation, and then multiply its derivatives by $-V^2 \rho\, d\, \tau$ [5]. Since our formula (7) is in agreement with POISSON's equations, the potential function must coincide with ω; therefore we have to assume as values of the force components

$$-V^2 \rho \frac{\partial \omega}{\partial x}\, d\, \tau, \text{ etc.} \tag{8}$$

If the forces, as it is claimed by MAXWELL's theory, shall be caused by the state of the aether, then it is probable that it depends on the dielectric displacement in the considered volume element. Indeed, when we consider (6), we can write for (8)

$$4\pi V^2 \mathfrak{d}_x \rho\, d\, \tau, \text{ etc.}$$

Therefore I will assume, that in *all* cases in which a dielectric displacement exists in element $d\tau$, the aether exerts a force with the mentioned components on ponderable matter located at this place, *i.e.* a force[6] which can be represented for the unit of charge by

$$\mathfrak{E}_1 = 4\pi V^2 \mathfrak{d}.$$

§ 10. Let two *stationary* ions with charges e and e' be given, whose dimensions are small in relation to distance *r*. To find the force that acts on the first one, we have to decompose it into space elements, to apply on any of them the previous theorem, and then to integrate. Thereby $\mathfrak{d}$ may be considered as composed of the dielectric displacements, that stem from the first and the second particle. We easily find that the first part of $\mathfrak{d}$ doesn't contribute anything to the total force. The second part has (within the first ion) everywhere the direction of *r* and the magnitude $e'/4\pi r^2$; so *e* will be repulsed by e' by the force

$$V^2 \frac{ee'}{r^2}.$$

As this is in agreement with COULOMB's laws, it is clear that the theory of ions, as regards the ordinary problems, leads back to the older way of treatment.

Electric currents in ponderable conductors.

§ 11. In a ponderable conductor, in which a current flows through, and in which innumerable ions are in motion according to our view, $\mathfrak{d}$, $\mathfrak{S}$ and $\mathfrak{H}$ are changing in an irregular way from point to point. Yet from equations (II) and (III) it follows

$$\begin{aligned} Div\, \bar{\mathfrak{H}} &= 0, \\ Rot\, \bar{\mathfrak{H}} &= 4\pi\bar{\mathfrak{S}}; \end{aligned}$$

since $\bar{\mathfrak{H}}$ coincides with $\mathfrak{H}$ in measurable distance from the conductor, and the action into the outside will only be determined by the average current $\bar{\mathfrak{S}}$. It is this current, with which the ordinary theory (which neglects molecular processes) is dealing.

By equation (4) we have

$$\bar{\mathfrak{S}} = \overline{\rho\mathfrak{v}} + \dot{\bar{\mathfrak{d}}}.$$

If the state of flow is stationary, then the observable magnitudes and also the averages are independent of time. Thus it will be

$$\bar{\mathfrak{S}} = \overline{\rho\mathfrak{v}},$$

i.e. only the convection currents cause the action into the outside.

By the definition given in § 4, the components of $\overline{\rho\mathfrak{v}}$ are

$$\frac{1}{I}\int \rho\mathfrak{v}_x \, d\,\rho, \text{ u. s. w.},$$

or, when ρ is different from zero only within the ions, and any ion is displaced without rotation

$$\frac{1}{I}\sum e\mathfrak{v}, \text{ etc.},$$

where e is the charge of an ion, and the sum is related to all charged particles contained in sphere I. We can easily see, that the result can be summarized in the formula

$$\bar{\mathfrak{S}} = \frac{1}{I}\sum e\mathfrak{v}$$

and this remains valid, when we don't interpret I just as a sphere, but as an *arbitrary* space, whose dimensions (albeit very small) are nevertheless much greater than the average distance of the ions. Of course, then the sum must be extended over the chosen space as well.

If there is a current within a lead wire with cross-section ω, then we can take for I the part, that lies between two cross-sections which are mutually distant by ds[7]. Since the magnitude of current will be determined by:

$$i = \omega \bar{\mathfrak{S}},$$

and $I = \omega\, d\, s$, thus we obtain

$$\sum e\mathfrak{v} = i\, d\, s,$$

where $i\, d\, s$ is to be considered as a vector in the direction of the current.

The second part of the force acting on ponderable matter

§ 12. A current element as the one previously considered, may be located in a magnetic field generated by external causes. According to a known law it suffers an electrodynamic force

$$[i\, d\, s . \mathfrak{H}],$$

for which we also can write now

$$\left[\sum e\mathfrak{v} . \mathfrak{H}\right],$$

or

$$\sum\{e[\mathfrak{v}.\mathfrak{H}]\}.$$

This action results (according to our view) from the forces, which will be exerted by the aether upon the ions of the current element. It is thus near at hand, to assume for the force acting on a single ion

$$e[\mathfrak{v}.\mathfrak{H}],$$

a hypothesis, which we still want to extend in a way, so that we *generally* assume a force acting on ponderable matter of the volume element $d\tau$

$$\rho d\,\tau[\mathfrak{v}.\mathfrak{H}]$$

In unit charge this would be

$$\mathfrak{E}_2 = [\mathfrak{v}.\mathfrak{H}]$$ [8].

By putting this vector together with $\mathfrak{E}_1$ that was considered earlier (§ 9), we obtain for the total force exerted on the unit of charge, *i.e.* for the *electric force*,

$$\mathfrak{E} = 4\pi V^2\mathfrak{d} + [\mathfrak{v}.\mathfrak{H}]. \tag{V}$$

We refuse to express the thus stated law by words. By elevating it to a general fundamental-law, we have completed the system of equations of motion (I)—(V), since the electric force, in connection with possible other forces, determines the motion of ions.

Concerning the latter, we still want to introduce the assumption, that the ions never rotate.[9]

The conservation of energy.

§ 13. To justify our hypotheses, it is necessary to show its agreement with the energy law. We consider an arbitrary system of ponderable bodies that contain ions, around which only the aether exists up to an infinite distance, and around it we put an arbitrarily closed surface σ. During an element of time dt, the work that affects ponderable matter and which stems from $\mathfrak{E}$, is

$$4\pi V^2 d\,t \int \rho\left(\mathfrak{d}_x \mathfrak{v}_x + \mathfrak{d}_y \mathfrak{v}_y + \mathfrak{d}_z \mathfrak{v}_z\right) d\,\tau,$$

where it is to be noted, that no work is done by the forces (which are derived from $\mathfrak{E}_2$), because they are always perpendicular to the direction of motion. Furthermore, if dA is the work of all other forces acting on matter, and L is the ordinary mechanical energy of that matter, then

$$d\,A = d\,L - 4\pi V^2 d\,t \int \rho\left(\mathfrak{d}_x \mathfrak{v}_x + \mathfrak{d}_y \mathfrak{v}_y + \mathfrak{d}_z \mathfrak{v}_z\right) d\,\tau. \quad (9)$$

The integral is related to the space filled with ponderable matter; but we can also extend it over the entire space enclosed by σ. All other space integrals in this § are to be understood in the latter sense.

We replace in (9), by (4) and (III)

$$4\pi\rho\mathfrak{v}_x. \text{ u. s. w.}$$

by

$$\frac{\partial \mathfrak{H}_z}{\partial y} - \frac{\partial \mathfrak{H}_y}{\partial z} - 4\pi \frac{\partial \mathfrak{d}_x}{\partial t}, \text{ u. s. w.,} \tag{10}$$

and transform the parts of the integral, that contain derivatives of $\mathfrak{H}_x, \mathfrak{H}_y, \mathfrak{H}_z$, by partial integration.

By consideration of equation (IV) we will find

$$d\,A = d(L + U) + V^2 d\,t \int [\mathfrak{d}.\,\mathfrak{H}]_n d\,\sigma, \tag{11}$$

where

$$U = 2\pi V^2 \int \mathfrak{d}^2 d\,\tau + \frac{1}{8\pi} \int \mathfrak{H}^2 d\,\tau \tag{12}$$

At first is should be assumed, that the electric motions are restricted to a certain finite space, and that surface σ is entirely outside of that space. Then at the surfaces it will be $\mathfrak{d} = 0, \mathfrak{H} = 0$, and

$$d\,A = d(L + U).$$

Therefore the magnitude *L* + *U* really applies, whose increase is equal to the work of the external forces, and which therefore is denoted by the expression "energy". It is composed of the ordinary mechanical energy *L* and the "electrical" energy *U*, and as regards the latter we find again the value given by MAXWELL.

The theorem of Pointing.

§ 14. Even if we abandon the previously made assumption about σ, formula (11) allows of a simple interpretation. With MAXWELL we not only assume that the electric force would have the value (12), but also, that it is really distributed over the space as it is expressed by the formula, *i.e.* that it amounts for unit volume

$$2\pi V^2 \mathfrak{d}^2 + \frac{1}{8\pi}\mathfrak{H}^2$$

In equations (11), L + U thus means the whole energy within surface σ, and therefore the view is near at hand, that a quantity of energy

$$V^2 d\,t \int [\mathfrak{d}.\mathfrak{H}]_n d\,\sigma$$

has traveled through the surface into the outside. It is most simple, if we put for the "energy flow" related to unit time and area

$$V^2 [\mathfrak{d}.\mathfrak{H}]_n. \tag{13}$$

By that we come to the known theorem formulated by POYNTING. Here, we don't discuss the subtle, related question concerning the localization of the energy. We can restrict ourselves with the fact, that the entire energy located in an arbitrary space — the "electric" portion calculated by formula (12) — always varies, *as if* the energy would travel according to the way determined by (13).

Tensions in the aether.

§ 15. The forces determined by our formula (V), not only require the motion of ions in ponderable bodies, but also in some circumstances can unify themselves to an action, that tends to set the body into motion. In this way all "ponderomotive" forces emerge, as for example the ordinary electrostatic and electrodynamic ones, as well as the pressure that is exerted by light rays on a body.

We want to consider the body as rigid, and calculate (by simple addition) all the forces that were exerted by the aether in the direction of the *x*-axis, *i.e.* the total force Ξ in this direction. The investigation should be based on the things said at the beginning of § 13.

We immediately obtain

$$\begin{aligned} \Xi & = 4\pi V^2 \int \mathfrak{d}_x \rho \, d\tau + \int \rho [\mathfrak{v} . \mathfrak{H}]_x \, d\tau = \\ & = 4\pi V^2 \int \mathfrak{d}_x \rho \, d\tau + \int \rho \left(\mathfrak{v}_y \mathfrak{H}_z - \mathfrak{v}_z \mathfrak{H}_y \right) \, d\tau, \end{aligned}$$

where the integrals only have to be extended over the ponderable body, but like in § 13, it should taken for the entire space enclosed by σ.

At first, we replace $4\pi\rho\mathfrak{v}_x$, etc. by the expressions (10), and, because of (I), ρ by

$$\frac{\partial \mathfrak{d}_x}{\partial x} + \frac{\partial \mathfrak{d}_y}{\partial y} + \frac{\partial \mathfrak{d}_z}{\partial z}$$

thus

$$\Xi = 4\pi V^2 \int \mathfrak{d}_x \left(\frac{\partial \mathfrak{d}_x}{\partial x} + \frac{\partial \mathfrak{d}_y}{\partial y} + \frac{\partial \mathfrak{d}_z}{\partial z} \right) d\,\tau + \\ + \frac{1}{4\pi} \int \left\{ \mathfrak{H}_z \left(\frac{\partial \mathfrak{H}_x}{\partial z} - \frac{\partial \mathfrak{H}_z}{\partial x} \right) - \mathfrak{H}_y \left(\frac{\partial \mathfrak{H}_y}{\partial x} - \frac{\partial \mathfrak{H}_x}{\partial y} \right) \right\} d\,\tau + \quad (14) \\ + \int \left(\mathfrak{H}_y \frac{\partial \mathfrak{d}_z}{\partial t} - \mathfrak{H}_z \frac{\partial \mathfrak{d}_y}{\partial t} \right) d\,\tau.$$

Furthermore, a partial integration and application of (IV) and (II) gives (when we denote the direction constants of the perpendicular to σ by χ, β, γ)

$$\begin{aligned} \int \mathfrak{d}_x \frac{\partial \mathfrak{d}_y}{\partial y} d\,\tau &= \int \beta \mathfrak{d}_x \mathfrak{d}_y \, d\,\sigma - \int \mathfrak{d}_y \frac{\partial \mathfrak{d}_x}{\partial y} d\,\tau = \\ &= \int \beta \mathfrak{d}_x \mathfrak{d}_y \, d\,\sigma - \int \mathfrak{d}_y \frac{\partial \mathfrak{d}_y}{\partial x} d\,\tau - \frac{1}{4\pi V^2} \int \mathfrak{d}_y \frac{\partial \mathfrak{H}_z}{\partial t} d\,\tau, \\ \int \mathfrak{d}_x \frac{\partial \mathfrak{d}_z}{\partial z} d\,\tau &= \int \gamma \mathfrak{d}_x \mathfrak{d}_z \, d\,\sigma - \int \mathfrak{d}_z \frac{\partial \mathfrak{d}_x}{\partial z} d\,\tau = \\ &= \int \gamma \mathfrak{d}_x \mathfrak{d}_z \, d\,\sigma - \int \mathfrak{d}_z \frac{\partial \mathfrak{d}_z}{\partial x} d\,\tau + \frac{1}{4\pi V^2} \int \mathfrak{d}_z \frac{\partial \mathfrak{H}_y}{\partial t} d\,\tau, \end{aligned}$$

$$\int \left(\mathfrak{H}_y \frac{\partial \mathfrak{H}_x}{\partial y} + \mathfrak{H}_z \frac{\partial \mathfrak{H}_x}{\partial z} \right) d\,\tau = \int \left(\beta \mathfrak{H}_x \mathfrak{H}_y + \gamma \mathfrak{H}_x \mathfrak{H}_z \right) d\,\sigma -$$

$$- \int \mathfrak{H}_x \left(\frac{\partial \mathfrak{H}_y}{\partial y} + \frac{\partial \mathfrak{H}_z}{\partial z} \right) d\,\tau = \int \left(\beta \mathfrak{H}_x \mathfrak{H}_y + \gamma \mathfrak{H}_x \mathfrak{H}_z \right) d\,\sigma +$$

$$+ \int \mathfrak{H}_x \frac{\partial \mathfrak{H}_x}{\partial x} d\,\tau.$$

If we substitute this value into (14), then several terms occur, that can be completely integrated, and eventually by a simple transformation we have

$$\Xi = 2\pi V^2 \int \left(2\mathfrak{d}_x \mathfrak{d}_n - \alpha \mathfrak{d}^2\right) \, d\,\sigma + \frac{1}{8\pi} \int \left(2\mathfrak{H}_x \mathfrak{H}_n - \alpha \mathfrak{H}^2\right) \, d\,\sigma + \tag{15}$$

$$+ \frac{d}{d\,t} \int \left(\mathfrak{H}_y \mathfrak{d}_z - \mathfrak{H}_z \mathfrak{d}_y\right) \, d\,\tau,$$

Two similar equations serve for the determination of the other components $\mathbf{H}$ and $\mathbf{Z}$ of the ponderomotive action.

Besides it is to be noticed, that Ξ, $\mathbf{H}$ and $\mathbf{Z}$ must vanish, as soon space τ doesn't contain ponderable matter. Then it would be

$$2\pi V^2 \int \left(2\mathfrak{d}_x \mathfrak{d}_n - \alpha \mathfrak{d}^2\right) \, d\,\sigma + \frac{1}{8\pi} \int \left(2\mathfrak{H}_x \mathfrak{H}_n - \alpha \mathfrak{H}^2\right) \, d\,\sigma = \tag{16}$$

$$= -\frac{d}{d\,t} \int \left(\mathfrak{H}_y \mathfrak{d}_z - \mathfrak{H}_z \mathfrak{d}_y\right) \, d\,\tau, \text{ u. s. w.}$$

§ 16. In some cases the space integral the remained in (15), will become independent of *t*, and if the last member vanishes, namely as soon as we have to deal with a stationary state, may it be with an electric charge, or may it be with a system of constant currents. Then, at least concerning the resultant *force*, the ponderomotive action can be calculated by integration over an arbitrary surface that encloses the body, and it is near at hand, to view them in a way, so that we (like *Maxwell* did) attribute to the aether a certain state of tension, and consider the tensions as the cause of the ponderomotive actions.[10] If we as usual understand by (X_n, Y_n, Z_n) the force related to unit area, that the aether exerts at the side (given by *n*) of an element $d\sigma$ upon the opposite aether, then by (15) we would have to put

$$X_n = 2\pi V^2 \left(2\mathfrak{d}_x\mathfrak{d}_n - \alpha\mathfrak{d}^2\right) + \frac{1}{8\pi}\left(2\mathfrak{H}_x\mathfrak{H}_n - \alpha\mathfrak{H}^2\right), \text{etc.} \quad (17)$$

From that, it is easy to derive the values of X_x, X_y, X_z, Y_x; then we exactly obtain the system of tensions that was given by MAXWELL.

§ 17. Since in (15) the space integral doesn't vanish, the assumption of tensions (17) doesn't generally lead to the action stated by us. If we would reject equation (V) as the basis of the calculation of the ponderomotive forces, and employ the tensions, then the case would in no way be finished by formulas (I)—(IV) and (17). One wouldn't even obtain the same values for Ξ, when we would apply the equation

$$\Xi = 2\pi V^2 \int \left(2\mathfrak{d}_x\mathfrak{d}_n - \alpha\mathfrak{d}^2\right)\, d\,\sigma + \frac{1}{8\pi}\int \left(2\mathfrak{H}_x\mathfrak{H}_n - \alpha\mathfrak{H}^2\right)\, d\,\sigma$$

on one area and then on the other area, that encloses the considered body. It is connected with the fact, that the tensions (17) wouldn't let the aether to be at rest.

Above we have found formulas (16) for a space which is free of ponderable matter. That it's correct, as long as the aether is at rest, can hardly be doubted, since for the derivation only generally taken equations come into play. From the formulas

$$Div\, \mathfrak{d} = 0$$

and

$$Rot\,\mathfrak{H} = 4\pi\dot{\mathfrak{d}}$$

it is given, namely, that the right-hand side of equation (14) is zero for the free aether; the application of (IV) and (II) then leads to the first of formulas (16).

Now, in those formulas, the forces (that follow from the tensions at surface σ) are on the left side, and thus the formulas say that the considered part of the aether *cannot* remain at rest under the influence of these forces. All who consider equations (17) as generally valid, must conclude that in all cases, where POYNTING's energy flow is variable with time[11], the aether as a whole will be set into motion. Thus it would be necessary to study the from of the emerging aether flows, and under consideration of them to again tackle the question after the ponderomotive actions.

The basics of a theory of the mentioned aether flows was already sketched by the masterhand of HERMANN VON HELMHOLTZ in one[12] of his last papers, which he was able to complete.

We cannot discuss the considered questions, because the fundamental assumption by which we started, gives another view. Indeed, why should we, since we assumed that the aether is not in motion, ever speak about a force acting on that medium? The most simple would by, to assume that on a volume element of the aether, considered as a whole, never acts a force, or even refuse to apply the concept of force on such an element that of course never moves from its place. Of course this view violates the equality of action and reaction —, since we have reason to say that the aether *exerts* forces on ponderable matter —; but, as far I can see, nothing forces us to elevate this theorem to an unrestricted fundamental law.

Once we have decided ourselves in favor of the previously discusses view, then we must refuse from the outset, to reduce the ponderomotive forces (that follow from (V)) to tensions of the aether.

Nevertheless we may apply equation (15) to simplify the calculation, and it won't cause a misunderstanding, when we express ourselves for brevities sake, as if the elements of the two first integrals would mean real tensions in the aether.

From these merely fictitious "tensions" we can, as we saw, directly derive the interaction between charged bodies and electrodynamic actions. It is also to be recommended, to operate with them, when the phenomena are periodic and when we only wish to know the averages of the ponderomotive forces during a full period; the last member of (15) namely doesn't contribute anything to these values.

In this way we come to MAXWELL's theorem on the pressure generated by motion of light.

The reversibility of motions and the mirror image of motion.

§ 18. For subsequent applications we include the following considerations at this place.

Let a system of moving ions be given, and ρ_1, $\mathfrak{v}_1$, $\mathfrak{d}_1$ and $\mathfrak{H}_1$ are the various relevant magnitudes within. We may denote the corresponding magnitudes for a second system by ρ_2, $\mathfrak{v}_2$, $\mathfrak{d}_2$ and $\mathfrak{H}_2$, and we want to imagine that in an arbitrary point, these magnitudes are at time +*t* in agreement with the magnitudes ρ_1, $-\mathfrak{v}_1$, $\mathfrak{d}_1$ and $-\mathfrak{H}_1$ at time -*t*.

We can easily see that, as regards ρ_2 and $\mathfrak{v}_2$, those conditions can be satisfied by a real motion of the ions, and namely the system of these ions must completely be in agreement with the first system; the same configurations with the same interval must occur one after the other, as in that first system, but in opposite order; in other words, we obtain the motions of the ions in the second system, when we reverse the motions given at first.

Furthermore, since $\mathfrak{d}_2$ and $\mathfrak{H}_2$ satisfy the conditions (I), (II), (III) and (IV), thus the condition of the aether as determined by these vectors, is in agreement with the motion of the ions.

Eventually it follows from equation (V), that in the second system at time +*t*, the forces exerted upon the ions have the same direction and magnitude, as the corresponding forces in the first system at time -*t*. Now, if also the remaining forces that act on the ions in both cases — and in the same instances — are the same, then we can conclude, that the second state of motion is realizable in any way.

By means of similar considerations the possibility of motion can be demonstrated, which is the "mirror image" of a given motion with respect to a fixed plane.

We call P_2 the mirror image of a point P_1 and denote the magnitudes that are valid for two system — namely for the first in P_1 and for the second in P_2 — by ρ_1, $\mathfrak{v}_1$, $\mathfrak{d}_1$, $\mathfrak{H}_1$ and ρ_2, $\mathfrak{v}_2$, $\mathfrak{d}_2$,

$\mathfrak{H}_2$. There it should constantly be $\rho_2 = \rho_1$, and the vectors $\mathfrak{v}_2$, $\mathfrak{d}_2$, $\mathfrak{H}_2$ should be the mirror images of the vectors $\mathfrak{v}_1$, $\mathfrak{d}_1$ and $-\mathfrak{H}_1$.

That the second state of motion can now conveniently be called "mirror image", requires no explanation. If the forces of non-electric origin are of such a manner, so that the vectors by which they can be represented in both cases behave like objects and their mirror images, then the second motion will be possible as soon as the first one is possible.

1. ↑ A prove of the designations employed can be found at the end of the treatise.
2. ↑ By that it is of course not excluded, that mutually separated ions can often have very different velocities.
3. ↑ The justification of this lies in equation (5)
4. ↑ We neglect special *magnetic* properties of ponderable matter — which by the way would be explained by the motion of ions. Consequently we don't have to distinguish between the magnetic force and the magnetic induction.
5. ↑ The factor V^2 must be added, because we use the electromagnetic system of units.
6. ↑ Since this force is the only one, which exists in relation to electrostatic phenomena, it can well be called *electrostatic* force, although in general it also depends on the motion of ions.
7. ↑ Here, this letter doesn't mean something infinitely small in the strict sense of the word, but a distance that is of course very small compared to the dimensions of the conductor, but nevertheless much greater than the distance of the molecules.
8. ↑ If we don't want to consider an ordinary electric current as a convection current, then we must substantiate this formula by the assumption, that a body in which a convection takes

place, experiences the same electrodynamic actions as a corresponding current conductor.

9. ↑ In an earlier published derivation of the equations of motion (La théorie électromagnétique de MAXWELL et son application aux corps mouvants), I have discussed the necessary conditions.
10. ↑ Also with respect to the resultant force couple, the ponderomotive action on a rigid body is equivalent to the system of tensions (17) on an arbitrary surface σ that encloses the body. If we also want to consider the ponderomotive actions on flexible or fluid bodies, then we would have to come back to volume elements. But this would lead too far.
11. ↑ Except the factor $-V^2$, the components of the energy flow are located on the right-hand side of equations (16) under the integral sign.
12. ↑ v. HELMHOLTZ. Folgerungen aus MAXWELL's Theorie über die Bewegungen des reinen Aethers. Berl. Sitz.-Ber., 5. Juli 1893; Wied. Ann., Bd. 53, p. 135, 1894.

Electric phenomena in ponderable bodies that are moving with constant velocity through the stationary aether.

Transformation of the fundamental equations.

§ 19. From now on it will be assumed that the bodies to be considered are moving at a steady velocity of translation $\mathfrak{p}$, under which we will have to understand in almost all applications, the speed of the earth in its motion around the sun. It would be interesting at first to further develop the theory for stationary bodies, but for brevity's sake let us immediately turn to the more general case. Besides, it may be still set $\mathfrak{p} = 0$.

The treatment of the problems that are now coming into play is most simple, when instead of the co-ordinate system used above, we introduce another one which is rigidly connected with ponderable matter and therefore shares its displacement.

While the coordinates of a point with respect to the fixed system were called *x, y, z*, let those, which refer to the moving system and which I call the *relative* coordinates, denoted by (*x*), *(y)*, *(z)* for the time being. Until now, all the variable parameters were seen as functions of *x, y, z, t*; furthermore $\mathfrak{d}_x$, $\mathfrak{d}_y$, etc. shall be seen as functions of *(x) (y)*, *(z)* and *t*.

Under a *fixed* point, we now understand one point, that has a steady position with respect to the new axis; in the same way, by *rest* or *motion* of a physical particle we shall mean the *relative* rest or the *relative* motion in relation to ponderable matter. With ions, which move in this sense of the word, we will have to do as soon as the displaced matter is the seat of electric motions.

By $\mathfrak{v}$ we shall not represent the real velocity, but the velocity of the previously mentioned relative motion. The real velocity is thus

$$\mathfrak{p} + \mathfrak{v},$$

and hereby $\mathfrak{v}$ is to be replaced in equations (4) and (V).

In addition, we have, instead of the derivatives with respect to *x*, *y*, *z* and *t*, to establish such with respect to *(x)*, *(y)*, *(z)* and *t*.

The first mentioned derivative I denote by

$$\frac{\partial}{\partial x}, \ \frac{\partial}{\partial y}, \ \frac{\partial}{\partial z}, \ \left(\frac{\partial}{\partial t}\right)_1,$$

however, the latter by

$$\frac{\partial}{\partial (x)}, \ \frac{\partial}{\partial (y)}, \ \frac{\partial}{\partial (z)}, \ \left(\frac{\partial}{\partial t}\right)_2.$$

Now we have, by application to an arbitrary function,

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial (x)}, \ \frac{\partial}{\partial y} = \frac{\partial}{\partial (y)}, \ \frac{\partial}{\partial z} = \frac{\partial}{\partial (z)},$$

$$\left(\frac{\partial}{\partial t}\right)_1 = \left(\frac{\partial}{\partial t}\right)_2 - \mathfrak{p}_x \frac{\partial}{\partial (x)} - \mathfrak{p}_y \frac{\partial}{\partial (y)} - \mathfrak{p}_z \frac{\partial}{\partial (z)}.$$

By that it follows, that we can write for $\boldsymbol{Div}\,\mathfrak{A}$ the expression

$$\frac{\partial \mathfrak{A}_x}{\partial (x)} + \frac{\partial \mathfrak{A}_y}{\partial (y)} + \frac{\partial \mathfrak{A}_z}{\partial (z)},$$

and for the components of $\boldsymbol{Rot}\,\mathfrak{A}$

$$\frac{\partial \mathfrak{A}_z}{\partial (y)} - \frac{\partial \mathfrak{A}_y}{\partial (z)}, \text{ etc.}$$

The expressions $\boldsymbol{Div}\,\mathfrak{A}$ and $\boldsymbol{Rot}\,\mathfrak{A}$ have still the meaning given in § 4, *g* and *h*, if, after having abandoned the old coordinates one and for all, for simplification we don't indicate the new ones with *(x), (y), (z),* but with *x, y, z*.

We also want, after we have passed to the new coordinates, use the sign $\frac{\partial}{\partial t}$ instead of $\left(\frac{\partial}{\partial t}\right)_2$ for a differentiation with respect to time at constant relative coordinates, so that

$$\left(\frac{\partial}{\partial t}\right)_1 = \frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} - \mathfrak{p}_y \frac{\partial}{\partial y} - \mathfrak{p}_z \frac{\partial}{\partial z} \tag{18}$$

The derivative with respect to time, which occurs in the basic equations (I) - (V), are all of the kind indicated by $\left(\frac{\partial}{\partial t}\right)_1$. We will maintain this sign as an abbreviation for the longer term (18).

In contrast, a point over a letter shall henceforth — such as $\partial/\partial t$ - indicate a differentiation with respect to time at constant relative coordinates. Thus the terms $\dot{\mathfrak{d}}$ and $\dot{\mathfrak{H}}$ in (4) and (IV) may not be left unaltered. By $\mathfrak{d}$, for example, we understood a vector with components

$$\left(\frac{\partial \mathfrak{d}_x}{\partial t}\right)_1, \text{etc.},$$

or

$$\left(\frac{\partial}{\partial t} - \mathfrak{p}_x \frac{\partial}{\partial x} - \mathfrak{p}_y \frac{\partial}{\partial y} - \mathfrak{p}_z \frac{\partial}{\partial z}\right) \mathfrak{d}_x, \text{etc.}$$

We can suitably write this vector

$$\left(\frac{\partial \mathfrak{d}}{\partial t}\right)_1,$$

while

$$\dot{\mathfrak{d}} \text{ or } \frac{\partial \mathfrak{d}}{\partial t}$$

will mean the vector with components

$$\frac{\partial \mathfrak{d}_x}{\partial t}, \text{etc.}$$

Based on the system of axes associated with ponderable matter, eventually the fundamental equations become

$$Div\, \mathfrak{d} = \rho, \qquad (I_a)$$

$$\mathfrak{S} = \rho(\mathfrak{p} + \mathfrak{v}) + \left(\frac{\partial \mathfrak{d}}{\partial t}\right)_1, \tag{4$_\mathrm{a}$}$$

$$Div\ \mathfrak{H} = 0, \tag{II$_\mathrm{a}$}$$

$$Rot\ \mathfrak{H} = 4\pi\mathfrak{S}, \tag{III$_\mathrm{a}$}$$

$$-4\pi V^2 Rot\ \mathfrak{d} = \left(\frac{\partial \mathfrak{H}}{\partial t}\right)_1, \tag{IV$_\mathrm{a}$}$$

$$\mathfrak{E} = 4\pi V^2 \mathfrak{d} + [\mathfrak{p} . \mathfrak{H}] + [\mathfrak{v} . \mathfrak{H}]. \tag{V$_\mathrm{a}$}$$

§ 20. For some purposes, a different form of some equations is more appropriate.

The first of the three (IV) summarized relations is namely

$$-4\pi V^2 \left(\frac{\partial \mathfrak{d}_z}{\partial y} - \frac{\partial \mathfrak{d}_y}{\partial z}\right) = \frac{\partial \mathfrak{H}_x}{\partial t} - \mathfrak{p}_x \frac{\partial \mathfrak{H}_x}{\partial x} - \mathfrak{p}_y \frac{\partial \mathfrak{H}_x}{\partial y} - \mathfrak{p}_z \frac{\partial \mathfrak{H}_x}{\partial z},$$

where, by equation (II$_\mathrm{a}$), we can write for the last three members

$$\left(\mathfrak{p}_x \frac{\partial \mathfrak{H}_y}{\partial y} - \mathfrak{p}_y \frac{\partial \mathfrak{H}_x}{\partial y}\right) - \left(\mathfrak{p}_z \frac{\partial \mathfrak{H}_x}{\partial z} - \mathfrak{p}_x \frac{\partial \mathfrak{H}_z}{\partial z}\right),$$

which is nothing else than the first component of

$$Rot\ [\mathfrak{p} . \mathfrak{H}].$$

Accordingly, we obtain instead of (IV$_\mathrm{a}$)

$$Rot\ \left\{4\pi V^2 \mathfrak{d} + [\mathfrak{p} . \mathfrak{H}]\right\} = -\dot{\mathfrak{H}}.$$

Furthermore, the current $\mathfrak{S}$ can be entirely eliminated. The first of equations (III_a) becomes, when we consider (4_a) and (I_a),

$$\frac{\partial \mathfrak{H}_z}{\partial y} - \frac{\partial \mathfrak{H}_y}{\partial z} = 4\pi\rho\left(\mathfrak{p}_x + \mathfrak{v}_x\right) + 4\pi\left(\frac{\partial \mathfrak{D}_x}{\partial t} - \mathfrak{p}_x\frac{\partial \mathfrak{D}_x}{\partial x} - \mathfrak{p}_y\frac{\partial \mathfrak{D}_x}{\partial y} - \right.$$

$$\left. -\mathfrak{p}_z\frac{\partial \mathfrak{D}_x}{\partial z}\right) = 4\pi\rho\mathfrak{v}_x + 4\pi\left\{\left(\mathfrak{p}_x\frac{\partial \mathfrak{D}_y}{\partial y} - \mathfrak{p}_y\frac{\partial \mathfrak{D}_x}{\partial y}\right) - \left(\mathfrak{p}_z\frac{\partial \mathfrak{D}_x}{\partial z} - \right.\right.$$

$$\left.\left. -\mathfrak{p}_x\frac{\partial \mathfrak{D}_z}{\partial z}\right)\right\} + 4\pi\frac{\partial \mathfrak{D}_x}{\partial t}.$$

By that it follows, if we define a new vector $\mathfrak{H}'$ by means of the equation

$$\mathfrak{H}' = \mathfrak{H} - 4\pi[\mathfrak{p}.\,\mathfrak{D}],$$

thus

$$Rot\ \mathfrak{H}' = 4\pi\rho\mathfrak{v} + 4\pi\dot{\mathfrak{D}}.$$

If we now introduce the sign $\mathfrak{F}$ for the electric force-action on *stationary* ions, we obtain the following set of formulas

$$Div\ \mathfrak{D} = \rho, \qquad (I_b)$$

$$Div\ \mathfrak{H} = 0, \qquad (II_b)$$

$$Rot\ \mathfrak{H}' = 4\pi\rho\mathfrak{v} + 4\pi\dot{\mathfrak{D}}, \qquad (III_b)$$

$$Rot\ \mathfrak{F} = -\dot{\mathfrak{H}}, \qquad (IV_b)$$

$$\mathfrak{F} = 4\pi V^2 \mathfrak{d} + [\mathfrak{p}.\mathfrak{H}], \tag{V$_b$}$$

$$\mathfrak{H}' = \mathfrak{H} - 4\pi[\mathfrak{p}.\mathfrak{d}], \tag{VI$_b$}$$

$$\mathfrak{E} = \mathfrak{F} + [\mathfrak{v}.\mathfrak{H}]. \tag{VII$_b$}$$

§ 21. From equations (I$_a$)—(V$_a$) (§ 19) also some formulas can be derived, any of them contains only one of the magnitudes $\mathfrak{d}_x$, $\mathfrak{d}_y$, $\mathfrak{d}_z$, $\mathfrak{H}_x$, $\mathfrak{H}_y$, $\mathfrak{H}_z$.

At first, it follows from (IV$_a$)

$$-4\pi V^2 Rot\ Rot\ \mathfrak{d} = Rot\left(\frac{\partial \mathfrak{H}}{\partial t}\right)_1 = \left(\frac{\partial Rot\ \mathfrak{H}}{\partial t}\right)_1.$$

If we consider here what has been said in § 4, *h*, as well as the relations (I$_a$), (III$_a$) and (4$_a$), we arrive at the three formulas

$$V^2 \Delta \mathfrak{d}_x - \left(\frac{\partial^2 \mathfrak{d}_x}{\partial t^2}\right)_1 = V^2 \frac{\partial \rho}{\partial x} + \left(\frac{\partial}{\partial t}\right)_1 \{\rho\,(\mathfrak{p}_x + \mathfrak{v}_x)\}\,,\ \text{etc.} \tag{A}$$

Similarly, we find

$$V^2 \Delta \mathfrak{H}_x - \left(\frac{\partial^2 \mathfrak{H}_x}{\partial t^2}\right)_1 = 4\pi V^2 \left[\frac{\partial}{\partial z}\{\rho\left(\mathfrak{p}_y + \mathfrak{v}_y\right)\} - \right.$$

$$\left. - \frac{\partial}{\partial y}\{\rho\,(\mathfrak{p}_z + \mathfrak{v}_z)\}\right],\ \text{etc.} \tag{B}$$

The last members of these six equations are completely known once we know how the ions are moving.

Application to electrostatics.

§ 22. We want to calculate by which forces the ions act on one another, when all of them are at rest with respect to ponderable matter. In this case a state occurs, where at every point $\mathfrak{d}$ and $\mathfrak{H}$ are independent of time. We have

$$\left(\frac{\partial}{\partial t}\right)_1 = -\left(\mathfrak{p}_x \frac{\partial}{\partial x} + \mathfrak{p}_y \frac{\partial}{\partial y} + \mathfrak{p}_z \frac{\partial}{\partial z}\right), \tag{19}$$

and equations (A) and (B) will be reduced, when for brevity's sake the operation

$$\Delta - \frac{1}{V^2}\left(\mathfrak{p}_x \frac{\partial}{\partial x} + \mathfrak{p}_y \frac{\partial}{\partial y} + \mathfrak{p}_z \frac{\partial}{\partial z}\right)^2$$

is indicated by Δ', to

$$\Delta' \mathfrak{d}_x = \frac{\partial \rho}{\partial x} - \frac{\mathfrak{p}_x}{V^2}\left(\mathfrak{p}_x \frac{\partial \rho}{\partial x} + \mathfrak{p}_y \frac{\partial \rho}{\partial y} + \mathfrak{p}_z \frac{\partial \rho}{\partial z}\right), \text{etc.} \tag{A'}$$

and

$$\Delta' \mathfrak{H}_x = 4\pi \left(\mathfrak{p}_y \frac{\partial \rho}{\partial z} - \mathfrak{p}_z \frac{\partial \rho}{\partial y}\right), \text{etc.} \tag{B'}$$

To fulfill these conditions, we determine a function ω by

$$\Delta' \omega = \rho$$

and put

$$\mathfrak{d}_x = \frac{\partial \omega}{\partial x} - \frac{\mathfrak{p}_x}{V^2}\left(\mathfrak{p}_x \frac{\partial \omega}{\partial x} + \mathfrak{p}_y \frac{\partial \omega}{\partial y} + \mathfrak{p}_z \frac{\partial \omega}{\partial z}\right), \text{etc.}, \tag{20}$$

$$\mathfrak{H}_x = 4\pi\left(\mathfrak{p}_y \frac{\partial \omega}{\partial z} - \mathfrak{p}_z \frac{\partial \omega}{\partial y}\right), \text{etc.}, \tag{21}$$

i.e., values that really satisfy the fundamental equations (I_a) - (IV_a).

From (V_a) it also follows

$$\mathfrak{E}_x = 4\pi\left(V^2 - \mathfrak{p}^2\right) \frac{\partial \omega}{\partial x}, \text{etc.}, \tag{22}$$

so that the sought forces are found.

Without prejudice to generality, we may assume that the translation happens in the direction of the *x*-axis. It is then $\mathfrak{p}_y = \mathfrak{p}_z = 0$, and the formula for the determination of ω will be transformed into

$$\left(1 - \frac{\mathfrak{p}^2}{V^2}\right) \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} + \frac{\partial^2 \omega}{\partial z^2} = \rho. \tag{23}$$

§ 23. To clearly define the meaning of the above formulas, we will compare the considered system S_1 with a second one S_2. The latter should *not* be moved, and it arises from S_1 by increasing all the dimensions that have the direction of the *x*-axis (therefore the relevant dimensions of the ions as well), in the ratio $\sqrt{V^2 - \mathfrak{p}^2}$ to V, or: between the coordinates *x, y, z* of a point of S_1 and the coordinates x', y', z' of the same corresponding point of S_2, we let remain the relations

$$x = x'\sqrt{1 - \frac{\mathfrak{p}^2}{V^2}},\ y = y',\ z = z' \tag{24}$$

In addition, the mutually corresponding volume elements, and therefore also the ions, shall have the same charges in S_1 and S_2.

If we apply to all magnitudes, which are related to the second system, a prime so they can be distinguished, then

$$\rho' = \rho\sqrt{1 - \frac{\mathfrak{p}^2}{V^2}},$$

and

$$\frac{\partial^2 \omega'}{\partial x'^2} + \frac{\partial^2 \omega'}{\partial y'^2} + \frac{\partial^2 \omega'}{\partial z'^2} = \rho' = \rho\sqrt{1 - \frac{\mathfrak{p}^2}{V^2}}.$$

Then the equation (23) can be written in the form

$$\frac{\partial^2 \omega}{\partial x'^2} + \frac{\partial^2 \omega}{\partial y'^2} + \frac{\partial^2 \omega}{\partial z'^2} = \rho$$

then

$$\omega = \frac{\omega'}{\sqrt{1 - \frac{\mathfrak{p}^2}{V^2}}},$$

and since in the second system

$$\mathfrak{E}'_x = 4\pi V^2 \frac{\partial \omega'}{\partial x'}, \text{ etc.}$$

thus

$$\mathfrak{E}_x = \mathfrak{E}'_x,\ \mathfrak{E}_y = \sqrt{1 - \frac{\mathfrak{p}^2}{V^2}}\mathfrak{E}'_y,\ \mathfrak{E}_z = \sqrt{1 - \frac{\mathfrak{p}^2}{V^2}}\mathfrak{E}'_z.$$

The same relations, as they exist between the components of $\mathfrak{E}$ and $\mathfrak{E}'$, also exist, since the charges in S_1 and S_2 are equal, between the force components acting on an ion.

If in the second system at certain places $\mathfrak{E}' = 0$, then $\mathfrak{E}$ vanishes at the corresponding points of the first system.

§ 24. Several implications of this theorem are obvious. From ordinary electrostatics, we know for example that an excess of positive (or negative) ions can be distributed over a conductor, namely over its surface Σ, so that in the interior no electric force is acting. If we take this distribution for the system S_2 and derive from it a system S_1 by the above-discussed transformation, then also in this one an excess of positive ions only exists at a certain surface Σ, while in all interior points the electric force $\mathfrak{E}$ vanishes. The fact that an electric charge is located at the *surface* of a conductor, won't be changed by the translation of ponderable matter.

Similar considerations apply to two or more bodies. If a conductor *C* is confronted with a charged body *K*, then there exists, according to a known theorem, always a certain amount of charge on the surface of *C*, which together with *K* exerts no action on the ions in the interior of the conductor. This theorem remains valid, if the ponderable matter is moving, and it is even still allowed to assume that, under the influence of *K*, an "induced" charge is formed by itself upon *C*, which just cancels the effect of *K* on the interior points.

Since by (22) the components of $\mathfrak{E}$ are proportional to the derivative of ω, we can also say that inducing and inducted charges together cause a constant ω at all points of *C*. It follows then by means of equations (20), (21) and (V_a), that also a moving ion in the interior of C does not experience any force-action from the two charges.

Finally, it should be noted that by our formulas, the distribution of a charge over a given conductor, as well as the attraction or repulsion of charged bodies by the motion of the earth, must be changed. But this influence is limited to the *second* order, namely if the fraction $\mathfrak{p}/V$ is called a magnitude of *first* order, and thus the fraction $\mathfrak{p}^2/V^2$ is called a magnitude of *second* order.

Since $\mathfrak{p}/V = 1/10000$, we may not hope, neglecting some very special cases, to find with respect to electrical and optical phenomena an influence of earth's motion that depends on $\mathfrak{p}^2/V^2$. The only thing that could be observed in relation to bodies at rest on earth, is the magnetic force (21). At first glance, we might expect a corresponding effect on the current elements. We will return to this question in § 26.

Values of $\mathfrak{d}$ and $\mathfrak{H}$ at a stationary current.

§ 25. On the basis of equations (A) and (B) we again tackle the problem treated in § 11. We consider, as there, the mean values and take into account that for them the simplification (19) is permitted in stationary states; moreover, we assume at first that the conductors do not have a significant charge, so that $\bar{\rho} = 0$.

It is near at hand to interpret the vector $\overline{\rho\mathfrak{v}}$ as being a "current". We think of it as solenoidally distributed and denote it by $\bar{\mathfrak{S}}$, where it remains, however, temporarily undecided whether this is also the mean value of the vector occurring in (4_a).

We now derive from (A) and (B)

$$V^2 \Delta' \bar{\mathfrak{d}}_x = -\left(\mathfrak{p}_x \frac{\partial}{\partial x} + \mathfrak{p}_y \frac{\partial}{\partial y} + \mathfrak{p}_z \frac{\partial}{\partial z}\right) \bar{\mathfrak{S}}_x, \text{ etc.,}$$

$$\Delta' \overline{\mathfrak{H}_x} = 4\pi \left(\frac{\partial \overline{\mathfrak{S}}_y}{\partial z} - \frac{\partial \overline{\mathfrak{S}}_z}{\partial y}\right), \text{ etc.}$$

If we determine thus the three auxiliary magnitudes χ_x, χ_y, χ_z[1] by means of the equations

$$\Delta' \chi_x = \overline{\mathfrak{S}}_x,\ \Delta' \chi_y = \overline{\mathfrak{S}}_y,\ \Delta' \chi_z = \overline{\mathfrak{S}}_z,$$

so everywhere we have

$$\overline{\mathfrak{d}_x} = -\frac{1}{V^2}\left(\mathfrak{p}_x \frac{\partial}{\partial x} + \mathfrak{p}_y \frac{\partial}{\partial y} + \mathfrak{p}_z \frac{\partial}{\partial z}\right)\chi_x\text{, etc.,} \tag{25}$$

$$\overline{\mathfrak{H}_x} = 4\pi\left(\frac{\partial \chi_y}{\partial z} - \frac{\partial \chi_z}{\partial y}\right)\text{, etc.,} \tag{26}$$

and by (V_a) the electric force acting on stationary ions,

$$\overline{\mathfrak{E}_x} = -4\pi \frac{\partial}{\partial x}\left(\mathfrak{p}_x \chi_x + \mathfrak{p}_y \chi_y + \mathfrak{p}_z \chi_z\right)\text{, etc.} \tag{27}$$

At first glance, it therefore seems as if a current that streams through a conductor, is acting on a stationary ion with a force of first order. However, on closer reflection we find that the force (27) is just being compensated by another force.

The values (27) are in fact in perfect agreement with the expressions (22), if we substitute

$$\omega = -\frac{\mathfrak{p}_x \chi_x + \mathfrak{p}_y \chi_y + \mathfrak{p}_z \chi_z}{V^2 - \mathfrak{p}^2} \tag{28}$$

By § 22, ω would belong to an electric charge, its density is

$$\rho = \Delta' \omega,$$

or by the given formulas

$$\rho = -\frac{\mathfrak{p}_x \overline{\mathfrak{S}_x} + \mathfrak{p}_y \overline{\mathfrak{S}_y} + \mathfrak{p}_z \overline{\mathfrak{S}_z}}{V^2 - \mathfrak{p}^2} \tag{29}$$

Let us imagine for a moment that the current does not exist, but there is a charge with the average density ρ. This would of course exist only in the conductor, and the total sum would be zero, as it follows from (29) and

$$\Delta\overline{\mathfrak{S}} = 0$$

Obviously this ion distribution would completely vanish, if it is left alone. This can also be expressed by saying that the charge will set them in motion by virtue of its action on resting ions, and that therefore eventually another charge with the average density -ρ occurs besides it, or

$$\frac{\mathfrak{p}_x\overline{\mathfrak{S}_x} + \mathfrak{p}_y\overline{\mathfrak{S}_y} + \mathfrak{p}_z\overline{\mathfrak{S}_z}}{V^2 - \mathfrak{p}^2}$$

Since the current that we considered initially, exactly acts on resting ions as the charge (29), it will also generate the charge *A* after a short time; this eliminates the effects on stationary ions, namely not only in the outer points, but also, at least with respect to the averages of the forces, in the interior of the conductor.

I want to call this charge *A* the *compensation charge*. Once generated, the conductor does not cause any electricity motion in a neighboring body. A stationary current in a wire moving with the Earth therefore exerts no inductive action on a circuit which is also at rest with respect to Earth, regardless of Earth's motion[2].

It should be noted now that in the finally occurring state of the system, ρ and $\mathfrak{d}$ have certain values of order $\mathfrak{p}$. Neglecting the magnitudes of second order, then it really follows from (4_a)

$$\overline{\mathfrak{S}} = \overline{\rho \mathfrak{v}}.$$

Interaction between a charged body *K* and a conductor.

§ 26. After the foregoing, we have to assume that in the conductor next to the current $\overline{\mathfrak{S}}$, a compensation charge does exist, and also (at the surface of the conductor) the electrostatic induction-charge *B* caused by *K*. For simplicity, we imagine that $\overline{\mathfrak{S}}$, A and B co-exist as independent ion systems[3]. Each of the four systems $\overline{\mathfrak{S}}$, A, B and K now forces a special state to the aether, and thus acts on any of the others. To shortly indicate these actions, we want to put $\left(\overline{\mathfrak{S}}, K\right)$ for those actions, which for example were exerted by $\overline{\mathfrak{S}}$ on K, where we have to notice that perhaps $\left(\overline{\mathfrak{S}}, K\right)$ and $\left(K, \overline{\mathfrak{S}}\right)$ are not equal and opposite, and that also actions such as $\left(\overline{\mathfrak{S}}, \overline{\mathfrak{S}}\right)$ may exist, namely forces which act on one of the ion systems due to condition changes in the aether, which were caused by itself.

In easily understandable symbols we can now write for the total action on *K*

$$(K, K) + (B, K) + (\overline{\mathfrak{S}}, K) + (A, K),$$

which, however, due to § 25

$$(\overline{\mathfrak{S}}, K) + (A, K) = 0$$

is reduced to the first two terms and thus becomes independent of the current.

On the other hand, the forces which act on the conductor, can be represented by a expression consisting of 12 members, since the action of K, $\overline{\mathfrak{S}}$, A and B on $\overline{\mathfrak{S}}$, A and B, has to be considered each time. It is now

$$(K, \overline{\mathfrak{S}}) + (B, \overline{\mathfrak{S}}) = 0, \ (K, A) + (B, A) = 0,$$

so that by the aforementioned expression it only remains

$$(K, B) + (B, B) + (A, \overline{\mathfrak{S}}) + (\overline{\mathfrak{S}}, \overline{\mathfrak{S}}). \tag{30}$$

Those forces represented by the first two members would also exist, when $\overline{\mathfrak{S}} = 0$, and the last two members are independent of the charged body *K*. An action of *K* exerted on the conductor as such, doesn't exist.

Besides, in each of the four members (30), the part that depends on $\mathfrak{p}$ is of second order. We already know this from $(K, B) + (B, B)$, since this represents an electrostatic effect. $(A, \overline{\mathfrak{S}})$ and $(\overline{\mathfrak{S}}, \overline{\mathfrak{S}})$, however, represent forces acting on a current, in which the mean electric density is zero. As it can be seen from (V_a), such forces are determined by the value of $\mathfrak{H}$, which belongs to the *acting* system. Inasmuch as $\mathfrak{H}$ (that belongs to $\overline{\mathfrak{S}}$) depends on $\mathfrak{p}$, it is of second order (§ 25), and the compensation charge *A* only produces by its velocity $\mathfrak{p}$ a magnetic force of second order, since its density already contains the factor $\mathfrak{p}/V$.

Electrodynamic actions.

§ 27. The question as to how these effects are influenced by earth's motion, can now easily be answered. If we denote the currents in two conductors by $\overline{\mathfrak{S}}$ and $\overline{\mathfrak{S}'}$, and the corresponding compensation charges by *A* and A', then the action exerted on the second conductor is

$$(\overline{\mathfrak{S}}, \overline{\mathfrak{S}'}) + (A, \overline{\mathfrak{S}'}) + (\overline{\mathfrak{S}}, A') + (A, A'),$$

in which the last two terms are mutually canceled. That $(A, \overline{\mathfrak{S}'})$ and the $\mathfrak{p}$-dependent part $(\overline{\mathfrak{S}}, \overline{\mathfrak{S}'})$ are of order $\mathfrak{p}^2/V^2$, follows from considerations such as those communicated above.

Induction in a linear conductor.

§ 28. A closed secondary wire from *B* will be displaced from B_1 into position B_2, while a primary conductor *A* at the same time passes from position A_1 to A_2, and the intensity of the primary current increases from i_1 to i_2. At the beginning and the end of time *T*, in which these processes take place, the two conductors shall be at rest and the primary current shall be constant; if no other electromotive forces acts on *B*, then this wire will eventually be, as before, without current. We want to determine the quantity of electricity, which has passed in time *T* through a cross section of the wire, and namely we will *only* consider *the convection current* at this place.

After the expiry of the whole process, the surface of *B* has nowhere a electric charge. It follows that the quantity of electricity that streamed through is the same for all cross sections, and that the conductor can be decomposed into infinitely thin current tubes, so that in each of them and equally through all cross-sections, the same quantity of electricity is streaming.

We consider in detail one of these tubes, and call *ds* an element of their length, ω is a vertical cross-section, *Ndt* the number of positive ions which pass through it during the time *dt* in the assumed positive direction *s*, *N'dt* the number of negative ions which move in the opposite direction, *e* is the charge of a positive and $-e'$ the charge of a negative ion. The total current through ω is then

$$i = \int (N\,e + N'\,e')\,d\,t. \tag{31}$$

Furthermore, $\mathfrak{E}_s$ and $\mathfrak{E}'_s$ are the electric forces acting in the direction of *ds*, which come into consideration for a positive or a negative ion. By Ohm's law we shall assume, that the motion of ions by these forces is thus determined, so that *N* and N' are proportional to its mean value; this and the proportionality to ω, we express by

$$N = p\overline{\mathfrak{E}_s}\omega,\ N' = q\overline{\mathfrak{E}'_s}\omega,$$

where *p* and *q* are constant factors.

It is now necessary to distinguish between the velocity of the considered conductor element and the relative velocity of an ion in the wire. The former shall be called $\mathfrak{v}$ and the latter $\mathfrak{w}$. From (V_a) it is given

$$\mathfrak{E} = 4\pi V^2\mathfrak{d} + [\mathfrak{p}\,.\,\mathfrak{H}] + [\mathfrak{v}\,.\,\mathfrak{H}] + [\mathfrak{w}\,.\,\mathfrak{H}].$$

Yet, the velocity $\mathfrak{w}$ has the direction of *ds*; consequently we have $[\mathfrak{w}\,.\,\mathfrak{H}]_s = 0$, and for positive as well as for negative ions

$$\mathfrak{E}_s = \mathfrak{E}'_s = 4\pi V^2\mathfrak{d}_s + [\mathfrak{p}\,.\,\mathfrak{H}]_s + [\mathfrak{v}\,.\,\mathfrak{H}]_s.$$

Finally, equation (31) transforms into

$$i = c\omega \int \left\{4\pi V^2\bar{\mathfrak{d}}_s + [\mathfrak{p}\,.\,\bar{\mathfrak{H}}]_s + [\mathfrak{v}\,.\,\bar{\mathfrak{H}}]_s\right\}\,d\,t,\; c = pe + qe'.$$

Let us divide by $c\,\omega$, multiply by *ds*, and integrate over the whole current-line. If we consider here, that *i* has everywhere the same value in the current-line, and if we put

$$\int \frac{d\,s}{c\,\omega} = \frac{1}{C},$$

we shall find

$$i = C \int \left\{4\pi V^2 \int \bar{\mathfrak{d}}_s\,d\,s + \int [\mathfrak{p}\,.\,\bar{\mathfrak{H}}]_s\,d\,s + \int [\mathfrak{v}\,.\,\bar{\mathfrak{H}}]_s\,d\,s\right\}\,d\,t. \quad (32)$$

§ 29. The following discussion is intended to derive the known fundamental law of induction from this formula. Imagine an area σ on which the current-line constantly is located during its motion, and consider the integral

$$\int \overline{\mathfrak{H}_n}\,d\,\sigma = P, \quad (33)$$

for the part that is cut by the line.

This quantity, which is usually called "the number of magnetic force-lines covered by *s*", changes over time, namely for two reasons. First, $\bar{\mathfrak{H}}$ varies at each point, and second, the area of integration changes.

During time *dt*, the first cause produces the following increase of *P*

$$d\,t\int\overline{\dot{\mathfrak{H}}_n}\,d\,\sigma.$$

As to the second variation, it should be noted that each element *ds* describes an infinitely small parallelogram on the surface, and that the value of the surface integral $\int\overline{\dot{\mathfrak{H}}_n}\,d\,\sigma$ of this parallelogram, by suitably chosen signs, goes into *dP*. This value is determined by the area of the parallelepiped, with $d\,s$, $\mathfrak{H}$ as its sides, and the distance $\mathfrak{v}\,d\,t$ in the direction of $\mathfrak{v}$. We will find for it

$$-d\,t[\mathfrak{v}.\bar{\mathfrak{H}}]_s\,d\,s,$$

and for the whole increase of (33)

$$d\,P=d\,t\int\overline{\dot{\mathfrak{H}}_n}\,d\,\sigma-d\,t\int[\mathfrak{v}.\bar{\mathfrak{H}}]_s\,d\,s,$$

or, if the relations (IV_b) and (V_b), as well as the theorem stated in (1) (§ 4, *h*), were considered,

$$-d\,t\int\left\{4\pi V^2\bar{\mathfrak{d}}_s+[\mathfrak{p}.\bar{\mathfrak{H}}]_s\right\}\,d\,s-d\,t\int[\mathfrak{v}.\bar{\mathfrak{H}}]_s\,d\,s.$$

Consequently, (32) transforms into

$$i = -C \int d\,P = C\,(P_1 + P_2)\,,$$

where P_1 and P_2 belong to the beginning and the end of the considered time.

The magnitude *P* depends on the different parts of $\mathfrak{H}$. Since an induced current neither exists at the beginning nor at the end of time *T*, we commit no mistake when we substitute into (33) for $\mathfrak{H}$ only the magnetic force generated by the primary current. The prime above the letter can be omitted here, and if the induced wire is very thin, we may calculate for all current-lines with the same *P*. Finally, if C_1 is the sum of all numbers *C* (*i.e.*, the conductivity of the induced electrical circuit), then the integral-current which we wished to calculate, becomes

$$I = C_1\,(P_1 - P_2)\,,$$

which is consistent with a known theorem.

The motion of Earth was never overlooked during the given derivation; consequently the formula admits of a conclusion about the influence of this motion on the phenomena of induction. There, only magnitudes of second order come into account. $\mathfrak{H}$, which should serve to determine the magnitude *P*, is indeed composed of the vector specified by (26) and the magnetic force which is generated by the compensation charge. The latter magnetic force is of order $\mathfrak{p}^2/V^2$, and since in equations (§ 25) that serve to determine χ_x, χ_y, χ_z, also just the square of $\mathfrak{p}$ is included, then the values (26) differ only to second order from the expressions that apply to a stationary earth.

By proving, that no first-order influence may be expected from the phenomena of induction, we have achieved the explanation for the negative result of Des Coudres[4].

1. ↑ These magnitudes are only different by a constant factor from the components of the vector potential, when $\mathfrak{p} = 0$.
2. ↑ It should be remembered that Mr. Budde (Wied. Ann., Vol 10, p. 553, 1880), on the basis of Clausius' law, reached the same conclusions, as it was drawn here by me. His value for the density of the compensation current even completely agrees with the above-found, if $\mathfrak{p}^2$ is neglected.
3. ↑ This mode of imagination, however, is in no way necessary. To show that the considerations communicated in the texts are correct, we don't need to assume, that the ions which form the charges A and B, were remaining at rest and were altogether uninfluenced by the adjacent existing current. We can also imagine that *all* ions are moving, similar to an electrolyte, in a most irregular manner. But a constant, non-zero mean value $\bar{ho}$ is very well possible; because this constitutes the charges designated by A and B (*i.e.*, $\bar{\rho}$ is composed of two terms of a sum $\bar{\rho_A}$ and $\bar{\rho_B}$), while the current $\overline{\mathfrak{S}}$ is determined by $\overline{\rho\mathfrak{v}}$. If in (A) and (B) all members are replaced by the mean values, one easily sees that each of the vectors $\overline{\mathfrak{d}}$ and $\overline{\mathfrak{H}}$ consists of two parts, where one of them only depends on $\bar{\rho}$ and the other one only depends on $\overline{\rho\mathfrak{v}}$. Now, as the actions to the outside were determined by those vectors, then they are just so, as if the charge and the current were not connected with each other at all. The same is true

for the actions *exerted* on the conductor. Namely, if $\mathfrak{d}$ and $\mathfrak{H}$ are the variations caused by external causes in the aether, then by (V_a) the force acting on a volume element is given by

$$4\pi V^2 \rho\mathfrak{d}\, d\tau + \rho[\mathfrak{p}.\mathfrak{H}]\, d\tau + \rho[\mathfrak{v}.\mathfrak{H}]\, d\tau.$$

The action, to which a noticeable part of the body is subjected, can thus be calculated in a manner, by which we put as unit volume

$$4\pi V^2 \bar{\rho}\mathfrak{d} + \bar{\rho}[\mathfrak{p}.\mathfrak{H}] + [\bar{\rho}\bar{\mathfrak{v}}.\mathfrak{H}],$$

which again decomposes into two parts $\bar{\rho}$ and $\overline{\rho\mathfrak{v}}$.

Strictly taken, also a *third* charge would have to be taken into account. The current can not exist without a potential gradient, and this cannot exist without electric charges of the parts of the conductor. These charges, however, play in the considered questions no essential role, and could even more be left out, as we can think of them as vanishingly small if we assume a very high conductivity.

4. ↑ Actually, we would have to consider now, under consideration of the Earth's motion, the effect of the induction of a galvanometer. In the experiments of Des Coudres (Wied. Ann., Vol 88, p. 71, 1889) an induction role was located between two successive connected primary roles, which have been streamed by the current, so that its effects are just compensated. Since, whatever influence the translation may have by the way, the galvanometer must remain at rest if *I* disappears, thus we may infer from the

theory that, neglecting magnitudes of second order, the compensation is not disturbed by Earth's motion.

Investigation of oscillations excited by oscillating ions.

General formulas.

§ 30. Once the motion of the ions is given, known functions of *x*, *y*, *z* and *t* appear on the right-hand side of equations (A) and (B) (§ 21); with respect to the last variable, these are periodic functions if the ions carry out oscillations with constant amplitude and a common oscillation interval *T*. It is easy to see, that in this case the equations are satisfied by values of $\mathfrak{d}_x$, $\mathfrak{d}_y$, $\mathfrak{d}_z$, $\mathfrak{H}_x$, $\mathfrak{H}_y$, $\mathfrak{H}_z$, which also have the period *T*. Therefore, the important and almost self-evident theorem is given:

If ion oscillations of period *T* take place in a light source, then $\mathfrak{d}$ and $\mathfrak{H}$ indicate the same periodicity at each point that shares the translation of the source.

The resolution of the equations leads to quite complicated expressions. For simplicity, it is advisable to calculate the components of the vector $\mathfrak{H}'$ (§ 20) at first.

According to (VI_b)

$$\mathfrak{H}'_x = \mathfrak{H}_x - 4\pi \left(\mathfrak{p}_y \mathfrak{d}_z - \mathfrak{p}_z \mathfrak{d}_y\right) .$$

Accordingly, we want to multiply the second and third of equations (A) by $4\pi\mathfrak{p}_z$ and $-4\pi\mathfrak{p}_y$ respectively, and then add them to the first of equations (B). We obtain in this way, under consideration of the importance of $\left(\frac{\partial}{\partial t}\right)_1$ (§ 19),

$$V^2\Delta\mathfrak{H}'_x - \left(\frac{\partial^2\mathfrak{H}'_x}{\partial t^2}\right)_1 = 4\pi V^2\left\{\frac{\partial(\rho\mathfrak{v}_y)}{\partial z} - \frac{\partial(\rho\mathfrak{v}_z)}{\partial y}\right\} +$$

$$+4\pi\mathfrak{p}_z\left\{\frac{\partial(\rho\mathfrak{v}_y)}{\partial t} - \mathfrak{p}_x\frac{\partial(\rho\mathfrak{v}_y)}{\partial x} - \mathfrak{p}_y\frac{\partial(\rho\mathfrak{v}_y)}{\partial y} - \mathfrak{p}_z\frac{\partial(\rho\mathfrak{v}_y)}{\partial z}\right\} -$$

$$-4\pi\mathfrak{p}_y\left\{\frac{\partial(\rho\mathfrak{v}_z)}{\partial t} - \mathfrak{p}_x\frac{\partial(\rho\mathfrak{v}_z)}{\partial x} - \mathfrak{p}_y\frac{\partial(\rho\mathfrak{v}_z)}{\partial y} - \mathfrak{p}_z\frac{\partial(\rho\mathfrak{v}_z)}{\partial z}\right\}.$$

§ 31. In the following calculation, magnitudes of order $\mathfrak{p}^2/V^2$ should be neglected. *First*, we neglect on the right-hand side the terms with *two* factors $\mathfrak{p}_x$, $\mathfrak{p}_y$ or $\mathfrak{p}_z$, since we find a similar term in V^2; and we therefore retain only

$$4\pi V^2\left\{\frac{\partial(\rho\mathfrak{v}_y)}{\partial z} - \frac{\partial(\rho\mathfrak{v}_z)}{\partial y}\right\} + 4\pi\left\{\mathfrak{p}_z\frac{\partial(\rho\mathfrak{v}_y)}{\partial t} - \mathfrak{p}_y\frac{\partial(\rho\mathfrak{v}_z)}{\partial t}\right\}.$$

Second, we write for the operation that has to be applied to $\mathfrak{H}'_x$,

$$V^2\Delta - \left(\frac{\partial}{\partial t} - \mathfrak{p}_x\frac{\partial}{\partial x} - \mathfrak{p}_y\frac{\partial}{\partial y} - \mathfrak{p}_z\frac{\partial}{\partial z}\right)^2 = \left(V^2\frac{\partial^2}{\partial x^2} + 2\mathfrak{p}_x\frac{\partial^2}{\partial x\,\partial t}\right) +$$

$$+\left(V^2\frac{\partial^2}{\partial y^2}+2\mathfrak{p}_y\frac{\partial^2}{\partial y\,\partial t}\right)+\left(V^2\frac{\partial^2}{\partial z^2}+2\mathfrak{p}_x\frac{\partial^2}{\partial z\,\partial t}\right)-\frac{\partial^2}{\partial t^2}=$$

$$=V^2\left(\frac{\partial}{\partial x}+\frac{\mathfrak{p}_x}{V^2}\frac{\partial}{\partial t}\right)^2+V^2\left(\frac{\partial}{\partial y}+\frac{\mathfrak{p}_y}{V^2}\frac{\partial}{\partial t}\right)^2+$$

$$+V^2\left(\frac{\partial}{\partial z}+\frac{\mathfrak{p}_z}{V^2}\frac{\partial}{\partial t}\right)^2-\frac{\partial^2}{\partial t^2}.$$

The form of this expression suggests the introduction of a new independent variable instead of t

$$t'=t-\frac{\mathfrak{p}_x}{V^2}x-\frac{\mathfrak{p}_y}{V^2}y-\frac{\mathfrak{p}_z}{V^2}z \tag{34}$$

and to consider $\mathfrak{H}'_x$, as well as $\rho\mathfrak{v}_y$ and $\rho\mathfrak{v}_z$, as functions of x, y, z and t'. We denote the derivative that corresponds to this view by

$$\left(\frac{\partial}{\partial x}\right)',\ \left(\frac{\partial}{\partial y}\right)',\ \left(\frac{\partial}{\partial z}\right)'\ \text{and}\ \frac{\partial}{\partial t'}$$

and give to the sign Δ' the meaning

$$\left(\frac{\partial^2}{\partial x^2}\right)'+\left(\frac{\partial^2}{\partial y^2}\right)'+\left(\frac{\partial^2}{\partial z^2}\right)'$$

It is now

$$\frac{\partial}{\partial x} = \left(\frac{\partial}{\partial x}\right)' - \frac{\mathfrak{p}_x}{V^2}\frac{\partial}{\partial t'}, \text{ etc.} \tag{35}$$

and

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'},$$

so that we find for the determination of $\mathfrak{H}'$

$$V^2\Delta'\mathfrak{H}'_x - \frac{\partial^2\mathfrak{H}'_x}{\partial t'^2} = 4\pi V^2\left[\left\{\frac{\partial(\rho\mathfrak{v}_y)}{\partial z}\right\}' - \left\{\frac{\partial(\rho\mathfrak{v}_z)}{\partial y}\right\}'\right], \text{ etc.}$$

A solution of these equations is easy to give. Namely, imagine three functions ψ_x, ψ_y, ψ_z that satisfy the conditions

$$V^2\Delta'\psi_x - \frac{\partial^2\psi_x}{\partial t'^2} = 4\pi V^2\rho\mathfrak{v}_x, \text{ etc.} \tag{36}$$

and put

$$\mathfrak{H}'_x = \left(\frac{\partial\psi_y}{\partial z}\right)' - \left(\frac{\partial\psi_z}{\partial y}\right)', \text{ etc.} \tag{37}$$

Once $\mathfrak{H}'$ is found by that, equation ($\mathrm{III_b}$) provides us with the value of $\dot{\mathfrak{d}}$ and thus also, as far as we don't use additive constants, the value of $\mathfrak{d}$. From ($\mathrm{VI_b}$) it also follows $\mathfrak{H}$; from ($\mathrm{V_b}$) and ($\mathrm{VII_b}$) it follows $\mathfrak{F}$ and $\mathfrak{E}$. That in this way really *all* the equations are satisfied, can be proven, but should not be discussed here for brevity.

In contrast, in the next section the value of ψ_x shall be given, and in § 33 the solution for a special case shall be further developed.

It should also be remarked before, that the variable t' can be regarded as a time, counting from an instant that depends on the location of the point. We can therefore call this variable the *local time* of this point, in contrast to the *general time t*. The transition from one time to another is provided by equation (34).

§ 32. The product $\rho\mathfrak{v}_x$ in the first of equations (36), as noted already, is a known function of *x*, *y*, *z* and t'. We accordingly set

$$\rho\mathfrak{v}_x = f(x, y, z, t')$$

and thus have

$$\psi_x = -\int \frac{1}{r} f\left(\xi, \eta, \zeta, t' - \frac{r}{V}\right) d\tau \tag{38}$$

a solution of (36)[1]. By that we have to imagine two points; *first*, the *fixed* point (*x*, *y*, *z*), for which we want to calculate ψ_x and which we call *P*; *second*, a *moving* point *Q*, which has to traverse the whole space, where $\rho\mathfrak{v}_x$ is different from zero. *r* represents the distance *QP*, and t' the local time of *P* at the instant for which we wish to calculate ψ_x; furthermore we have to understand by ξ, η, ζ, the coordinates of *Q*, and by *dτ* an element of the just mentioned space. The function $f\left(\xi, \eta, \zeta, t' - \frac{r}{V}\right)$ is the value of $\rho\mathfrak{v}_x$ in this element, namely, if the local time that is valid at this place, is $t' - \frac{r}{V}$.

A single luminous molecule.

§ 33. To excite electric oscillations, a single molecule with oscillating ions shall serve; let Q_0 be an arbitrary fixed point in it — for brevity, we say, "the molecule is present in Q_0" —, and for *P* a place is chosen, whose distance from Q_0 is much larger than the dimensions of the molecules. For distinction, $Q_0P = r_0$.

We now want to replace the various distances *r*, that are present in formula (38), by r_0 and also neglect the differences of local times at the various points of the molecule. In this way,

$$\psi_x = -\frac{1}{r_0}\int \rho\mathfrak{v}_x d\tau,$$

where all occurring $\rho\mathfrak{v}_x$ are related to the *same* instant, namely to the instant when

$$t' = \frac{r_0}{V}$$

is the local time of Q_0.

Since $\mathfrak{v}_x$ is equal for all points of an ion, then, if we write *e* for the charge of such a particle, the last integral transforms into

$$\Sigma e\mathfrak{v}_x.$$

The sum is extending over all ions of the molecule.

Furthermore, if $\mathfrak{q}$ is now the displacement of an ion from its equilibrium position, then

$$\mathfrak{v}_x = \frac{d\mathfrak{q}_x}{dt},$$

and

$$\Sigma e\mathfrak{v}_x = \frac{d}{dt}\Sigma e\mathfrak{q}_x.$$

This has a simple meaning. We can conveniently call the vector $\Sigma e\mathfrak{q}$ the *electric moment* of the molecule and denote it by $\mathfrak{m}$. Then it is

$$\Sigma e\mathfrak{q}_x = \mathfrak{m}_x,$$

$$\psi_x = -\frac{1}{r_0}\frac{d\mathfrak{m}_x}{dt} = -\frac{\partial}{\partial t}\left(\frac{\mathfrak{m}_x}{r_0}\right);$$

after the things said here, we have to take the value of the derivative for the instant when the local time in Q_0 is $t' = \frac{r_0}{V}$. Obviously we can also write

$$\psi_x = -\frac{\partial}{\partial t'}\left(\frac{\mathfrak{m}_x}{r_0}\right),$$

where $\mathfrak{m}_x$ means the first component of the electric moment in that very instant. After (by that and by two equations of the same from) we have found ψ_x, ψ_y, ψ_z for the point (x, y, z) and the local time t' at this place, the study of the propagating oscillations is very simple. The equations (37) give

$$\mathfrak{H}'_x = \frac{\partial}{\partial t'}\left(\frac{\partial}{\partial y}\right)'\left(\frac{\mathfrak{m}_x}{r_0}\right) - \frac{\partial}{\partial t'}\left(\frac{\partial}{\partial z}\right)'\left(\frac{\mathfrak{m}_y}{r_0}\right), \text{etc.}, \quad (39)$$

and because we seek the value of $\mathfrak{d}$ outside the molecule, ($\mathrm{III_b}$) is transformed into

$$4\pi\dot{\mathfrak{d}} = Rot\ \mathfrak{H}',$$

or, due to (35), it is transformed into

$$4\pi\dot{\mathfrak{d}}_x = \left(\frac{\partial\mathfrak{H}'_z}{\partial y}\right)' - \left(\frac{\partial\mathfrak{H}'_y}{\partial z}\right)' - \frac{\mathfrak{p}_y}{V^2}\frac{\partial\mathfrak{H}'_z}{\partial t} + \frac{\mathfrak{p}_z}{V^2}\frac{\partial\mathfrak{H}'_y}{\partial t}, \text{etc.}$$

If we bring the last two terms on the left side, then we just obtain $\frac{1}{V^2}\dot{\mathfrak{F}}_x$ or $\frac{1}{V^2}\frac{\partial\mathfrak{F}_x}{\partial t'}$, as it can be seen by ($\mathrm{V_b}$); since $\mathfrak{H}$ and $\mathfrak{H}'$ only differ by magnitudes of order $\mathfrak{p}$, we may replace the vector product ($\mathrm{V_b}$) by $[\mathfrak{p}.\mathfrak{H}']$.

From

$$\frac{\partial\mathfrak{F}_x}{\partial t'} = V^2\left[\left(\frac{\partial\mathfrak{H}'_z}{\partial y}\right)' - \left(\frac{\partial\mathfrak{H}'_y}{\partial z}\right)'\right], \text{etc.}$$

we obtain $\mathfrak{F}$ by integration; constants were omitted by us, since we are only dealing with vibrations.

We substitute the values (39) and put

$$\left(\frac{\partial}{\partial x}\right)'\left(\frac{\mathfrak{m}_x}{r_0}\right)+\left(\frac{\partial}{\partial y}\right)'\left(\frac{\mathfrak{m}_y}{r_0}\right)+\left(\frac{\partial}{\partial z}\right)'\left(\frac{\mathfrak{m}_z}{r_0}\right)=S.$$

It is then

$$\mathfrak{F}_x=V^2\left\{\left(\frac{\partial S}{\partial x}\right)'-\Delta'\left(\frac{\mathfrak{m}_x}{r_0}\right)\right\}, \text{etc.}, \tag{40}$$

and namely, $\mathfrak{m}_x$, $\mathfrak{m}_y$, $\mathfrak{m}_z$ are still related to the instant given above.

As to how the other magnitudes occurring in (I_b)-(VII_b) can be determined, can immediately be seen.

§ 34. Just some words on the error committed in the above calculation. That in (38) the factor $\frac{1}{r}$ was replaced by $\frac{1}{r_0}$, needs surely no justification. But we also haven't taken the values of $\rho\mathfrak{v}$ for the function *f* at the the correct times. Once we have replaced $t'-\frac{r}{V}$ by $t'-\frac{r_0}{V}$ in (38), then in the time when *l* is one of the dimensions of the molecule, we have committed an error of order $\frac{l}{V}$, secondly, the inequality of the local times at the various locations of the molecule were not considered, and in this lies an error of order $\frac{l\mathfrak{p}}{V^2}$ by (34). But even then, if we want to keep magnitudes of the order $\frac{\mathfrak{p}}{V}$, we don't need to care about this second error, when already the first may be neglected. Now this is indeed the case when the dimensions of the molecule are much smaller than the wavelength of TV. Then also l/V is considerably smaller than T, and the state in the molecule will not noticeably change in the time l/V.

§ 35. The formulas for the propagation of *oscillations* is obtained, if goniometric functions of time are substituted into the equations (39) and (40) for $\mathfrak{m}_x$, $\mathfrak{m}_y$,$\mathfrak{m}_z$. If, for example,

$$\mathfrak{m}_y = 0,\ \mathfrak{m}_z = 0,$$

and, as a function of local time which is valid for the location of the molecule,

$$\mathfrak{m}_x = a \cos 2\pi \frac{t'}{T}, \text{ (a constant)},$$

thus at an external point in the distance r and for the local time t' that belongs to it

$$\mathfrak{H}'_x = 0,\ \mathfrak{H}'_y = \frac{\partial}{\partial t'}\left(\frac{\partial \chi}{\partial z}\right)',\ \mathfrak{H}'_z = -\frac{\partial}{\partial t'}\left(\frac{\partial \chi}{\partial y}\right)',$$

$$\mathfrak{F}_z = -V^2\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)'\chi,\ \mathfrak{F}_y = V^2\left(\frac{\partial^2 \chi}{\partial x\, \partial y}\right)',\ \mathfrak{F} = V^2\left(\frac{\partial^2 \chi}{\partial x\, \partial z}\right)'.$$

$$\chi = \frac{a}{r} \cos \frac{2\pi}{T}\left(t' - \frac{r}{V}\right).$$

If we eventually want to consider a stationary light source once, so we simply have to omit all accents. The formulas then are in accordance with the expressions, by which Hertz[2] represented the oscillations in the vicinity of his vibrator.

The direction of the wave normal.

§ 36. Now we shall examine the oscillations in such distances from the luminous molecules, which are considerably larger than the wavelength. It should be noted that in (39) and (40), $\mathfrak{m}_x$, $\mathfrak{m}_y$, $\mathfrak{m}_z$ are goniometric functions of

$$t' - \frac{r}{V};$$

we namely want to write from now on *r* instead of r_0. The assumption made about the length of this line justifies to consider only the variability of the argument of any goniometric function for all differentiations with respect to x, y, z, but to consider as constant all factors such as $\frac{1}{r}$, or $\cos(r, x)$, by which these functions are multiplied.

For any of the magnitudes $\mathfrak{H}'_x$, $\mathfrak{H}'_y$, $\mathfrak{H}'_z$, $\mathfrak{F}_x$, $\mathfrak{F}_y$, $\mathfrak{F}_z$ - we will call them φ — it can therefore be found an expression of the form

$$\phi = A \cos \frac{2\pi}{T} \left(t' - \frac{r}{V} + B \right), \tag{41}$$

where *A* and *B* are indeed dependent on the length and the direction of line Q_0P — Q_0 is the location of the molecule, and *P* is the considered external point —, but, if *r* were just big enough, it may be regarded as constant in a space that comprises many wavelengths. While *x*, *y*, *z* are the coordinates of *P*, we denote by ξ, η, ζ the coordinates of Q_0, and by b_x, b_y, b_z the direction constants of the connection-line Q_0P. If we now replace in the formula (41) *r* by

$$b_x(x-\xi)+b_y(y-\eta)+b_z(z-\zeta),$$

and t' by the value (34), we obtain

$$\phi = A\cos\frac{2\pi}{T}\left\{t-\left(\frac{b_x}{V}+\frac{\mathfrak{p}_x}{V^2}\right)x-\left(\frac{b_y}{V}+\frac{\mathfrak{p}_y}{V^2}\right)y-\right.$$
$$\left.-\left(\frac{b_z}{V}+\frac{\mathfrak{p}_z}{V^2}\right)z+C\right\}, \tag{42}$$

$$C = B+\frac{1}{V}(b_x\xi+b_y\eta+b_z\zeta).$$

In an area that isn't too extended, we may also regard b_x, b_y, b_z as constant, and thus regard the motion as a system of plane waves. The direction constants b'_x, b'_y, b'_z of the wave normal are obviously to be determined from the condition

$$b'_x : b'_y : b'_z = \left(b_x+\frac{\mathfrak{p}_x}{V}\right) : \left(b_y+\frac{\mathfrak{p}_y}{V}\right) : \left(b_z+\frac{\mathfrak{p}_z}{V}\right) \tag{43}$$

For $\mathfrak{p}=0$, b'_x, b'_y, b'_z fall into b_x, b_y, b_z, and the waves are perpendicular to Q_0P. This is not the case if the light source is moving. From (43) follows, that the waves are perpendicular to the line that connects *P* with that point at which the light source was at the moment, when the light was sent that reaches *P* at time *t*.

The law of Doppler.

§ 37. In a point that moves together with the luminous molecule — and thus also for an observer who shares the translation — the values of $\mathfrak{d}_x, \ldots \mathfrak{H}_x, \ldots$ are changing, as we have seen (§ 30), as often in unit time as it corresponds to the actual period of oscillation T of the ions.

We can also examine, with which frequency these values in a *stationary* point are changing their sign. This frequency causes *the oscillation period for a stationary observer*. The question can be solved immediately, if instead of x, y, z we introduce new coordinates $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$, which refer to a *stationary* system of axes. If the two systems have the same directions of axes and the same origin at time $t = 0$, then

$$x = \mathbf{x} - \mathfrak{p}_x t,\ y = \mathbf{y} - \mathfrak{p}_y t,\ z = \mathbf{z} - \mathfrak{p}_z t, \tag{44}$$

and by (42) for $\mathfrak{d}_x, \ldots \mathfrak{H}_x, \ldots$ we obtain expressions of the form

$$A \cos \frac{2\pi}{T} \left\{ t + \frac{\mathfrak{p}_r}{V} t - \left(\frac{b_x}{V} + \frac{\mathfrak{p}_x}{V^2} \right) \mathbf{x} - \text{etc.} \ldots + C \right\},$$

where

$$\mathfrak{p}_r = b_x \mathfrak{p}_x + b_y \mathfrak{p}_y + b_z \mathfrak{p}_z$$

is the component of $\mathfrak{p}$ with respect to the connection line $Q_0 P$.

The "observed" period of oscillation is thus

$$T' = \frac{T}{1 + \frac{\mathfrak{p}_r}{V}} = T \left(1 - \frac{\mathfrak{p}_r}{V} \right),$$

what is in agreement with the known law of Doppler[3]. If the law, as it is usually applied, should be given, it must of course still be assumed, *that the translation does not change the actual period of oscillation of the luminous particles*. I must abstain from giving an account of this hypothesis, since we know nothing about nature of the molecular forces that determine the oscillation period.

§ 38. The case that the light source is at rest and the observer progresses, allows of a similar treatment. If namely, as above, $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ are the coordinates based on stationary axes, then in a distant point *P*, any of the magnitudes $\mathfrak{d}_x, \ldots \mathfrak{H}_x, \ldots$ shall now be represented by

$$A \cos \frac{2\pi}{T} \left\{ t - \frac{b_x \mathbf{x} + b_y \mathbf{y} + b_z \mathbf{z}}{V} + C \right\}. \tag{46}$$

We most conveniently describe the perception of motion by means of a co-ordinate system, which shares the translation $\mathfrak{p}$ of the observer. Here, again the relations (44) are applicable, and (46) transforms into

$$A \cos \frac{2\pi}{T} \left\{ t - \frac{\mathfrak{p}_r}{V} t - \frac{b_x x + b_y y + b_z z}{V} + C \right\},$$

from which it is given for the "observed" period of oscillation

$$T' = \frac{T}{1 - \frac{\mathfrak{p}_r}{V}} = T \left(1 + \frac{\mathfrak{p}_r}{V} \right).$$

1. ↑ The proof for this can be found, for example, in my treatise: La théorie électromagnétique de MAXWELL et son application aux corps mouvants.
2. ↑ HERTZ. Wied. Ann., Bd. 36, p. 1, 1889.
3. ↑ The derivation given here can easily be generalized so that it can be applied to all similar cases, for example also to sounding bodies. An arbitrary body A move with constant velocity $\mathfrak{p}$ in a medium that either remains at rest, or comes into a *stationary* state of motion. In this latter case (which also encloses the former one) we find at any point P, *which translates with the body A*, always the same state of motion, and it can be said, that the whole figure representing the distribution of velocities in the vicinity of A, shares the translation $\mathfrak{p}$.

   Furthermore, imagine now that the parts of the body perform simple oscillations of period T and of constant amplitude. It seems clear without further ado, when a sufficiently long time has elapsed since the beginning of this motion, that in the just-mentioned point P, the deviation from equilibrium or rather from the stationary state of flow, must necessarily have the period T. If we now introduce the co-ordinates x, y, z with respect to a system of axes progressing with the body (relative coordinates), and if we restrict ourselves to a space, that is so far from A and so small that we can speak of plane waves in it, then the above deviation can be represented by expressions of the form

$$\phi = A\cos\frac{2\pi}{T}\left\{t - \frac{a_x x + a_y y + a_z z}{V} + p\right\}. \quad (45)$$

Here, a_x, a_y, a_z are the direction constants of the wave normal, while *V* is the velocity of propagation.

If we now want to know, by which frequency φ (in a *stationary* point) its sign is varying, then we have to introduce coordinates $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ with respect to stationary axes. By using the relations (44), (45) transforms into

$$\phi = A \cos \frac{2\pi}{T} \left\{ t + \frac{\mathfrak{p}_n}{V} t - \frac{a_x \mathbf{x} + a_y \mathbf{y} + a_z \mathbf{z}}{V} + p \right\},$$

where

$$\mathfrak{p}_n = a_x \mathfrak{p}_x + a_y \mathfrak{p}_y + a_z \mathfrak{p}_z$$

are the components of $\mathfrak{p}$ with respect to the wave normal

For the observed oscillation period we now obtain

$$T' = \frac{T}{1 + \frac{p_n}{V}} = T \left(1 - \frac{\mathfrak{p}_n}{V} \right).$$

What we have already stated without proof, namely that the period *T* exists throughout in the medium, is nothing else than what Petzval, in his attacks against Doppler's theory, called the law of the immutability of the oscillation period (Wiener Sitz.-Ber., vol 8, p. 134, 1852). He only forgot to notice, that this law only would apply, if we consider the phenomena as a function of *t* and the *relative* coordinates.

The proof of the theorem is, by the way, easy to give, when the oscillations are infinitely small, and when we have to do with homogeneous linear differential equations.

As regards the acoustic phenomena, the problem was discussed in detail by WAS (Het beginsel van DOPPLER in de geluidsleer, Leiden, Engels, 1881).

The equations of motion of light for ponderable bodies.

Equations for the aether enclosed in ponderable bodies.

§ 39. Let us now turn to the motion of light in ponderable, dielectric, and completely transparent bodies. It shall be assumed, that they are moving with velocity $\mathfrak{p}$ in an arbitrary direction, and that, as already said, the molecules contain ions that are connected with certain equilibrium position.

For one of these particles we again denote the charge by *e*, and the displacement from the equilibrium position by $\mathfrak{q}$. The components $\mathfrak{q}_x$, $\mathfrak{q}_y$,$\mathfrak{q}_z$, as well as the velocities $\dot{\mathfrak{q}}_x$,$\dot{\mathfrak{q}}_y$,$\dot{\mathfrak{q}}_z$ we consider as infinitely small; *i.e.* besides magnitudes that only contain one of these components as factor, we neglect terms in which two such factors occur.

Any of the considered bodies shall be homogeneous. However, for that the cases of reflexion and refraction are not excluded, we imagine two different bodies, they may (Fig. 1) either sharply mutually separated at a surface Σ, or steadily go into one another at a thin limiting layer, such as between the surfaces Σ_1, and Σ_2, (Fig. 2). If we speak in the latter case about a "limiting surface", then we shall mean by that, for example, a surface Σ halfway between Σ_1 and Σ_2.

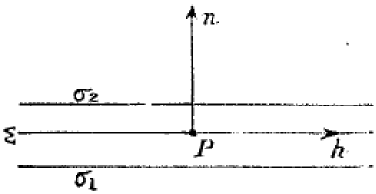

Fig. 1.

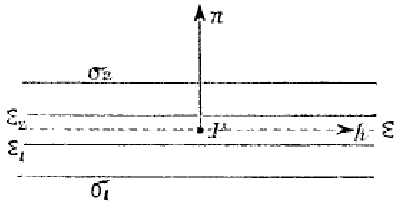

Fig. 2.

We will always calculate by averages, and namely not only by those defined in §4 *l*, but sometimes also by others, that come into consideration when the relevant magnitude only exist in a single point *Q*, for example in one point of the various molecules, or if we have reason to consider only the values of a function in such points. Such a average of *second* kind we distinguish from the averages of *first* kind by a double horizontal prime, and besides we follow a similar calculation rule as during the last calculation. Namely we understand under the value of $\overline{\overline{\phi}}$ in a point *P* the arithmetic average of the values of φ in the points Q, so far as they are present within the sphere *I* around *P* (as mentioned in §4, *l*).

By the assumption made about the radius *R* (§4), all "rapid" variations are vanished from the averages; however, concerning the velocity of the remaining variations, we have to distinguish between the interior of the body and the border. If we are positioning in Figures 1 and 2 the surfaces σ_1 and σ_2 in such a way, so that in the first figure they are both distant from Σ by *R*, while in the second this distance exists, first, between Σ_1 and σ_1, and second, between Σ_2 and σ_2, then for the calculation of $\overline{\overline{\phi}}$ or $\overline{\phi}$ in the points, that are outside of the layer (σ_1, σ_2), only the values of φ come into play. While the averages, although in a completely steady way, can be considerably different from σ_1 to σ_2, we want to assume, that the variations from point to point are much slower in the interior of the body. This will be indeed satisfied in the problems to be considered, when only the wavelength λ is many times greater than the distance a of σ_1 and σ_2

We even want to assume, that between λ and a, we can also introduce the distance l, so that λ/l and l/a become very great. The purpose of this assumption will become clear soon.

If the limiting surface Σ is curved, then the radii of curvature shall be greater than λ, or at least of the same order.

§ 40. We already have spoken about the electric moment of a molecule in § 33. We want also now to retain the definition given there, and in similar manner call the vector

$$\frac{1}{l}\Sigma e\mathfrak{q}, \tag{47}$$

where the sum is extended over all ions in the interior of sphere I, the *moment of unit volume*. More precisely we say, that (47) may indicate the value of this moment in the center of the sphere. If we choose for this new vector the sign $\mathfrak{M}$, then

$$\mathfrak{M}_x = \frac{1}{l}\Sigma e\mathfrak{q}_x, \text{ etc.} \tag{48}$$

With this $\mathfrak{M}$, another magnitude is most closely connected. During the displacement of the ions from the equilibrium positions, a fixed surface will namely be interspersed, which we may call a "convection current through the surface". If $d\sigma$ is an element of surface, with P as its center and n as its perpendicular, then the charge ϵ that passed through it into the side designated by n, will depend on the location of P, if we specify the magnitude $d\sigma$ and the direction of n once and for all. Let $d\sigma$ be very small in relation to the molecular distances, but as great, so that we don't have to consider the cases in which an ion just is in contact with the borderline. Obviously some locations of *P* will exist, where the element won't intercept any ion at all, and others where it will intersect the path $\mathfrak{q}$ of an ion. In the first case $\epsilon = 0$, in the latter it is equal to the positive and negative calculated charge of the ion.

Since ϵ depends on the location of *P*, we can form the average $\bar{\epsilon}$ in the ordinary way; it is now, as it shall be shown in the next §,

$$\mathfrak{M}_n d\sigma.$$

§ 41. The rule contained in the formula

$$\bar{\epsilon} = \frac{1}{l}\int \epsilon d\mathfrak{r}$$

can be express somewhat differently. Namely we shall choose for the point *P* an infinite amount, we want to say *k*, locations that are uniformly distributed over the sphere *I*, and take the arithmetic mean of the values of ϵ that are valid for these locations, *i.e.* we put

$$\bar{\epsilon} = \frac{1}{k}\Sigma\epsilon \tag{49}$$

Any ion, that has its equilibrium position in the interior of *I*, will (during its displacement) now pass through some positions that are connected with the element $d\sigma$ and thus add some terms to the sum $\Sigma\epsilon$. We obtain the whole sum, if we at first add to one another the terms that stem from a certain ion, and than sum over all ions.

Let *Q* be the equilibrium position of the considered ions, and *Q* the new position; so $QQ' = \mathfrak{q}$. The length and the direction of this line are given, as well as the direction and magnitude of $d\sigma$. Whether the particle hits the surface element and provides the part *e* for the sought sum, only depends on the relative positions of *P* and *Q*. Thus we can, instead of giving *k* positions in the sphere *l* to *P*, also let remain the point at its position and lead point *Q* around a sphere *I*. As QQ' now hits the fixed element $d\sigma$, when Q lies in a certain, easily specifiable cylinder of area $\mathfrak{q}_n d\sigma$, then the number of "relevant" positions is related to the integer k, as the area of that cylinder is related to the area of sphere I. This number is thus

$$\frac{k}{I}\mathfrak{q}_n d\sigma,$$

and the sum $\Sigma\epsilon$, as far it is caused by the ion Q,

$$\frac{k}{I}e\mathfrak{q}_n d\sigma.$$

Eventually in formula (49) we obtain

$$\Sigma\epsilon = \frac{k}{I}\Sigma e\mathfrak{q}_n \cdot d\sigma,$$

where the sum is extended over all ions of sphere I, and

$$\bar{\epsilon} = \frac{k}{I}\Sigma eq_n \cdot d\sigma,$$

or by (48)

$$\bar{\epsilon} = \mathfrak{M}_n d\sigma.$$

§ 42. Equations (I_b) — (VII_b) (§ 20) may form the initial point for the subsequent considerations. At first we notice, that the first of them is equivalent to

$$\int \mathfrak{d}_n d\sigma = E,$$

for an arbitrary closed surface (n is to be drawn into the outside), when E is the electric charge enclosed by it. If now in one element $d\mathfrak{r}$ of the inner space in equilibrium state, a density ρ_0 exists, and if (for an element of the surface) ϵ has the meaning given above, then

$$E = \int \rho_0 d\mathfrak{r} - \Sigma\epsilon,$$

where the sum is related to all elements $d\sigma$.

By that

$$\int \mathfrak{d}_n d\sigma + \Sigma\epsilon = \int \rho_0 d\mathfrak{r}.$$

From the definition of the average we easily find now

$$\int \overline{\mathfrak{d}_n} d\sigma + \Sigma\overline{\epsilon} = \int \overline{\rho_0} d\mathfrak{r}.$$

Since now

$$\overline{\rho_0} = 0,$$

and

$$\overline{\epsilon} = \mathfrak{M}_n d\sigma.$$

it is eventually given

$$\int \left(\overline{\mathfrak{d}_n} + \mathfrak{M}_n\right) d\sigma = 0.$$

We now want to define a new vector $\mathfrak{D}$ by the equation

$$\mathfrak{D} = \overline{\mathfrak{d}} + \mathfrak{M},$$

and call it the *dielectric polarization*.

This vector, that goes over for the free aether (where $\mathfrak{M} = 0$) into $\mathfrak{d}$, is exactly that, what Maxwell calls "dielectric displacement". Its basic property is according to the above, that for any closed surface

$$\int \mathfrak{D}_n \mathfrak{d}\sigma = 0 \tag{50}$$

and also in the interior of any body

$$Div\,\mathfrak{D} = 0. \tag{I_c}$$

§ 43. Formula (50) leads to an important limiting-condition, if we apply it to a surface, that lies partly in the first, and partly in the second body. Around a certain point *P* of the limiting-surface Σ(Fig. 1 and 2) we shall lay a cylinder-surface *C* that is parallel to the perpendicular in *P*, and choose for the mentioned area the surface of the space that is cut from layer (σ_1, σ_2). If now the dimensions of the parts limited in σ_1 and σ_2 are of order *l* (§ 39), then we may consider the parts as elements that are equal, parallel and plane, and as they are much greater than the part of *C* that lies between σ_1 and σ_2, we can omit the integral taken over the latter

$$\int \mathfrak{D}_n d\sigma$$

Thus we find, if we mutually distinguish the values that are valid in σ_1 and σ_2 by the indices 1 and 2, and draw either at σ_1 as well as at σ_2 the perpendicular *n* from the first to the second body,

$$\mathfrak{D}_{n(1)} = \mathfrak{D}_{n(2)} \tag{51}$$

In relation to this, we have to notice one thing. In any medium, $\mathfrak{D}_x, \mathfrak{D}_y, \mathfrak{D}_z$ can be represented as slowly (§ 39) varying functions of coordinates, and we would have to substitute in these functions the coordinates of a point of σ_1 or σ_2, to obtain $\mathfrak{D}_{n(1)}$ and $\mathfrak{D}_{n(2)}$. Instead of this we can without noticeable error — due to the small distance of the surfaces — introduce the coordinates of the point *P* that lies in Σ. Thus it is allowed to say, that $\mathfrak{D}_{n(1)}$ and $\mathfrak{D}_{n(2)}$ are the values *at the limiting-surface* and that the previous formula *expresses the continuity of* $\mathfrak{D}_n$.

Similar formulas as equations (I_c) and (51) are emerging from (II_b); namely for the interior of a body

$$Div\,\overline{\mathfrak{H}} = 0,$$

and for the limiting-surface

$$\overline{\mathfrak{H}}_{n(1)} = \overline{\mathfrak{H}}_{n(2)}.$$

§ 44. From fundamental equation (III_b) we derive

$$Rot\,\overline{\mathfrak{H}'} = 4\pi\overline{\rho\mathfrak{v}} + 4\pi\dot{\overline{\mathfrak{d}}},$$

or, be means of the definition

$$\overline{\rho\mathfrak{v}} = \frac{1}{l}\Sigma e\mathfrak{v} = \frac{1}{l}\Sigma e\dot{\mathfrak{q}} = \dot{\mathfrak{M}},$$

$$Rot\,\overline{\mathfrak{H}'} = 4\pi\dot{\mathfrak{D}}.$$

This derivation is true for the interior of a body. To arrive at the limiting condition, we note at first, that (§ 4, *h*) (by the equation (III_b)

for an arbitrary surface σ, with the borderline *s')*

$$\int \mathfrak{H}'_s ds = 4\pi \int \left(\rho\mathfrak{v}_n + \dot{\mathfrak{d}}_n\right) d\sigma$$

and thus also

$$\int \mathfrak{H}'_s ds = 4\pi \int \left(\overline{\rho\mathfrak{v}_n} + \overline{\dot{\mathfrak{d}}_n}\right) d\sigma = 4\pi \int \dot{\mathfrak{D}}_n d\sigma \qquad (52)$$

Now we lay through the point *p* (Fig. 1 and 2) a plane, that contains the perpendicular of the borderline and the arbitrary direction *h* tangential to Σ, and choose as surface σ the part of this plane, that lies between σ_1 and σ_2 and which is limited by two lines parallel to that perpendicular. If the length of this layer in the direction *h* is of order *l* (§ 39), then we may neglect all magnitudes of order *a* and we obtain from (52)

$$\overline{\mathfrak{H}'}_{h(1)} = \overline{\mathfrak{H}'}_{h(2)}.$$

where the indices 1 and 2 have the same meaning as above. For the two components of $\overline{\mathfrak{H}'}$ we may take at this place the values in point *P* again, and thus the equations says, that the *tangential components of vector* $\overline{\mathfrak{H}'}$ *were steady*.

§ 45. Equation (IV_b) admits of a similar application. Before, I give the remark that no magnetic forces exists, as long the ions are at rest, thus that $\mathfrak{H}$ is of same order as the velocities $\mathfrak{v}$. In (VII_b) we can consequently neglect the last term; it becomes $\mathfrak{F} = \mathfrak{E}$, consequently by ($IV_b$) for the interior of a body

$$Rot\ \overline{\mathfrak{E}} = -\dot{\overline{\mathfrak{H}}},$$

and for the borderline

$$\overline{\mathfrak{E}}_{h(1)} = \overline{\mathfrak{E}}_{h(2)}.$$

At last it still follows from (V_b) and (VI_b)

$$\overline{\mathfrak{E}} = 4\pi V^2 \overline{\mathfrak{d}} + [\mathfrak{p}.\overline{\mathfrak{H}}], \tag{53}$$

and

$$\overline{\mathfrak{H}}' = \overline{\mathfrak{H}} - 4\pi[\mathfrak{p}.\overline{\mathfrak{d}}] \tag{54}$$

The equations of motion for ions.

§ 46. So far everything was quite simple. Yet great difficulties arise, when we also want to form the equations of motion for the oscillating ions themselves. To express in these equations the relations, which are the basis for dispersion, birefringence, and circular polarization, it would be required an understanding of molecular processes that wasn't achieved by us by far. We want to restrict ourselves, to derive from a very simple presupposition the most probable shape of the sought relations, and then help on ourselves as good as possible. It is of course an advantage, that for this task we have to consider the interior of the homogeneous body, since (regarding the borderlines) the already derived equations enclose all required conditions.

The mentioned assumption is now, that any of the mutual completely equal molecules, only contains a single movable ion, while all others are fixed.

Let m be the mass of a movable ion, $\mathfrak{K}$ the total force acting on it, N the number of molecules in unit volume. For the equations

$$m\frac{d^2\mathfrak{q}_x}{dt^2} = \mathfrak{K}_x, \text{ etc.}$$

it follows, when we take the averages of second kind and multiply them by *eN*

$$m\frac{\partial^2\mathfrak{M}_x}{\partial t^2} = eN\overline{\overline{\mathfrak{K}_x}}, \text{ etc.}$$

As regards $\mathfrak{K}$, it is at first to note, that by our assumption the fixed parts of the molecule are acting upon the ion by a certain force, that is exactly caused by the displacement $\mathfrak{q}$. Let the components of this force be linear, homogeneous functions of $\mathfrak{q}_x, \mathfrak{q}_y, \mathfrak{q}_z$, or rather, since only this is relevant for the following, let the averages of those components be given by

$$\left.\begin{aligned} &-\left(s_{1.1}\overline{\overline{\mathfrak{q}_x}} + s_{1.2}\overline{\overline{\mathfrak{q}_y}} + s_{1.3}\overline{\overline{\mathfrak{q}_z}}\right), \\ &-\left(s_{2.1}\overline{\overline{\mathfrak{q}_x}} + s_{2.2}\overline{\overline{\mathfrak{q}_y}} + s_{2.3}\overline{\overline{\mathfrak{q}_z}}\right), \\ &-\left(s_{3.1}\overline{\overline{\mathfrak{q}_x}} + s_{3.2}\overline{\overline{\mathfrak{q}_y}} + s_{3.3}\overline{\overline{\mathfrak{q}_z}}\right), \end{aligned}\right\} \tag{55}$$

in which certain constants are denoted by *s*.

We also assume for these forces, that they won't be changed by the translation $\mathfrak{p}$, at least not as regards magnitudes of first order.

§ 47. In consequence of the electric motions, also the aether exerts an action upon the ions. This can be derived from formula (V_b), since $\mathfrak{E} = \mathfrak{F}$ as we have seen. If it would be allowed, to put for the electric force $\mathfrak{E}$ everywhere the average $\overline{\mathfrak{E}}$, that has the same magnitude and direction at all points of an ion, then we would have to add into the expressions (55) only the terms

$$e\overline{\mathfrak{E}}_x,\ e\overline{\mathfrak{E}}_y,\ e\overline{\mathfrak{E}}_z \tag{56}$$

But this matter isn't all that simple. First, the oscillating ion itself causes a value of $\mathfrak{E}$, that is not the same in all points of the particle, so that we could find the corresponding part of $\mathfrak{K}$ only by an integration over the space in which the ion is located. Second, even if we could neglect this, for the calculation of $\overline{\overline{\mathfrak{K}}}$ the average $\overline{\overline{\mathfrak{E}}}$ is of relevance, not the average $\overline{\mathfrak{E}}$, and it is not allowed, to mutually interchange both. Of course nothing would be in the way, in so far the motions of ions that cause the electric force $\mathfrak{E}$, take place in the distance P from the considered point that is much greater as the distance of the molecules, but $\mathfrak{E}$ is partly caused by molecules that are located more nearly — we want to say, by the oscillations within the sphere I drawn around P — and an inequality $\overline{\mathfrak{E}}$ and $\overline{\overline{\mathfrak{E}}}$ is very well possible for a irregular distribution of the thus produced states in the aether.

When we now, in agreement with these remarks and to obtain $\overline{\overline{\mathfrak{K}}}$, add to the expressions (55) not only the values (56) but also certain supplementary terms

$$\mathfrak{k}_x,\ \mathfrak{k}_y,\ \mathfrak{k}_z$$

and thus put

$$m\frac{\partial^2\mathfrak{M}_x}{\partial t^2} = -eN\left(s_{1.1}\overline{\overline{\mathfrak{q}_x}} + s_{1.2}\overline{\overline{\mathfrak{q}_y}} + s_{1.3}\overline{\overline{\mathfrak{q}_z}}\right) + e^2N\overline{\mathfrak{E}_x} + eN\mathfrak{k}_x \tag{57}$$

,etc.

then we can maintain for the magnitudes $\mathfrak{k}$, that they only depend on processes within sphere I. Additionally it is given, that also the supplementary terms only exist during the displacement of the ions from their equilibrium positions and — since $\mathfrak{q}$ can be considered as infinitely small — they must be linear, homogeneous functions of the magnitudes $\mathfrak{q}, \dot{\mathfrak{q}}$, etc., or rather of their averages. In consequence of equations (48), also $\mathfrak{k}$ are homogeneous, linear functions of the values of $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z, \dot{\mathfrak{M}}_x$, etc. in the various points of the spherical space *I*. Eventually we still have to consider, that all these values can be expressed by application of TAYLOR's theorem by the values, which will be assumed by $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z, \dot{\mathfrak{M}}_x$, etc., and the derivatives with respect to *x*, *y*, *z* in the considered point *P*, the center of the sphere. All these values thus are linearly included into the expressions for $\mathfrak{k}_x, \mathfrak{k}_y, \mathfrak{k}_z$.

To which extend these latter ones must contain the translation velocity $\mathfrak{p}$, remains undecided for now. In any case, since we neglect magnitudes of second order, only the first powers of $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$ will occur. If we also consider now, that in formulas (57) the magnitudes $eN\overline{\mathfrak{q}_x}$, etc., could be replaced by $\mathfrak{M}_x$, etc., and if we think of these equations as solved with respect to $\overline{\mathfrak{E}_x}$, etc., then we can see, that these components of the electric force can be represented as linear, homogeneous functions of $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z$ and their derivatives with respect *x, y, z, t,* and that the coefficients in these functions can linearly contain the velocities $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$.

For brevity, the equations that would result in a completely developed theory for $\overline{\mathfrak{E}_x}, \overline{\mathfrak{E}_y}, \overline{\mathfrak{E}_z}$, may be summarized in the formula

$$\overline{\mathfrak{E}} = F(\mathfrak{M}, \dot{\mathfrak{M}}, \ddot{\mathfrak{M}}, \ldots, \mathfrak{p}) \tag{58}$$

As regards any of the vectors $\mathfrak{M}, \dot{\mathfrak{M}}, \ddot{\mathfrak{M}}$, ..., we also have to consider the derivatives of its components with respect to the coordinates.

If we now eventually let fall our simplifying presupposition, and consider any molecule as a formation of, maybe, very complicated structure that contains several movable ions, then it is near at hand to assume, that still a relation like the one represented in (58) does exist. Our next task shall be, to simplify as much as possible the relation by means of certain, general considerations.

Simplification for transparent bodies.

§ 48. If a certain motion exists in a system, then, as it was shown in § 18, also the inverse motion is possible, as soon as forces of non-electric origin are the same for a certain location of the ions as well, as in the original case. From this it directly follows, that all motions in a body, that besides ions also contain uncharged mass particles, can be reversed, in case all molecular forces are determined by the configurations and not, for example, depending on the velocities.

During the inversion of motions all velocities obtain an opposite direction, thus also the translation $\mathfrak{p}$. Furthermore we can easily see, — look at formulas of §§ 43 and 44 —, that in the new state at time *t*, the vectors

$$\mathfrak{M},\ \overline{\mathfrak{H}} \text{ und } \overline{\mathfrak{E}}$$

have the same direction and magnitude, as the vectors

$$\mathfrak{M},\ -\overline{\mathfrak{H}} \text{ und } \overline{\mathfrak{E}}$$

in the original state at time -*t*.

Obviously, the *transparent* bodies, namely only those,[1] in which the light motions are reversible in the alluded sense, and it may be clearly emphasized, that the circular polarizing substances form no exception from this rule.[2]

We now want to see, which simplification of equation (58) is obtained from this reversibility; there, terms without and with $\mathfrak{p}$ shall be considered separately.

§ 49. If $\mathfrak{p} = 0$, then it must be possible to express $\overline{\mathfrak{E}}_x, \overline{\mathfrak{E}}_y, \overline{\mathfrak{E}}_z$ as homogeneous, linear functions of the magnitudes $\mathfrak{M}_x, \dot{\mathfrak{M}}_x, \ddot{\mathfrak{M}}_x$, etc., and their derivatives with respect to the coordinates; the relations that serve for this, must stay unchanged, when we pass to the inverse motion. As to this motion we now have (at time *t*) $\overline{\mathfrak{E}}_x, \overline{\mathfrak{E}}_y, \overline{\mathfrak{E}}_z$, and also the components $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z$, as well as their derivatives with respect to the coordinates, have the same value and the same sign as with the original motion (at time -*t*). The same is true for all *even* derivatives with respect to time. The *uneven* derivatives with respect to *t* have, however, the same magnitude as regards the two motions, but opposite signs, and thus these derivatives cannot occur in the relations between $\overline{\mathfrak{E}}$ and $\mathfrak{M}$. To indicate this, we replace (58) for resting bodies by

$$\overline{\mathfrak{E}} = F_1(\mathfrak{M}, \ddot{\mathfrak{M}}, \ldots) \tag{59}$$

If we again allow the translation, then we have to add to F_1 still another vector, whose components are linear and homogeneous functions of $\mathfrak{M}, \dot{\mathfrak{M}}, \ddot{\mathfrak{M}}$, ..., and which contain in any term one of the factors $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$; also this new vector must stay unchanged when passing to the inverse motion. As in this case the components $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$ contain opposite signs, thus they can only be multiplied by such magnitudes which also change the sign, *i.e.* by uneven derivatives with respect to time. The equations (58) therefore generally assume the form

$$\overline{\mathfrak{E}} = F_1(\mathfrak{M}, \ddot{\mathfrak{M}}, \ldots) + F_2(\mathfrak{M}, \dot{\mathfrak{M}}, \ldots \mathfrak{p}) \tag{60}$$

An additional simplifications we can achieved, by considering a certain kind of homogeneous light, *i.e.* by considering goniometric functions of time of a certain period *T*. Then

$$\ddot{\mathfrak{M}} = -\left(\frac{2\pi}{T}\right)^2 \mathfrak{M},\ \dddot{\mathfrak{M}} = -\left(\frac{2\pi}{T}\right)^2 \dot{\mathfrak{M}}, \text{ etc.} \tag{61}$$

If we in (60) express in this way all even derivatives by $\mathfrak{M}$ and all uneven by $\dot{\mathfrak{M}}$, it will be given

$$\overline{\mathfrak{E}} = \boldsymbol{F}_1(\mathfrak{M}) + \boldsymbol{F}_2(\dot{\mathfrak{M}}, \mathfrak{p}) \tag{62}$$

The components of $\boldsymbol{F}_1$ are now homogeneous functions of $\mathfrak{M}_x, \mathfrak{M}_y, \mathfrak{M}_z$ and its derivatives with respect to *x, y, z*, while $\boldsymbol{F}_2$ depends in a similar way on $\dot{\mathfrak{M}}$. The coefficients of this function may well depend on the oscillation period *T*, since we have introduced the values (61) into (60).

The dispersion of light.

§ 50. There are two ways of attempting to explain the dispersion of colors, either by (like Cauchy) considering from location to location the variation of the equilibrium disturbance, or by considering as relevant the variation with respect to time. In one case it is the wave length, in the other one the oscillation period, that *directly* determines the propagation velocity, although at the end both have the same result.

If we would take the first path and also reproduce the explanation given by Cauchy — in its mathematical form — in our theory, then we would have to assume, that the equations summarized in (59) likely contain derivatives with respect to *x, y, z*, but not such with respect to *t*, and that namely, due to the smallness of *m*, the first term in (57) would vanishes. It is clear, that the propagation velocity must change with wave length, as soon as, for example, $\mathfrak{M}_x$, and $\frac{\partial^2 \mathfrak{M}_x}{\partial y^2}$ are standing next to one another. Namely, the latter magnitude gains with respect to the first a greater influence, the smaller the wavelength.

The straight opposite assumption would be, that only derivatives with respect to *t*, but none with respect to *x, y, z* occur in formula (59). Now, in so far, that the only magnitude of the first kind (whose introduction has proven to be necessary) is the term

$$m \frac{\partial^2 \mathfrak{M}_x}{\partial t^2}$$

in equation (57), we can say that the second mentioned view reduces the phenomenon to the mass of the co-oscillating ion.

That this explanation can really be *achieved* now, was already proven by v. HELMHOLTZ and earlier also by me. The new form that I now give to the theory, makes no difference in this respect.

As we know, mainly the phenomena of anomalous dispersion speak in favor of the assumption of co-oscillating masses. On the other hand, as regards the derivative with respect to *x, y, z*, it is the question, whether the terms in which they occur, are really great enough to exert a considerable influence. As we saw, the mentioned terms can only stem from the fact, that the electric moment $\mathfrak{M}$ doesn't have in all points of sphere *I* the same magnitude and direction. Since the radius is much smaller than the wave length, thus the differences are surely very insignificant, and we won't hesitate to neglect them, if it is about an action upon a distant point. Anyway, is would be premature to claim that also this small variation of $\mathfrak{M}$ couldn't have an influence on the phenomena in the interior of the sphere. The rotation of the polarization plane, to which we will return too, which presumable can't be understood without the aid of derivatives with respect to *x, y, z*, must prevent us from denying from the outset an influence of such terms on dispersion.

With more justification we can derive *from the phenomena* the insignificance of that influence. Namely, if we retain in equations (59) the derivatives with respect to *x*, *y*, *z*, and then simplify the equations, so far it is possible due to the known symmetry relations of crystals, then we are lead to laws for the motion of light, which are more complicated than the ones actually applied, and only go over into them by further simplification of the formulas, for which we cannot give any reason. For example, according to these laws the regular crystals wouldn't be isotropic, but must show a peculiar kind of birefringence. [3]

The things said may justify, that we, while preliminarily the circular-polarizing media remain excluded, assume for the other transparent bodies that the relation (62) contains no derivative with respect to *x*, *y*, *z*. We thus put

$$\left\{\begin{aligned} \overline{\mathfrak{E}}_x &= \sigma_{1.1}\mathfrak{M}_x + \sigma_{1.2}\mathfrak{M}_y + \sigma_{1.3}\mathfrak{M}_z + (\dot{\mathfrak{M}}, \mathfrak{p})_x, \\ \overline{\mathfrak{E}}_y &= \sigma_{2.1}\mathfrak{M}_x + \sigma_{2.2}\mathfrak{M}_y + \sigma_{2.3}\mathfrak{M}_z + (\dot{\mathfrak{M}}, \mathfrak{p})_y, \\ \overline{\mathfrak{E}}_z &= \sigma_{3.1}\mathfrak{M}_x + \sigma_{3.2}\mathfrak{M}_y + \sigma_{3.3}\mathfrak{M}_z + (\dot{\mathfrak{M}}, \mathfrak{p})_z, \end{aligned}\right. \tag{63}$$

and here we understand by $(\dot{\mathfrak{M}}, \mathfrak{p})_x$, $(\dot{\mathfrak{M}}, \mathfrak{p})_y$, $(\dot{\mathfrak{M}}, \mathfrak{p})_z$ expressions, which are linear and homogeneous with respect to $\dot{\mathfrak{M}}_x$, $\dot{\mathfrak{M}}_y$, $\dot{\mathfrak{M}}_z$ as well as to $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$. The coefficients in these expressions, as well as the factors σ are to be viewed as functions of *T*.

Now I will prove, that for a very general class of bodies, the terms $(\dot{\mathfrak{M}}, \mathfrak{p})_x$, etc. will vanish; at the same time we reach on that occasion also a simplification of the terms independent from $\mathfrak{p}$.

Bodies with three mutually perpendicular planes of symmetry.

§ 51. Let *A* be an arbitrary body, and A' a second body that is the mirror image of the first one with respect to a certain plane *E*, and namely down to the smallest parts, thus also for the distribution of the smallest particles. If the molecular forces depend in such way from the configurations, that the vectors, by which they were represented in *A* and A', behave like objects and their mirror images, then ion motions can occur in the two bodies in connection with state changes of aether (§ 18), so that also regarding these phenomena, one system is forever the mirror image of the other one. When passing from the first system to the second, the vectors $\overline{\mathfrak{E}}, \mathfrak{M}$ and $\mathfrak{p}$ are transformed into their mirror images.

The inner construction of body *A* can only be such, by appropriate choice of the plane *E*, so that *A* and A' with respect to the *same* coordinate system have the same properties, *i.e.* that the properties in *A* and A' can be expressed by the *same* equations, without change of a constant or sign. In this case we call *E* a *plane of symmetry*. The bodies which we now consider, and to which we will restrict ourselves preliminarily, are those, for which three mutually perpendicular symmetry planes of this kind exist.

We give the coordinate planes the direction with of the symmetry-planes, and consider at first the mirror image with respect to the *yz*-plane. When passing to this image, $\overline{\mathfrak{E}}_x, \mathfrak{M}_x$ and $\mathfrak{p}_x$ change their sign, while the other components of $\overline{\mathfrak{E}}, \mathfrak{M}$ and $\mathfrak{p}$ remain completely unchanged. This is only possible, when (after $(\dot{\mathfrak{M}}, \mathfrak{p})_x$, etc. are represented as functions of $\dot{\mathfrak{M}}_x$, $\dot{\mathfrak{M}}_y$, $\dot{\mathfrak{M}}_z$, $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$) the index *x* appears in every term of the first formula once, or in every term of the second and third either not at all, or two times. To a similar conclusion we come also with respect to indices *y* and *z*. If we additionally consider the mirror images with respect to the *zx*- and the *xy*-plane, then we find, that not a single term as $(\dot{\mathfrak{M}}, \mathfrak{p})_x$ is applicable, and that from the nine coefficients σ, only $\sigma_{1.1}$, $\sigma_{2.2}$ and $\sigma_{3.3}$ can be different from zero.

Thus we obtain

$$\overline{\mathfrak{E}}_x = \sigma_{1.1}\mathfrak{M}_x,\ \overline{\mathfrak{E}}_y = \sigma_{2.2}\mathfrak{M}_y,\ \overline{\mathfrak{E}}_z = \sigma_{3.3}\mathfrak{M}_z, \tag{64}$$

or

$$\frac{4\pi V^2}{\sigma_{1.1}}\overline{\mathfrak{E}}_x = 4\pi V^2\mathfrak{M}_x, \text{ u.s.w.}$$

If we add these formulas to the three summarized in (53), and put

$$1 + \frac{4\pi V^2}{\sigma_{1.1}} = \varkappa_1,\ 1 + \frac{4\pi V^2}{\sigma_{2.2}} = \varkappa_2,\ 1 + \frac{4\pi V^2}{\sigma_{3.3}} = \varkappa_3,$$

then

$$\varkappa_1\overline{\mathfrak{E}}_x = 4\pi V^2\mathfrak{D}_x + [\mathfrak{p}.\mathfrak{H}]_x$$

where, for a certain type of light, $\varkappa_1$, $\varkappa_1$ and $\varkappa_3$ are constants.

Summary of the equations.

§ 52. Neglecting the primes over the letters — since was continue to only speak about averages — we summarize the equations of motion now in the following way.

In the interior of any body it is given

$$Div\, \mathfrak{D} = 0, \tag{I_c}$$

$$Div\, \mathfrak{H} = 0, \tag{II_c}$$

$$Div\, \mathfrak{H}' = 4\pi\dot{\mathfrak{D}}, \tag{III_c}$$

$$Div\, \mathfrak{E} = -\dot{\mathfrak{H}}, \tag{IV_c}$$

$$\left.\begin{array}{c} \varkappa_1 \mathfrak{E}_x = 4\pi V^2 \mathfrak{D}_x + [\mathfrak{p}.\mathfrak{H}]_x,\ \varkappa_2 \mathfrak{E}_y = 4\pi V^2 \mathfrak{D}_y + [\mathfrak{p}.\mathfrak{H}]_y \\ \varkappa_3 \mathfrak{E}_z = 4\pi V^2 \mathfrak{D}_z + [\mathfrak{p}.\mathfrak{H}]_z \end{array}\right\} \tag{V_c}$$

and

$$\mathfrak{H}' = \mathfrak{H} - \frac{1}{V^2}[\mathfrak{p}.\mathfrak{E}], \tag{VI_c}$$

since, neglecting magnitudes of second order, we may replace, by the relation (53), $4\pi\bar{\mathfrak{d}}$ by $\mathfrak{E}/V^2$ in equation (54).

At the borderline the conditions apply

$$\mathfrak{D}_{n(1)} = \mathfrak{D}_{n(2)},\ \mathfrak{H}_{n(1)} = \mathfrak{H}_{n(2)},\ \mathfrak{E}_{h(1)} = \mathfrak{E}_{h(2)},\ \mathfrak{H}'_{h(1)} = \mathfrak{H}'_{h(2)} \tag{$VIII_c$}$$

,

If there is no translation, then $\mathfrak{H}'$ falls into $\mathfrak{H}$; then the equations (III_c) and (V_c) go over into

$$Rot\, \mathfrak{H} = 4\pi\dot{\mathfrak{D}}, \tag{III'_c}$$

$$\varkappa_1 \mathfrak{E}_x = 4\pi V^2 \mathfrak{D}_x, \text{ etc.} \tag{V'_c}$$

and the last of the limiting conditions ($VIII_c$) into

$$\mathfrak{H}_{h(1)} = \mathfrak{H}_{h(2)}.$$

Thus for this case, the known equations of motion and limiting conditions of the electromagnetic theory of light are given. From formulas $(I_c), (II_c), (III_c'), (IV_c)$ and (V_c') we derive (when $\varkappa_1, \varkappa_2, \varkappa_3$ are different from each other) the laws of light motion in crystals of two axes, while the assumption $\varkappa_1 = \varkappa_2 = \varkappa_3$ leads back to isotropic bodies. Besides, since $\varkappa_1, \varkappa_2$ and $\varkappa_3$ depend on the oscillation period, also the explanation of the dispersion of light is contained in the formulas.

Also the case of the pure aether is not excluded. Since no electric moments $\mathfrak{M}$ exist in it, then we have to set by (64) $\sigma_{1.1} = \sigma_{2.2} = \sigma_{3.3} = \infty$ and thus $\varkappa_1 = \varkappa_2 = \varkappa_3 = 1$. The equations V_c) and (V_c') thereby are transformed into

$$\mathfrak{E} = 4\pi V^2 \mathfrak{D} + [\mathfrak{p} . \mathfrak{H}],$$

$$\mathfrak{E} = 4\pi V^2 \mathfrak{D} .$$

We can easily see, that the equations which we obtain in this way for the aether, are in agreement with formulas (I)-(V) or (I_b)-(VII_b)

It is self-evident, as regards the interior of the pure aether, that the connection between the various magnitudes is always the same, the ponderable matter may be in motion or not.

Circular polarizing media.

§ 53. Bodies, which turn the polarization plane, were excluded above. It's not feasible to form a thorough theory for them until now; nevertheless some general consideration, as required by our purpose, may find their place here.

Since the rotation of the polarization plane is connected with the fact, that the medium is not in accordance in all its properties with its mirror image, then the things said in § 51 are not applicable anymore. Nevertheless, everything becomes quite easy when we restrict ourselves to *isotropic* media.

If we assume, in the relation between $\overline{\mathfrak{E}}$ and $\mathfrak{M}$, that no derivatives with respect to *x, y, z* are present, then we have to understand under $F_1(\mathfrak{M})$ of equation (62), a vector that is completely determined even by $\mathfrak{M}$, namely the isotropy requires that the figure consisting of $\mathfrak{M}$ and $F_1(\mathfrak{M})$ can be rotated in an arbitrary manner, without that $F_1(\mathfrak{M})$ ceases to fit to $\mathfrak{M}$. If we now choose the direction of $\mathfrak{M}$ itself as the rotation axis, then $\mathfrak{M}$ always remains the same vector; thus $F_1(\mathfrak{M})$ must remain unchanged, which is only possible when this vector has the direction of $\mathfrak{M}$. With respect to the linear character of the sought relation, we consequently have to set

$$F_1(\mathfrak{M}) = \sigma\mathfrak{M}, \tag{65}$$

where σ is a scalar constant.

The second vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$ occurring in (62), has the following properties. First, its components are homogeneous, linear functions of $\dot{\mathfrak{M}}_x$, $\dot{\mathfrak{M}}_y$, $\dot{\mathfrak{M}}_z$ as well as from $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$. Second, after an arbitrary rotation of the figure consisting of the three vectors $\dot{\mathfrak{M}}, \mathfrak{p}$ and $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$, $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$ must still fit to $\dot{\mathfrak{M}}$ and $\mathfrak{p}$. By that we derive[4]

$$F_2(\dot{\mathfrak{M}}, \mathfrak{p}) = k[\dot{\mathfrak{M}}. \mathfrak{p}], \tag{66}$$

where *k* is a positive or negative constant, which by the way, as σ above, can also depend on the oscillation period *T*.

§ 54. The presupposition, that no derivatives with respect to *x*, *y*, *z* occur, has led us to equation (65), from which the rotation of the polarization plane does *not* arise. Thus it is necessary, as it was already indicated earlier to assume (at least in the expression $F_1(\mathfrak{M})$) derivatives with respect to the coordinates. The most simple is, to add to the second term of (65) another vector $\mathfrak{R}$, whose components do linearly and homogeneously depend on the *first* derivatives of $\mathfrak{M}_x$, $\mathfrak{M}_y$, $\mathfrak{M}_z$. Magnitude and direction will now again be closely determined by isotropy. Namely, if we imagine at any point of space a line, that represents the vector $\mathfrak{M}$, and in addition in the considered point the vector $\mathfrak{R}$, then after an arbitrary rotation of that entire figure, $\mathfrak{R}$ must still fit to $\mathfrak{M}$. Only the assumption[5]

$$\mathfrak{R} = j\ Rot\ \mathfrak{M},$$

is in agreement with this, where *j* is a certain constant and which we want to add for resting bodies (65) to

$$F_1(\mathfrak{M}) = \sigma\mathfrak{M} + j\ Rot\ \mathfrak{M}$$

Now, we could introduce (into the term $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$) derivatives with respect to *x*, *y*, *z*; however, we will omit this, since the things already said are sufficient for our purpose. By that we have (when we omit the prime over $\mathfrak{E}$ from now on) to put for isotropic, circular-polarizing media

$$\mathfrak{E} = \sigma\mathfrak{M} + j\ Rot\ \mathfrak{M} + k[\dot{\mathfrak{M}}.\mathfrak{p}] \tag{68}$$

§ 55. It is not without interest, to consider for a moment the mirror image of a motion to which the found equation applies. The vectors that apply to this new motion, which may be called $\mathfrak{E}'$, $\mathfrak{M}'$, $\dot{\mathfrak{M}}'$ and $\mathfrak{p}'$, are mirror images of the vectors $\mathfrak{E}$, $\mathfrak{M}$, $\dot{\mathfrak{M}}$ and $\mathfrak{p}$. From that if follows, that the mirror images of $Rot\ \mathfrak{M}$ and $[\dot{\mathfrak{M}}.\mathfrak{p}]$ don't fall into $Rot\ \mathfrak{M}'$

and $[\dot{\mathfrak{M}}'.\mathfrak{p}']$, but into $-\boldsymbol{Rot}\,\mathfrak{M}'$ and $-[\dot{\mathfrak{M}}'.\mathfrak{p}']$. Now, since the linear relation between four vectors expressed in (68), also then remains when we replace any of them by its mirror image, hence

$$\mathfrak{E}' = \sigma\mathfrak{M}' - j\,\boldsymbol{Rot}\,\mathfrak{M}' - k[\dot{\mathfrak{M}}'.\mathfrak{p}']$$

By that we see, that the processes that can occur in the mirror image of the considered body, don't satisfy the relation (68) anymore, but a relation in which the terms with *j* and *k* have difference signs. Thus it is confirmed, that these terms are likely be connected with the fact, that body and its mirror image have different properties; we may expect, that a rotation of the polarization plane will actually be in agreement with them.

I postpone the details about this. Here, it only shall be remarked that the magnitude $j\,\boldsymbol{Rot}\,\mathfrak{M}$ (we will make that the natural rotation of the polarization plane will depend on it) has much similarity with the terms, that were assumed by various physicists in the equations of motion of light, to explain circular-polarization. Indeed I regard, in the absence of a theory that explains the phenomenon more deeply, the introduction of the term $j\,\boldsymbol{Rot}\,\mathfrak{M}$ as neither better nor worse than the hypotheses of those physicists.

The last term in (68) has a peculiar meaning. Namely a rotation of the polarization plane would correspond to it, that would be caused in a body (that is different from its mirror image) by the motion of earth[6].

1. ↑ If we would reverse the motions in an *absorbing* medium, then a state would arise, at which the amplitude would be increased in the direction of propagation.
2. ↑ The *magnetic* rotation of the polarization plane remains excluded from our considerations
3. ↑ See my earlier considerations (Over het verband tauchen de voortplantingsanelheid van het licht en de dichtheid en

samenstelling der middenstoffen. Verhandelingen der Akad. van Wet. te Amsterdam, Deel 18, pp. 68—77; Wied. i., Bd. 9, p. 656).

4. ↑ If we decompose $\mathfrak{p}$ into two components $\mathfrak{p}_1$ and $\mathfrak{p}_2$, then it follows from the first mentioned property of $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$

$$F_2(\dot{\mathfrak{M}}, \mathfrak{p}) = F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1) + F_2(\dot{\mathfrak{M}}, \mathfrak{p}_2)$$

It is assumed, that $\mathfrak{p}_1$ falls into the direction of $\dot{\mathfrak{M}}$, and $\mathfrak{p}_2$ is perpendicular to it. If we now rotate the figure (consisting of $\dot{\mathfrak{M}}, \mathfrak{p}_1$ and $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1)$) around an axis that falls into $\dot{\mathfrak{M}}$, $\dot{\mathfrak{M}}$ and $\mathfrak{p}_1$ stay were they are, and thus $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1)$ may not change as well. Consequently, this vector must have the direction of $\dot{\mathfrak{M}}$ and $\mathfrak{p}_1$. That

$$F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1) = 0 \tag{67}$$

can be shown in addition, by means of a rotation of 180° around an axis perpendicular to $\dot{\mathfrak{M}}$ and $\mathfrak{p}_1$. In the course of this rotation, the vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_1)$ would obtain the opposite direction; yet it shouldn't be changing, because *both* vectors $\dot{\mathfrak{M}}$ and $\mathfrak{p}_1$ change their sign.

To find out the direction of $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_2)$, we turn the figure (which is formed by this vector with $\dot{\mathfrak{M}}$ and $\mathfrak{p}_2$) around an axis perpendicular to the plane $(\dot{\mathfrak{M}}, \mathfrak{p}_2)$ or $(\dot{\mathfrak{M}}, \mathfrak{p})$, namely around 180°. Here, $\dot{\mathfrak{M}}$ and $\mathfrak{p}_2$ go over into $-\dot{\mathfrak{M}}$ and $-\mathfrak{p}_2$; the vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_2)$ thus may not be changed, which is only possible when it has the direction of the axis.

Thus the vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p}_2)$ — and thus by (67) also the vector $F_2(\dot{\mathfrak{M}}, \mathfrak{p})$ — is perpendicular to the plane $(\dot{\mathfrak{M}}, \mathfrak{p})$; its magnitude is proportional to the values of $\dot{\mathfrak{M}}$ and $\mathfrak{p}_2$. Both we have expressed in (66).

5. ↑ After a rotation of the mentioned figure we want, as we are really free to do this, to apply again the *original* coordinate axis for the decomposition of the vectors and the formation of the derivatives. At first, only a rotation of 180° around the axis takes place. Here, $\mathfrak{R}_x$ remains unchanged; consequently in the expression for this component only these derivatives of $\mathfrak{M}_x$, $\mathfrak{M}_y$, $\mathfrak{M}_z$ can occur, which do *not* change the sign. These are

$$\frac{\partial \mathfrak{M}_x}{\partial x}, \frac{\partial \mathfrak{M}_y}{\partial y}, \frac{\partial \mathfrak{M}_y}{\partial z}, \frac{\partial \mathfrak{M}_z}{\partial y}, \frac{\partial \mathfrak{M}_z}{\partial z}.$$

If we further notice, that in the course of a rotation of 180° around the *y*- or *z*-axis, $\mathfrak{R}_x$ assumes the opposite direction, and that also those derivatives are excluded, which retain the same sign during one of these rotations, the we find, that $\mathfrak{R}_x$ must be of the form

$$j\frac{\partial \mathfrak{M}_z}{\partial y} + j\frac{\partial \mathfrak{M}_y}{\partial z}$$

Eventually we imagine still another rotation around 90° around the *x*-axis, whereby *OY* is transformed into *OZ*. After that rotation, $\frac{\partial \mathfrak{M}_z}{\partial y}$ and $\frac{\partial \mathfrak{M}_y}{\partial z}$ have the values, that previously belonged to $-\frac{\partial \mathfrak{M}_y}{\partial z}$ and $-\frac{\partial \mathfrak{M}_z}{\partial y}$; however, since $\mathfrak{R}_x$ hasn't changed, then $j' = -j$. From $\mathfrak{R}_x$ we find $\mathfrak{R}_y$ and $\mathfrak{R}_z$ by permutation of the letters.

6. ↑ The following consideration might be sufficient, to make the existence of the electric force $k[\dot{\mathfrak{M}}.\mathfrak{p}]$ somewhat probably, for which only the possibility was shown in the text. Since a molecule of a circular-polarizing substance must have a so-called "helical" structure, then the particles from which it consists may be mutually connected, so that the displacement of one of them produces a circular motion of one or many others. Let, for example, a positive ion *A* be in motion along the line *G*, and by that the moment $\mathfrak{M}$ shall be produced, so that the velocity is proportional to $\dot{\mathfrak{M}}$, and this motion may be accompanied by the rotation (in a circle with *G* as its axis) of some other ions *B* that are also positive. Between the velocities of *A* and *B* there is a constant relation. The motion of particle *B* then forms a circular electric current, proportional to $\dot{\mathfrak{M}}$, and this produces in the molecule and in its vicinity a *local* magnetic force, which in *A* falls into line *G* and thus also into $\dot{\mathfrak{M}}$, and which is proportional to $\dot{\mathfrak{M}}$. If we combine, in accordance with the last term of fundamental equation (V), this magnetic force with the velocity $\mathfrak{p}$, then we obtain an electric force like $k[\dot{\mathfrak{M}}.\mathfrak{p}]$.

Application to optical phenomena.

Reduction to a resting system.

§ 56. The specification of the influence, that the motion of ponderable bodies exerts on the phenomena of light, can be achieved in a very simple manner, if we neglect circular polarization, as it will always take place in this section.

Namely we want, as we did it earlier (§ 31) already, by continuing omission of magnitudes of second order, to introduce (instead of *t*) the "local time"

$$t' = t - \frac{1}{V^2}\left(\mathfrak{p}_x x + \mathfrak{p}_y y + \mathfrak{p}_z z\right)$$

as an independent variable; besides we want (instead of $\mathfrak{D}$) consider a new vector $\mathfrak{D}'$, which we define by the formula

$$4\pi V^2 \mathfrak{D}' = 4\pi V^2 \mathfrak{D} + [\mathfrak{p}.\mathfrak{H}] \qquad \text{(IX)}$$

If we consider an arbitrary magnitude as a function of *x*, *y*, *z* and *t'* , then (as before (§81)) we denote the partial derivative by

$$\left(\frac{\partial}{\partial x}\right)', \left(\frac{\partial}{\partial y}\right)', \left(\frac{\partial}{\partial z}\right)', \frac{\partial}{\partial t'}.$$

Furthermore, according to this notation, we shall understand by

$$Div' \mathfrak{A}$$

the expression

$$\left(\frac{\partial \mathfrak{A}_x}{\partial x}\right)' + \left(\frac{\partial \mathfrak{A}_y}{\partial y}\right)' + \left(\frac{\partial \mathfrak{A}_z}{\partial z}\right)',$$

and by

$$Rot' \mathfrak{A}$$

a vector with the components

$$\left(\frac{\partial \mathfrak{A}_z}{\partial y}\right)' - \left(\frac{\partial \mathfrak{A}_y}{\partial z}\right)' \text{ etc.}$$

The introduction of t' and $\mathfrak{D}'$ gives the advantage, that (as I will show now) the equations (I_c) — (V_c) assume the same form as the formulas that apply to $\mathfrak{p} = 0$.

§ 57. At first we obtain, by consideration of formulas (35),

$$Div\, \mathfrak{D} = Div'\, \mathfrak{D} - \frac{1}{V^2}\left(\mathfrak{p}_x \dot{\mathfrak{D}}_x + \mathfrak{p}_y \dot{\mathfrak{D}}_y + \mathfrak{p}_z \dot{\mathfrak{D}}_z\right),$$

or by (III_c), if we replace (in the terms multiplied by $\mathfrak{p}_x, \mathfrak{p}_y, \mathfrak{p}_z$) $\mathfrak{H}'$ by $\mathfrak{H}$ and *Div* by Div'

$$Div\,\mathfrak{D} = Div'\,\mathfrak{D} - \frac{1}{4\pi V^2}\left\{\mathfrak{p}_x[Rot\,\mathfrak{H}]_x + \mathfrak{p}_y[Rot\,\mathfrak{H}]_y + \mathfrak{p}_z[Rot\,\mathfrak{H}]_z\right\} =$$

$$= Div'\,\mathfrak{D} + \frac{1}{4\pi V^2} Div\,[\mathfrak{p}.\,\mathfrak{H}] = Div'\,\mathfrak{D}'\,.$$

Hence the equation (I_c) becomes

$$Div'\,\mathfrak{D}' = 0 \tag{I_d}$$

In a similar way

$$Div\,\mathfrak{H} = Div'\,\mathfrak{H} - \frac{1}{V^2}\left(\mathfrak{p}_x\dot{\mathfrak{H}}_x + \mathfrak{p}_y\dot{\mathfrak{H}}_y + \mathfrak{p}_z\dot{\mathfrak{H}}_z\right)$$

i.e., by (IV_c),

$$Div\,\mathfrak{H} = Div'\,\mathfrak{H} + \frac{1}{V^2}\left\{\mathfrak{p}_x[Rot\,\mathfrak{E}]_x + \mathfrak{p}_y[Rot\,\mathfrak{E}]_y + \mathfrak{p}_z[Rot\,\mathfrak{E}]_z\right\} =$$

$$= Div'\,\mathfrak{H} - \frac{1}{V^2} Div[\mathfrak{p}.\,\mathfrak{E}] = Div'\,\mathfrak{H}',$$

so that it can be written for (II_c)

$$Div'\,\mathfrak{H}' = 0 \tag{IId}$$

Now let us turn to formula (III_c). In this one, three equations are summarized, namely in the first of them on the left side, the expression

$$\frac{\partial \mathfrak{H}'_z}{\partial y} - \frac{\partial \mathfrak{H}'_y}{\partial z}.$$

is stated. For that, we can write with respect to (35)

$$[Rot'\ \mathfrak{H}']_x - \frac{1}{V^2}\left\{\mathfrak{p}_y \frac{\partial \mathfrak{H}'_z}{\partial t'} - \mathfrak{p}_z \frac{\partial \mathfrak{H}'_y}{\partial t'}\right\},$$

and thus for the equation itself

$$[Rot'\ \mathfrak{H}']_x = 4\pi \frac{\partial \mathfrak{D}_x}{dt'} + \frac{1}{V^2}\frac{\partial}{dt'}\left\{\mathfrak{p}_y \mathfrak{H}_z - \mathfrak{p}_z \mathfrak{H}_y\right\} = 4\pi \frac{\partial \mathfrak{D}'_x}{dt'}$$

The two other equations admit of a similar transformation, and therefore we have

$$Rot'\ \mathfrak{H}' = 4\pi \frac{\partial \mathfrak{D}'}{\partial t'} \qquad (III_d)$$

Furthermore, as regards the first of equations IV_c), this one goes over, since

$$\frac{\partial \mathfrak{E}_z}{\partial y} - \frac{\partial \mathfrak{E}_y}{\partial z} = [Rot'\ \mathfrak{E}]_x - \frac{1}{V^2}\left\{\mathfrak{p}_y \frac{\partial \mathfrak{E}_z}{\partial t'} - \mathfrak{p}_z \frac{\partial \mathfrak{E}_y}{\partial t'}\right\}$$

into

$$[Rot'\ \mathfrak{E}]_x = -\frac{\partial \mathfrak{H}_x}{\partial t'} + \frac{1}{V^2}\frac{\partial}{\partial t'}\left\{\mathfrak{p}_y \mathfrak{E}_z + \mathfrak{p}_z \mathfrak{E}_y\right\} = -\frac{\partial \mathfrak{H}'_x}{\partial t'},$$

so that (IV_c) is equivalent with

$$Rot' \ \mathfrak{E} = -\frac{\partial \mathfrak{H}'}{\partial t'} \tag{IV_d}$$

Eventually it follows from

$$\varkappa_1 \mathfrak{E}_x = 4\pi V^2 \mathfrak{D}'_x, \ \varkappa_2 \mathfrak{E}_y = 4\pi V^2 \mathfrak{D}'_y, \ \varkappa_3 \mathfrak{E}_z = 4\pi V^2 \mathfrak{D}'_z \ (V_d)$$

§ 58. To introduce the new variables also into the *limiting conditions*, we consider the perpendicular *n* for the considered point, and also two directions *h* and *k* that are perpendicular to one another and to *n*. There, the direction *n* shall correspond to a rotation by a right angle from *h* to *k*. Consequently it follows from (IX) (§ 56)

$$4\pi V^2 \mathfrak{D}'_n = 4\pi V^2 \mathfrak{D}_n + [\mathfrak{p}.\mathfrak{H}]_n = 4\pi V^2 \mathfrak{D}_n + [\mathfrak{p}.\mathfrak{H}']_n =$$
$$= 4\pi V^2 \mathfrak{D}_n + \mathfrak{p}_h \mathfrak{H}'_k - \mathfrak{p}_k \mathfrak{H}'_h.$$

Now, since $\mathfrak{D}_n$, $\mathfrak{H}'_k$ and $\mathfrak{H}'_h$ are steady, then this must also be so for $\mathfrak{D}'_n$.

In a similar manner we derive from the continuity of $\mathfrak{H}_n$, $\mathfrak{E}_h$ and $\mathfrak{E}_k$, by means of the relation to be derived from (VI_c)

$$\mathfrak{H}'_n = \mathfrak{H}_n - \frac{1}{V^2}[\mathfrak{p}.\mathfrak{E}]_n = \mathfrak{H}_n - \frac{1}{V^2}[\mathfrak{p}_h \mathfrak{E}_k - \mathfrak{p}_k \mathfrak{E}_h],$$

the continuity of $\mathfrak{H}'_n$.

If we also notice the other equations $(VIII_c)$, then it is clear, that all limiting conditions are contained in the formulas

$$\mathfrak{D}'_{n(1)} = \mathfrak{D}'_{n(2)}\,,\ \mathfrak{E}_{h(1)} = \mathfrak{E}_{h(2)}\,,\ \mathfrak{H}'_{(1)} = \mathfrak{H}'_{(2)}\,, \qquad (VIII_d)$$

in which *h* can be now any arbitrary direction in the border surface.

§ 59. The equations $(I_d) - (V_d)$ and $(VIII_d)$ differ from the equations which apply to stationary bodies by § 52, only by the fact that

$$t',\ \mathfrak{D}' \text{ und } \mathfrak{H}'$$

has taken the place of

$$t,\ \mathfrak{D} \text{ and } \mathfrak{H}$$

This coincidence opens for as a way, to treat problems regarding the influence of Earth's motion on optical phenomena, in a very simple way.

Namely, if a state of motion for a system of stationary bodies is known, where

$$\mathfrak{D}_x, \mathfrak{D}_y, \mathfrak{D}_z,\ \mathfrak{E}_x, \mathfrak{E}_y, \mathfrak{E}_z,\ \mathfrak{H}_x, \mathfrak{H}_y, \mathfrak{H}_z. \qquad (69)$$

are certain functions of x, y, z and t, then in the same system, if it is displaced by the velocity $\mathfrak{p}$*, there can exist a state of motion, where*

$$\mathfrak{D}'_x, \mathfrak{D}'_y, \mathfrak{D}'_z,\ \mathfrak{E}'_x, \mathfrak{E}'_y, \mathfrak{E}'_z,\ \mathfrak{H}'_x, \mathfrak{H}'_y, \mathfrak{H}'_z. \qquad (70)$$

are exactly the same functions of x, y, z and t' [that is, $t - \frac{1}{V^2}\left(\mathfrak{p}_x x + \mathfrak{p}_y y + \mathfrak{p}_z z\right)$*].*

Although we have given (in the previous consideration) to the coordinate axes the directions of the symmetry axis, the derived theorem applies to any right-angled coordinate system. We can easily recognize this, when we consider, that for local time t' it can also be written

$$t - \frac{\mathfrak{p}_r r}{V^2},$$

where *r* is the line drawn from the coordinate origin to the point (*x*, *y*, *z*), and t' is independent of the *direction* of the coordinate axes.

We may remember the fact, that in a moving system we always have to understand by *x*, *y*, *z* the coordinates with respect to the axes that share the translation.

If the magnitudes (70) are known as functions of *x*, *y*, *z* and *t'* , thus also as functions of *x*, *y*, *z* and *t*, then $\mathfrak{D}_x, \mathfrak{D}_y, \mathfrak{D}_z,\ \mathfrak{H}_x, \mathfrak{H}_y, \mathfrak{H}_z$ can be calculated fron the equations (IX) and (VI_c).

Different applications.

§ 60. We want to call the two states of motion — in the stationary and in the moving system of bodies —, of which we have spoken so far, *corresponding* states. Now, they shall be mutually compared more precisely.

a. If in a stationary system the magnitudes (69) are periodic functions of *t* with the period *T*, then in the other system the magnitudes (70) have the same period with respect to t', thus also with respect to *t*, when we let *x*, *y*, *z* remain constant. When interpreting this result, we have to consider, that two periods must be distinguished in the case of translation (see §§ 37 and 38), which we accordingly can call *absolute* and *relative* period. We are dealing with the absolute one, when we consider the temporal variations in a point that has a fixed position against the aether; but we are dealing with the relative one, when we consider a point that moves together with ponderable matter. The things found above can now be expressed as follows:

If a state of oscillation in the moving system shall correspond to a state in the stationary system, then the relative oscillation period in the first mentioned case must be equal to the oscillation period in the second mentioned case.

b. In the stationary system, no motion of light may take place at an arbitrary location, *i.e.*, $\mathfrak{D}$, $\mathfrak{E}$ and $\mathfrak{H}$ may vanish at this place. At the corresponding location of the moving bodies it is consequently $\mathfrak{D}' = 0, \mathfrak{E} = 0, \mathfrak{H}' = 0$, thus also $\mathfrak{D} = 0, \mathfrak{H} = 0$, *so that at this place the motion of light is missing as well*.

From that it directly follows, that a surface *that forms the border of a space filled with light within a stationary body, can have the same meaning when the body is moving.*

In a stationary, homogeneous medium, for example, light bundles are possible which are limited by cylindric surfaces, if it is only assumed that the dimensions of the bisections are much greater than the wave length. *By our theorem, such bundles also can exist in a moving system.*

The described lines of the mentioned cylindric surfaces we call *light rays*, and in the case of translation: *relative* light rays. The cylinders we have to imagine as rigidly connected with ponderable matter; thus they form the paths for the propagation of light relative to that matter.

c. A cylindric light bundle falls upon a plane limiting-surface in a stationary system, and it will be mirrored and refracted by it, — for generality we want to say: bi-refracted. The new light bundles have a cylindric border as well. If we now apply the things said under *a* and *b* to the corresponding case of the moving system, then we come to the theorem:

In the moving system, relative light rays of relative oscillation period T were mirrored and refracted by the same laws, as rays of the oscillations period T in the stationary system.

d. Let in the stationary system be a transparent body of arbitrary form, that was hit by a cylindric light bundle, and by that an arbitrary interference- or diffraction-phenomenon occurs. If dark strips do occur on that occasion, then they must appear in the corresponding state of the moving system at exactly the same locations.

An extreme case of a diffraction-phenomenon is the unification of all light in a *focus*. *By the preceding, the laws by which a light ray of certain cylindric limitation is concentrated by a telescope objective, won't be changed at all by a translation.*

e. While in corresponding states the *lateral limitation* of a light ray is the same, the *wave normals* have different directions. If it is set, for example, that plane waves are propagating with the velocity *W* in the stationary system whose perpendicular has the direction (b_x, b_y, b_z), so that the deviation from equilibrium is a function of

$$t - \frac{b_x x + b_y y + b_z z}{W}$$

then for the moving system, similar functions of

$$t' - \frac{b_x x + b_y y + b_z z}{W} = t - \left\{ \left(\frac{b_x}{W} + \frac{\mathfrak{p}_x}{V^2} \right) x + \left(\frac{b_y}{W} + \frac{\mathfrak{p}_y}{V^2} \right) y + \left(\frac{b_z}{W} + \frac{\mathfrak{p}_z}{V^2} \right) z \right\}$$

occur. The direction constants b'_x, b'_y, b'_z of the wave normal will thus be determined for this system by the condition

$$b'_x : b'_y : b'_z = \left(b_x + \frac{W\,\mathfrak{p}_x}{V^2} \right) : \left(b_y + \frac{W\,\mathfrak{p}_y}{V^2} \right) : \left(b_z + \frac{W\,\mathfrak{p}_z}{V^2} \right),$$

or, in the case of a propagation in pure aether, by

$$b'_x : b'_y : b'_z = \left(b_x + \frac{\mathfrak{p}_x}{V} \right) : \left(b_y + \frac{\mathfrak{p}_y}{V} \right) : \left(b_z + \frac{\mathfrak{p}_z}{V} \right).$$

From this equation is is given in reverse

$$b_x : b_y : b_z = \left(b'_x - \frac{\mathfrak{p}_x}{V} \right) : \left(b'_y - \frac{\mathfrak{p}_y}{V} \right) : \left(b'_z - \frac{\mathfrak{p}_z}{V} \right). \quad (71)$$

The aberration of light.

§ 61. Let b'_x, b'_y, b'_z be the direction constants of the line drawn from a stationary celestial body to earth, thus also the direction constants of the perpendicular with respect to the plane waves that arrive in the vicinity of earth. So when we, to investigate the following path of propagation, relate the motion of light to a coordinate system, that shares the motion of earth, then of course the direction constants of the wave normal remain $\mathfrak{b}'_x, \mathfrak{b}'_y, \mathfrak{b}'_z$, while that one comes into play as the relative oscillation period T' (§ 37), which was modified by DOPPLER's law. As we have seen, the motion (as regards the lateral limitation of a light bundle cut out by a diaphragm, the concentration through lenses, and the passage through other transparent bodies) will correspond to a motion in a stationary system, for which the oscillation period is T', and the perpendicular to the incident waves has the direction constants $\mathfrak{b}_x, \mathfrak{b}_y, \mathfrak{b}_z$ that are to be determined by (71).

Thus all phenomena happen exactly in such a manner, as if the earth were at rest, the oscillation period ware T', and the celestial body, as seen from earth, would be located not in the direction ($-\mathfrak{b}'_x, -\mathfrak{b}'_y, -\mathfrak{b}'_z$), but in the direction ($-\mathfrak{b}_x, -\mathfrak{b}_y, -\mathfrak{b}_z$).

Now, *aberration* exactly consists of the latter. That the magnitude and direction, which we find for it, actually corresponds to the known rule which is in accordance with observations, follows immediately from equation (71). Namely, we obtain a vector of direction ($\mathfrak{b}_x, \mathfrak{b}_y, \mathfrak{b}_z$), when we compose a vector of direction ($\mathfrak{b}'_x, \mathfrak{b}'_y, \mathfrak{b}'_z$), whose length represents the velocity of light, with a second one which is equal and opposite to Earth's velocity $\mathfrak{p}$.

By the way, in our theorem also lies the explanation for the fact, that during the observation by a lens system, always that aberration arises which is determined by the previously mentioned rule[1], as well as the explanation for the known experiments of ARAGO[2] by a prism, and for the experiment proposed by BOSCOVICH and executed by AIRY, in which the tube of a telescope was filled with water[3].

Observations by sun light.

§ 62. The trajectory of Earth deviates as little from a circle, that, when we are dealing with sun rays, we can neglect the velocity component $\mathfrak{p}_r$, on which the variation of the oscillation period depends (§ 37). Experiments with these rays must have the result, as if the earth were at rest, and the sun were in a direction changed by aberration, and *would emanate types of light of the same oscillation period, as in reality*[4]

From that it immediately follows, that (as regards a certain line of FRAUNHOFER during a refraction in a prism, or the diffraction through a lattice) we *don't* register *any influence of Earth's motion*, thus it cannot make any difference, whether the direction of light (that falls upon the prism of the lattices) would form this or that angle with the translation of earth. As regards the lattice-spectra, this result was confirmed by the careful experiments of MASCART[5]. This physicist has additionally demonstrated by certain experiments[6], that as regards the mentioned spectra, the deflection for a certain FRAUNHOFER line fully agrees with the deflection for the corresponding rays of a terrestrial light source[7].

Moving light sources.

§ 63. Above, in § 61, the celestial body was assumed to be at rest. Yet also for a moving body we arrive at a simple result. We already know (§ 36), that the perpendicular to the waves arriving at Earth, is directed to location *P*, where the light source was present in the instant when the light was emitted. Now the motion of Earth causes, that we observe the star not at this place *P*, but at another place P', namely the displacement from *P* to *P'* can be derived by the ordinary rule for aberration. By the consideration of § 61 its prove is at hand.

Eventually a simple figure shows, *that P' falls into the true place at the time of observation, as soon as the velocity of the light source agrees in magnitude and direction with that of earth.*

Experiments with terrestrial light sources.

§ 64. From the results previously obtained it directly follows, that we will see a distant terrestrial object always in the direction, where it is actually located. We also have already seen, that for a light sources rigidly connected with earth, no difference exists between the true and the observed oscillation period.

In general, the motion of Earth will never have an influence of first order on the experiments with terrestrial light sources.

To justify this theorem, we want at first (by application of the superposition principle (§ 7)) derive from the formulas of § 33 other ones, which are valid for an arbitrary system of luminous molecules. On that occasion we assume, that they have the common translation $\mathfrak{p}$, and we choose the local time t' specified by (34), and the relative coordinates (§ 19), as independent variables.

Let

$$(\xi_1, \eta_1, \zeta_1), \quad (\xi_2, \eta_2, \zeta_2), \text{ etc.}$$

be the locations of the molecules, and

$$\left.\begin{array}{c}\mathfrak{m}_{x(1)}=f_1(t'),\ \mathfrak{m}_{y(1)}=g_1(t'),\ \mathfrak{m}_{z(1)}=h_1(t'),\\ \mathfrak{m}_{x(2)}=f_2(t'),\ \mathfrak{m}_{y(2)}=g_2(t'),\ \mathfrak{m}_{z(2)}=h_2(t'),\\ \text{etc.}\end{array}\right\}\qquad(72)$$

or

$$\left.\begin{array}{c}\mathfrak{m}_{x(1)}=f_1\left(t-\frac{\mathfrak{p}_x}{V^2}\xi_1-\frac{\mathfrak{p}_y}{V^2}\eta_1-\frac{\mathfrak{p}_z}{V^2}\zeta_1\right),\ \text{etc.},\\ \mathfrak{m}_{x(2)}=f_2\left(t-\frac{\mathfrak{p}_x}{V^2}\xi_2-\frac{\mathfrak{p}_y}{V^2}\eta_2-\frac{\mathfrak{p}_z}{V^2}\zeta_2\right),\ \text{etc.},\\ \text{etc.}\end{array}\right\}\qquad(73)$$

be the electric moments that occur within.

The condition that was caused by a single molecule in the point (x,y,z) of the aether, will be determined by equations (39) and (40). The latter one we additionally want to transform (to subsequently apply the theorem of § 59 more conveniently) by introducing the expressions $\mathfrak{D}$ and $\mathfrak{D}'$ for the aether. For this medium, as we know, $\mathfrak{D}$ is equal to $\mathfrak{d}$, and thus by (IX) (§ 56), $4\pi V^2\mathfrak{D}'$ is equal to

$$4\pi V^2\mathfrak{d}+[\mathfrak{p}.\mathfrak{H}].$$

By means of equation V_b) we may replace $\mathfrak{F}$ by $4\pi V^2\mathfrak{D}'$ in (40).

Furthermore, if we denote by Σ the sum of terms, any of them stemming from a luminous molecule, then we obtain from (39) and (40) the following formulas for the condition in the aether caused by ion oscillations (72):

$$\left.\begin{aligned}\mathfrak{H}'_x &= \frac{\partial}{\partial t'}\left(\frac{\partial}{\partial y}\right)'\left\{\sum\left(\frac{\mathfrak{m}_z}{r}\right)\right\} - \frac{\partial}{\partial t'}\left(\frac{\partial}{\partial z}\right)'\left\{\sum\left(\frac{\mathfrak{m}_y}{r}\right)\right\}, \text{ etc.,}\\ 4\pi\mathfrak{D}'_x &= \left(\frac{\partial S}{\partial x}\right)' - \Delta'\left\{\sum\left(\frac{\mathfrak{m}_x}{r}\right)\right\}, \text{ etc.,}\\ S &= \left(\frac{\partial}{\partial x}\right)'\left\{\sum\left(\frac{\mathfrak{m}_x}{r}\right)\right\} + \left(\frac{\partial}{\partial y}\right)'\left\{\sum\left(\frac{\mathfrak{m}_y}{r}\right)\right\} + \left(\frac{\partial}{\partial z}\right)'\left\{\sum\left(\frac{\mathfrak{m}_z}{r}\right)\right\}.\end{aligned}\right\} \quad (74)$$

Here, r denotes the distance of point (x, y, z) from the location (ξ, η, ζ) of one of the luminous molecules, while $\mathfrak{m}_x, \mathfrak{m}_y, \mathfrak{m}_z$ represent the moments of this molecule at local time $t' - \frac{r}{V}$. The two first members of the sum

$$\sum\left(\frac{\mathfrak{m}_x}{r}\right)$$

are for example

$$\frac{1}{r_1} f_1\left(t' - \frac{r_1}{V}\right) \text{ und } \frac{1}{r_2} f_2\left(t' - \frac{r_2}{V}\right),$$

when r_1 and r_2 are the distances between (x, y, z) and the two first molecules.

§ 65. From the preceding formulas, others immediately arise, which apply to a *stationary* light source when we simply erase all accents. If in this case in the luminous molecules the moments exist

$$\left.\begin{array}{c}\mathfrak{m}_{x(1)} = f_1(t),\ \mathfrak{m}_{y(1)} = g_1(t),\ \mathfrak{m}_{z(1)} = h_1(t),\\ \mathfrak{m}_{x(2)} = f_2(t),\ \mathfrak{m}_{y(2)} = g_2(t),\ \mathfrak{m}_{z(2)} = h_2(t),\\ \text{etc.,}\end{array}\right\} \tag{75}$$

then we have in the aether

$$\left.\begin{array}{c}\mathfrak{H}_x = \frac{\partial}{\partial t}\frac{\partial}{\partial y}\left\{\sum\left(\frac{\mathfrak{m}_z}{r}\right)\right\} - \frac{\partial}{\partial t}\frac{\partial}{\partial z}\left\{\sum\left(\frac{\mathfrak{m}_y}{r}\right)\right\},\ \text{etc.,}\\ 4\pi\mathfrak{D}_x = \frac{\partial S}{\partial x} - \Delta\left\{\sum\left(\frac{\mathfrak{m}_x}{r}\right)\right\},\ \text{etc.,}\\ S = \frac{\partial}{\partial x}\left\{\sum\left(\frac{\mathfrak{m}_x}{r}\right)\right\} + \frac{\partial}{\partial y}\left\{\sum\left(\frac{\mathfrak{m}_y}{r}\right)\right\} + \frac{\partial}{\partial z}\left\{\sum\left(\frac{\mathfrak{m}_z}{r}\right)\right\}.\end{array}\right\} \tag{76}$$

where $\mathfrak{m}_x, \mathfrak{m}_y, \mathfrak{m}_z$ are now the moments of a molecule at time $t - \frac{r}{V}$, so that *e.g.* the two first members of the sum

$$\sum\left(\frac{\mathfrak{m}_x}{r}\right)$$

have the values

$$\frac{1}{r_1} f_1\left(t - \frac{r_1}{V}\right) \text{ and } \frac{1}{r_2} f_2\left(t - \frac{r_2}{V}\right)$$

Of course, ξ, η, ζ, *x*, *y*, *z* are now the coordinates related to *stationary* axes.

§ 66. The two cases considered in §§ 64 and 65 (*with* or *without* translation) shall be compared to one another. Here, we imagine that the spatial arrangement of the luminous molecules is the same in the two cases, *i.e.* that all ξ, η, ζ have the same value; the latter we also assume for *x*, *y*, *z*, with the result that we consider the state of the aether in a point that has a particular location with respect to the light source. Eventually we understand by f_1, g_1, h_1, f_2 etc. the same function-sign for both cases.

A look upon the formulas (74) and (76) let us recognize, that we are dealing with *corresponding* states, on which the theorem of § 59 is applicable. If the light is incident on a non-transparent screen with one opening, then the limitation of light and shadow, or the location of dark diffraction fringes behind of it, will be the same in both cases. Also no difference in the spatial distribution of light and dark will be seen, when the rays were mirrored or refracted at an arbitrary transparent body, or when a lens concentrates them, or when some interference phenomena occur.

Of course, motions that are present in the light source itself, which generate these corresponding states, are not quite the same. In one case they will be determined by (73), and in the other case by (75). If we put

$$f_1\left(t - \frac{\mathfrak{p}_x}{V^2}\xi_1 - \frac{\mathfrak{p}_y}{V^2}\eta_1 - \frac{\mathfrak{p}_z}{V^2}\zeta_1\right) = f_1'(t), \text{ etc.,}$$

then we may thus also say:

A moving light source, in which ion motions as represented by

$$\left.\begin{array}{c} \mathfrak{m}_{x(1)} = f_1'(t),\ \mathfrak{m}_{y(1)} = g_1'(t),\ \mathfrak{m}_{z(1)} = h_1'(t) \\ \textbf{etc.} \end{array}\right\} \qquad (77)$$

take place, generates the same phenomena as a stationary light source, to which the formulas

$$\left.\begin{aligned}\mathfrak{m}_{x(1)} &= f_1'\left(t+\frac{\mathfrak{p}_x}{V^2}\xi_1+\frac{\mathfrak{p}_y}{V^2}\eta_1+\frac{\mathfrak{p}_z}{V^2}\zeta_1\right),\\ \mathfrak{m}_{y(1)} &= g_1'\left(t+\frac{\mathfrak{p}_x}{V^2}\xi_1+\frac{\mathfrak{p}_y}{V^2}\eta_1+\frac{\mathfrak{p}_z}{V^2}\zeta_1\right),\\ &\text{etc.}\end{aligned}\right\} \tag{78}$$

apply.

If we are dealing with oscillations, then the difference between (77) and (78) is reduced to a *variation of the phases*, namely this will be determined for an arbitrary molecule by

$$\frac{\mathfrak{p}_x}{V^2}\xi+\frac{\mathfrak{p}_y}{V^2}\eta+\frac{\mathfrak{p}_z}{V^2}\zeta$$

consequently it is *not equal for the various molecules*.

It is now to be noticed, that the molecules of a light source, *e.g.* a flame, must be considered as totally independent from one another, so that, as it is ordinarily expressed, the rays emanated by two of these particles cannot mutually interfere. From that if follows, that arbitrary variations in the phases of the single molecules cannot have any influence on the observable phenomena. The stationary light source with motions (78) will give nothing other than a stationary source (also at rest) with motions (77), and thus we may claim:

If we set a light source into translation, without changing anything of the oscillations of their ions, then the observable phenomena in bodies rigidly connected with them, remain as they were.

§ 67. Numerous experiments have proven, that when using terrestrial light sources, the phenomena are indeed independent of the orientation of the devices with respect to the direction of Earth's motion. Here, the observations of RESPIGHI,[8] HOEK,[9] KETTELER[10] and MASCART[11] on refraction do belong, as well as the experiments of the three last mentioned physicists on interference phenomena.[12] We are indebted to KETTELER for an investigation on the inner reflection and the refraction at calcite prisms.[13] and to MASCART for an investigation[14] on the interference fringes that appear at calcite plates in polarized light.

The entrainment of light waves by ponderable matter.

§ 68. In a stationary, isotropic or anisotropic body a bundle of plane light waves propagate, as to which the components of $\mathfrak{D}$ and $\mathfrak{H}$ can be expressed by expressions of the form

$$A \cos \frac{2\pi}{T}\left(t - \frac{\mathfrak{b}_x x + \mathfrak{b}_y y + \mathfrak{b}_z z}{W} + B\right) \tag{79}$$

thus W is the velocity of propagation. This magnitude can depend on $\mathfrak{b}_x, \mathfrak{b}_y, \mathfrak{b}_z$, and T. After we have given to the body the velocity $\mathfrak{p}$, a state of motion can occur in it, for which expressions like

$$A \cos \frac{2\pi}{T}\left(t' - \frac{\mathfrak{b}_x x + \mathfrak{b}_y y + \mathfrak{b}_z z}{W} + B\right)$$

or

$$A \cos \frac{2\pi}{T}\left\{t - \frac{\mathfrak{p}_x x + \mathfrak{p}_y y + \mathfrak{p}_z z}{V^2} - \frac{\mathfrak{b}_x x + \mathfrak{b}_y y + \mathfrak{b}_z z}{W} + B\right\} \tag{80}$$

apply. The direction constants b'_x, b'_y, b'_z of the wave normal are now proportional to the magnitudes

$$\frac{b_x}{W}+\frac{\mathfrak{p}_x}{V^2},\ \frac{b_y}{W}+\frac{\mathfrak{p}_y}{V^2},\ \frac{b_z}{W}+\frac{\mathfrak{p}_z}{V^2}$$

If we consequently put

$$\frac{b_x}{W}+\frac{\mathfrak{p}_x}{V^2}=\frac{b'_x}{W'},\ \frac{b_y}{W}+\frac{\mathfrak{p}_y}{V^2}=\frac{b'_y}{W'},\ \frac{b_z}{W}+\frac{\mathfrak{p}_z}{V^2}=\frac{b'_z}{W'} \quad (81)$$

,

then (80) becomes

$$A\ \cos\frac{2\pi}{T}\left\{t-\frac{b'_x x+b'_y y+b'_z z}{W'}+B\right\}$$

for which we can see, that W' is the velocity by which the waves of *relative* oscillation period *T* are propagating in the direction (b'_x, b'_y, b'_z) within the moving body.

From (81) we find

$$\frac{1}{W'^2}=\frac{1}{W^2}+2\frac{b_x\mathfrak{p}_x+b_y\mathfrak{p}_y+b_z\mathfrak{p}_z}{W\,V^2},$$

and for that we can write, neglecting magnitudes of second order,

$$\frac{1}{W'^2}=\frac{1}{W^2}+2\frac{b'_x\mathfrak{p}_x+b'_y\mathfrak{p}_y+b'_z\mathfrak{p}_z}{W\,V^2}=\frac{1}{W^2}+2\frac{p_n}{W\,V^2}.$$

Here, $\mathfrak{p}_n$ is the component of velocity into the direction of the wave normal, with which W' is related. Eventually

$$W' = W - \mathfrak{p}_n \frac{W^2}{V^2} \tag{82}$$

§ 69. Up to now, the investigation was general. Now it shall be assumed, that the body be isotropic. The velocity W is thus independent of the direction of the waves, and also the ratio

$$\frac{V}{W} = N,$$

the absolute refraction index of the stationary body, only depends on T.

When interpreting formula (82), which now passes to

$$W' = W - \frac{\mathfrak{p}_n}{N^2}, \tag{83}$$

we have to remember, that we have used a coordinate system for the description of the phenomena, that moves together with ponderable matter. Thus (83) is the velocity of the light waves *relative to that matter*. If we wish it know the relative velocity W'' *with respect to the aether*, we have to compose the velocity (83), which has the direction of the wave normal, with the component $\mathfrak{p}_n$ of the translation velocity (that exactly falls in that direction). By that we obtain

$$W'' = W + \left(1 - \frac{1}{N^2}\right) \mathfrak{p}_n, \tag{84}$$

which is in agreement with the known assumption of FRESNEL.

As regards this result, two things shall be remarked. *First*, the given derivation applies to every value of *T*, thus *for every kind of light*, and *second*, this has to be understood, that the substitution of the values of *N* and *W*, which belong in the stationary body to a particular *T*, gives the value of W'' for the relative oscillation period *T*.[15]

§ 70. If the considered body is birefringent, than it may not be forgotten, that *W* and *W'* in equation (82) are related to *different* directions of the wave normal, namely *W* to direction (b_x, b_y, b_z), and *W* to direction (b'_x, b'_y, b'_z). Concerning the question, as to how the velocities in stationary and in moving bodies are mutually different *for a given direction of the waves*, the equation doesn't directly give an answer. To a simple theorem, however, leads the introduction of *light rays*.

In a stationary birefringent body, to any direction of the wave normal (as soon as one of the two possible oscillation directions is chosen) belongs a particular direction for the light-rays, *i.e.*, for the describing lines of a cylindric limiting-surface of a light bundle. For the points of such a line, it is now, when c_x, c_y, c_z are the direction constants, and *s* means the distance of a fixed point (x_0, y_0, z_0) of the line,

$$x = x_0 + c_x s,\ y = y_0 + c_y s,\ z = z_0 + c_z s. \tag{85}$$

By that, when we put

$$\frac{W}{b_x c_x + b_y c_y + b_z c_z} = U$$

and understand by B' a new constant, the expression (79) is transformed into

$$A \cos \frac{2\pi}{T}\left(t - \frac{s}{U} + B'\right).$$

The magnitude U is, what we usually call the *velocity of the light ray*.

If we now pass to the corresponding motion in the progressing body, then the considered line *remains* (§ 60, *b*) a light ray, and we obtain for the determination of the deviations of equilibrium, in the different points of it, expressions as

$$A \cos \frac{2\pi}{T}\left(t' - \frac{s}{U} + B'\right).$$

or, by (34) and (85),

$$A \cos \frac{2\pi}{T}\left(t - \frac{\mathfrak{p}_s s}{V^2} - \frac{s}{U} + B''\right), \tag{86}$$

where $\mathfrak{p}_s$ is the component of $\mathfrak{p}$ in the direction of the light ray, while the new constant B'' has the value

$$B' - \frac{\mathfrak{p}_x x_0 + \mathfrak{p}_y y_0 + \mathfrak{p}_z z_0}{V^2}$$

The expression (86) goes over into

$$A \cos \frac{2\pi}{T}\left(t' - \frac{s}{U'} + B''\right).$$

and here, U' *is the velocity of the light ray in the moving body*, when we put

$$\frac{1}{U'} = \frac{1}{U} + \frac{\mathfrak{p}_s}{V^2}.$$

From that we conclude

$$U' = U - \mathfrak{p}_s \frac{U^2}{V^2}, \tag{87}$$

a formula, whose shape agrees with (82), in which U and U' are now related to *light rays of the same direction*.

§ 71. Formula (84) has found a nice confirmation by the experiment, that were first executed by Fizeau and later repeated by Michelson and Morley[16], on the propagation of light in streaming water. The arrangement of them should be sufficiently known, so that we can restrict ourselves to compare (still more deeply than it is usually happening) the results with the theory.

To apply the formula, we first have to derive the relative period from the experimental conditions, and then (from the dispersion formula for stationary water) the refraction exponent *N* corresponding with this period. The value of *V/N* calculated in this way, we eventually have to substitute into (82) for *W*. However, as regards the relative period, a more closer consideration is required.

It's known that, as regards these experiment, two tubes were used which are closed by glass plates and which are lying next to one another, through which the water was flowing with the same velocity, but in different direction; since the base tubes were present entirely at the edges, we may assume, that at all places (at least in the middle parts of the bisection) the same velocity $\mathfrak{p}$ occurred[17]. The two light bundles, which should mutually interfere, passed through the device, so that one was propagated in *both* tubes in the direction of the water stream, and the other one steadily in the opposite direction.

We now consider a fixed point P in the interior of one of the tubes. The conditions, under which the light is propagating from the source to this point, obviously remain — when the water stream is stationary — constantly the same, and namely this applies to *both* ways, on which the rays can reach point P. Impulses, which emanate by certain periods from the source, will arrive with the same periods in P, and when T is the oscillation period of the light source, then this is also the absolute oscillation period in P.

From that if follows for the *relative* oscillation period related to the water

$$\left(1 \pm \frac{\mathfrak{p}}{W'}\right) T, \tag{88}$$

where W' is exactly the sought velocity of the waves, while (as also in the following formulas) the above or below sign is to be applied, depending on whether the considered light bundle is propagating in the direction of the water motion, or in the opposite direction.

We always neglect magnitudes of second order and thus we may put instead of (88)

$$\left(1 \pm \frac{\mathfrak{p}}{W}\right) T \tag{89}$$

Under W in equation (82) — and also in this expression (89) itself — we have now to understand the value, which belong to period (89) in the stationary body. The corresponding refraction exponent is

$$n \pm \frac{\mathfrak{p}}{W} T \frac{dn}{dT},$$

in case we denote the refraction exponent for period *T* by *n*; consequently we have to substitute

$$W = \frac{V}{n \pm \frac{\mathfrak{p}}{W} T \frac{dn}{dT}} = \frac{V}{n} \mp \frac{\mathfrak{p}}{n^2} \frac{V}{W} T \frac{dn}{dT},$$

or, when we replace *W* by $\dfrac{V}{n}$ in the last member,

$$W = \frac{V}{n} \mp \frac{\mathfrak{p}}{n} T \frac{dn}{dT}.$$

Furthermore it is in (82)

$$\mathfrak{p}_n = \pm \mathfrak{p},$$

so that we find

$$W' = \frac{V}{n} \mp \frac{\mathfrak{p}}{n^2} \mp \frac{\mathfrak{p}}{n} T \frac{dn}{dT},$$

and for the relative velocity with respect to the aether, thus also with respect to the closing plates of the tubes,

$$W'' = \frac{V}{n} \mp \mathfrak{p}\left(1 - \frac{1}{n^2}\right) \mp \frac{\mathfrak{p}}{n} T \frac{dn}{dT}. \tag{90}$$

§ 72. The mentioned physicists have compared their observations, not with that formula, but with another in which the last term is missing; a satisfying agreement occurred at this place. Namely, if we put

$$W'' = \frac{V}{n} \pm \mathfrak{p}\epsilon,$$

thus the coefficient ε can be derived from the experiments. Now, while MICHELSON and MORLEY found in this manner

$$\epsilon = 0,434,$$

"with a possible error of $\pm 0,02$", $1 - \frac{1}{n^2}$ for *D*-light has the value 0,438.

By our theory it should be

$$\epsilon = 1 - \frac{1}{n^2} - \frac{1}{n} T \frac{dn}{dT}$$

or, if we consider *n* as a function of wave length λ in air,

$$\epsilon = 1 - \frac{1}{n^2} - \frac{1}{n} \lambda \frac{dn}{d\lambda}$$

For the FRAUNHOFER line *D*, this becomes

0,451.

Thus formula (90) somewhat further deviates from the observations than the simpler equation

$$W'' = \frac{V}{n} \pm \mathfrak{p}\left(1 - \frac{1}{n^2}\right); \tag{91}$$

however, the observation were possibly not as exact for allowing us to put weight to this condition.

If it should be achieved, which namely appears to be difficult but not impossible, to experimentally distinguish between the equations (90) and (91), and if the first one should be justified, then we would have observed the DOPPLER variation of the oscillation period for an artificially generated velocity. It is only by consideration of this variation, that we have derived equation (90).

§ 73. It is hardly necessary to recall at this place the importance of the role, which is played by formula (84) in the theory of aberration and the related phenomena. FRESNEL based his explanation of ARAGO's prism experiment upon the value $1 - \frac{1}{N^2}$ of the dragging coefficient. Subsequent scientists have applied this equations to many other cases, and have derived from it, that the motion of earth, as regards most of experiments with terrestrial light sources, is without influence, and that experiments with the light of a celestial body must give a result, as if the direction altered by aberration would be the real one. How easy the theoretical considerations are formed, when we look, not upon the direction of the waves, but on the *path of light rays*, I have demonstrated (following the example of VELTMANN[18]) in my treatise of the year 1887.[19] At that time, I restricted myself to isotropic bodies, since it wasn't known to me yet, how to extend FRESNEL's law for crystals.

Now, since it was demonstrated, that the propagation velocity of light rays obeys in these bodies the simple law expressed in formula (87), it is easy to show, *that also the birefringence of rays is independent of Earth's motion*.[20] For this purpose we can start with a simple theorem that follows from the principle of HUYGENS, and I allow myself to shortly state it at this place.

Let *A* and *B* two arbitrary, which may lie within different mutually adjacent media. In general, only a restricted amount of light rays can travel from one to another. If we now form (for one such ray, as well as for other ways between *A* and *B* with only small deviations) the integral

$$\int \frac{ds}{U},$$

in which *U* means the velocity for a light rays that follows the line element *ds*, then (by the referred theorem) the integral for the light ray is a minimum.

However, I neither want to dwell more closely on these considerations, nor on further applications of formulas (82) and (87), since we have solved the question concerning the influence of Earth's motion in different cases, already above in a much simpler way.

Closer consideration of light bundles of plane waves.

§ 74. In the application of the general theorem found in § 59, I was as brief as possible and I didn't dwell more into the details, as it just was required. For further explanation is seems justified, however, to show by some examples, as to how all details of the light motions are given from that theorem as well.

At first, we consider a light bundle of plane waves, that propagates in the aether, after it went through an additional opening in a non-transparent screen which is connected to Earth. For a moment we neglect the motion of Earth. Let:

l, *m*, *n* the direction constants of the wave normal,
q a constant,
f, *g*, *h* the direction constants of the dielectric displacement,
a the "amplitude" of the latter.

Consequently, the light motion can be represented by the equations

$$\mathfrak{d}_x = af\ \cos\psi,\ \mathfrak{d}_y = ag\ \cos\psi,\ \mathfrak{d}_z = ah\ \cos\psi, \tag{92}$$

$$\mathfrak{H}_x = 4\pi aV(mh - ng)\cos\psi,\ \mathfrak{H}_y = 4\pi aV(nf - lh)\cos\psi$$

$$, \tag{93}$$

$$\mathfrak{H}_z = 4\pi aV(lg - mf)\cos\psi,$$

$$\psi = \frac{2\pi}{T}\left(t - \frac{lx + my + nz}{V} + q\right), \tag{94}$$

with the condition

$$lf + mg + nh = 0. \tag{95}$$

We can easily see, that these values satisfy all equations of motion. The vectors $\mathfrak{d}$ and $\mathfrak{H}$ are perpendicular to one another and to the wave normal; the direction of the light rays (§ 60, *b*) falls into the latter.

§ 75. If the Earth is moving, then by the theorem of § 59 a condition is possible, which (related to a moving coordinate system), will be represented by

$$\mathfrak{d}'_x = af\ \cos\psi',\ \mathfrak{d}'_y = ag\ \cos\psi',\ \mathfrak{d}'_z = ah\ \cos\psi', \qquad (96)$$

$$\mathfrak{H}'_x = 4\pi aV(mh - ng)\cos\psi',\ \mathfrak{H}'_y = 4\pi aV(nf - lh)\cos\psi'$$

,

$$\mathfrak{H}'_z = 4\pi aV(lg - mf)\cos\psi', \qquad (97)$$

$$\psi' = \frac{2\pi}{T}\left(t - \frac{\mathfrak{p}_x x + \mathfrak{p}_y y + \mathfrak{p}_z z}{V^2} - \frac{lx + my - nz}{V} + q\right) \qquad (98)$$

,

By $\mathfrak{d}'$ we have to understand a vector $\mathfrak{D}'$ for the pure aether, which is defined by (IX) (§ 56).

While the light rays, which determine the lateral limitation of the bundle, have still the direction (*l*, *m*, *n*), the wave normal deviates from it. Its direction constants *l'*, *m'*, *n'* satisfy, as it can be seen from (98), the conditions

$$l' : m' : n' = \left(l + \frac{\mathfrak{p}_x}{V}\right) : \left(m + \frac{\mathfrak{p}_y}{V}\right) : \left(n + \frac{\mathfrak{p}_z}{V}\right).$$

We will neglect all magnitudes of second order again. Then, by denoting the components of $\mathfrak{p}$ in the direction of the rays by $\mathfrak{p}_s$, we have

$$l'\left(1 + \frac{\mathfrak{p}_s}{V}\right) = l + \frac{\mathfrak{p}_x}{V}, \text{etc.} \qquad (99)$$

by which (98) is transformed into

$$\psi' = \frac{2\pi}{T}\left\{t - \left(1 + \frac{\mathfrak{p}_s}{V}\right)\frac{l'x + m'y - n'z}{V} + q\right\}.$$

While T is now the *relative* oscillation period, we find for the *absolute* one (§§ 60 and 37)

$$T' = T\left(1 - \frac{\mathfrak{p}_s}{V}\right).$$

For the determination of $\mathfrak{d}$ and $\mathfrak{H}$, the formulas (IX) (§ 56) and (VI_b) (§ 20) can serve, which we may replace by

$$4\pi V^2\mathfrak{d} = 4\pi V^2\mathfrak{d}' - [\mathfrak{p}.\mathfrak{H}']$$

and

$$\mathfrak{H} = \mathfrak{H}' + 4\pi[\mathfrak{p}.\mathfrak{d}']$$

If follows

$$\mathfrak{d}_x = a\left\{f - \frac{\mathfrak{p}_y}{V}(lg - mf) + \frac{\mathfrak{p}_z}{V}(nf - lh)\right\}\cos\psi', \text{ etc.} \quad (100)$$

$$\mathfrak{H}_x = 4\pi a\left\{V(mh - ng) + (\mathfrak{p}_y h - \mathfrak{p}_z g)\right\}\cos\psi', \text{ etc.} \quad (101)$$

or, if we put by (99)

$$\frac{\mathfrak{p}_x}{V} = l'\left(1 + \frac{\mathfrak{p}_s}{V}\right) - l, \text{ etc.}$$

and if we consider (95).

$$\mathfrak{d}_x = a\left(1+\frac{\mathfrak{p}_s}{V}\right)\{-m'(lg-mf)+n'(nf-lh)\}\cos\psi' \tag{102}$$

, etc.

$$\mathfrak{H}_x = 4\pi a V\left(1+\frac{\mathfrak{p}_s}{V}\right)(m'h-n'g)\cos\psi', \text{ etc.}$$

By that we see, that $\mathfrak{d}$ and $\mathfrak{H}$ are both perpendicular to the wave normal, as it was expected. Additionally, both vectors are mutually perpendicular, which can be seen most easily, when by replace (100) by

$$\mathfrak{d}_x = a\left\{f-\frac{\mathfrak{p}_y}{V}(l'g-m'f)+\frac{\mathfrak{p}_z}{V}(n'f-l'h)\right\}\cos\psi', \text{ etc.}$$

Furthermore, we can conclude, that the vector $[\mathfrak{d}.\mathfrak{H}]$ which is present in POYNTING's theorem, falls into the wave normal. We can easily convince ourselves, that it has the direction, in which the waves are propagating, and we find for its magnitude

$$4\pi a^2(V+2\mathfrak{p}_s)\cos^2\psi'.$$

The energy flux through a plane which is parallel to the waves, thus amounts for the unit of area and time

$$4\pi a^2V^2(V+2\mathfrak{p}_s)\cos^2\psi' \tag{103}$$

§ 76. From a light bundle as the one considered above, others of the same kind can arise by refraction and mirroring at plane limiting surfaces. Here, we only consider such ones, that are again propagating in the aether, and we represent (for the case that the earth is at rest) one of the bundles, which emerge from the incident motion considered in § 74, by the following formula

$$\mathfrak{d}_{x(1)} = a_1 f_1 \ \cos\psi_1, \ \mathfrak{d}_{y(1)} = a_1 g_1 \ \cos\psi_1, \ \mathfrak{d}_{z(1)} = a_1 h_1 \ \cos\psi_1,$$

$$\mathfrak{H}_{x(1)} = 4\pi a_1 V(m_1 h_1 - n_1 g_1)\cos\psi_1, \ etc.,$$

$$\psi_1 = \frac{2\pi}{T}\left(t - \frac{l_1 x + m_1 y - n_1 z}{V} + q_1\right).$$

§ 77. With this motion only that corresponds, which (in case Earth is moving together with the reflecting or refracting body) emerges from the light represented by (96)-(98). For this new state of motion we can thus write

$$\mathfrak{d}'_{x(1)} = a_1 f_1 \ \cos\psi'_1, \ \mathfrak{d}'_{y(1)} = a_1 g_1 \ \cos\psi'_1, \ \mathfrak{d}'_{z(1)} = a_1 h_1 \ \cos\psi'_1,$$

$$\mathfrak{H}'_{x(1)} = 4\pi a_1 V(m_1 h_1 - n_1 g_1)\cos\psi'_1, \text{ etc.},$$

$$\psi'_1 = \frac{2\pi}{T}\left(t - \frac{\mathfrak{p}_x x + \mathfrak{p}_y y + \mathfrak{p}_z z}{V^2} - \frac{l_1 x + m_1 y - n_1 z}{V} + q_1\right),$$

from which it again follows — see (100) and (101) —

$$\mathfrak{d}_{x(1)} = a_1 \left\{ f_1 - \frac{\mathfrak{p}_y}{V}(l_1 g_1 - m_1 f_1) + \frac{\mathfrak{p}_z}{V}(n_1 f_1 - l_1 h_1) \right\} \cos\psi_1',$$

etc.,

$$\mathfrak{H}_{x(1)} = 4\pi a_1 \left\{ V(m_1 h_1 - n_1 g_1) + (\mathfrak{p}_y h_1 - \mathfrak{p}_z g_1) \right\} \cos\psi_1',$$

etc.

In these equations l_1, m_1, n_1 determine the direction of the rays, which we also want to denote by s_1.

§ 78. In the course of mirroring or refraction, the *absolute* period will be changed in general, while, as it nearly goes without saying and as it is also expressed by our formulas, the *relative* period is the same for all relevant light bundles. The absolute period of the incident motion is (§ 75)

$$T\left(1 - \frac{\mathfrak{p}_s}{V}\right).$$

Also, as regards the bundle considered in the previous paragraph it becomes

$$T\left(1 - \frac{\mathfrak{p}_{s1}}{V}\right).$$

Thus it has changed in the ratio of 1 to $1 + \dfrac{\mathfrak{p}_s - \mathfrak{p}_{s1}}{V}$.

If *e.g.* the rays are falling perpendicular upon a plate, which retreats by the velocity $\mathfrak{p}$ in the direction of the perpendicular, then for the incident light $\mathfrak{p}_s = \mathfrak{p}$, and for the reflected light $\mathfrak{p}_{s1} = -\mathfrak{p}$. The variation of the absolute oscillation period during reflection will consequently be determined by the ratio $1 + \dfrac{2\mathfrak{p}}{V}$.

Also in the ratio between the amplitudes of the incident and the mirrored or refracted light, an influence of Earth's motion can be seen. The amplitude of the dielectric displacement $\mathfrak{d}$ is namely with respect to the states of motion considered in §§ 74, 75, 76 and 77

$$a,\ a\left(1+\frac{\mathfrak{p}_s}{V}\right),\ a_1,\ a_1\left(1+\frac{\mathfrak{p}_{s1}}{V}\right).$$

The ratio just mentioned is

$$\frac{a_1}{a},$$

in case the earth is at rest, and

$$\frac{a_1}{a}\left(1+\frac{\mathfrak{p}_{s1}-\mathfrak{p}_s}{V}\right),$$

if it is moving.

In the case previously considered, where the rays are falling perpendicular to the retreating plate, the latter expression becomes

$$\frac{a_1}{a}\left(1-\frac{2\mathfrak{p}}{V}\right);$$

the reflected light will thus be weakened by the motion of the plate. Of course, the opposite motion would strengthen it.

Now the important question emerges, whether these variations of intensity are in accordance with the conservation of energy. To decide this matter, we have to consider, that the aether (due to the motion of light), is acting by certain forces on the mirroring or refracting body (§ 17), and that these forces do some work, as soon as the body is displaced by the velocity $\mathfrak{p}$.

Now, we imagine (limited by plane surfaces and surrounded by aether) a transparent body *K*, upon which a system of plane waves is falling, and from which reflected and refracted light-bundles are emanating again. Let us put a *fixed*, closed surface σ around it, and calculate for a time interval which is equal to the *relative* period *T*,

1°. the amount of energy *A*, which is flows rather in- than outwards through σ,

2°. the growth *B* of the electric energy within the surface, and

3°. the work *C* of the forces mentioned above.

For simplification we assume, that the amplitudes be constant, and that the body will be continuously hit by rays in the same way, which is the case, when the light source, or the diaphragm that serves to limit a bundle of sunlight, shares the translation of *K*. After expiration of time *T*, the energy itself has again the original value within the body, and even the energy located in σ wouldn't be changed, when also the surface would be displaced by the velocity $\mathfrak{p}$. As regards the calculation of *B*, consequently only the energy in certain parts of space that lie in the direct vicinity of σ, come into consideration.

Eventually, we will find

$$A = B + C, \tag{104}$$

by which its is proven, that we were always (as regards our developments) in agreement with the energy theorem.

However, I don't want to hinder myself with the verification of equation (104), since it might be preferable to treat the question more generally.

The conservation of energy in a more general case.

§ 79. An arbitrary transparent body *K* shall be hit by a homogeneous light motion, whose intensity remains constant; consequently, a certain motion arises in the body and in the aether in its vicinity.

Here, when Earth is at first imagined as stationary, the components of $\mathfrak{d}$ and $\mathfrak{H}$ in the aether are certain functions of *x,y,z,t*, and namely as regards the last variable, goniometric functions with the period *T*. During a complete period, *e.g.* in the time interval from $t_0 - T$ to t_0, equal quantities of energy must flow in- and outwards through an arbitrary surface σ that surrounds the surface, which can be expressed by Poynting's theorem by

$$\int_{t_0-T}^{t_0} dt \int [\mathfrak{d}.\mathfrak{H}]_n d\sigma = 0 \tag{105}$$

By assuming, that this condition is fulfilled, we want to show, that also state of motion that corresponds with that above, which can exist in the case of a translation $\mathfrak{p}$, satisfies the energy theorem.

If we replace in the functions, which apply to $\mathfrak{d}_x$, $\mathfrak{H}_x$, etc. when the Earth is at rest, the time *t* by the "local time" t' (§ 31), and if we understand in those functions by *x*, *y*, *z* the coordinates with respect to a movable system, then we obtain values of $\mathfrak{d}'_x$, $\mathfrak{H}'_x$, etc. for the new state. From (105) it thus directly follows, that

$$\int_{t_0-T}^{t_0} dt \int [\mathfrak{d}'.\mathfrak{H}']_n d\sigma = 0 \tag{106}$$

if it is presupposed, that we choose for σ a surface, which shares the motion of the body.

§ 80. However, now the flux of energy through a fixed surface σ shall now be considered. The energy flux related to its unit shall be

$$V^2[\mathfrak{p}.\mathfrak{H}]_n,$$

or, as we find from the formulas (IX) and (VI_b) (§§ 56 and 20), under continuing omission of magnitudes of second order,

$$V^2[\mathfrak{d}'.\mathfrak{H}']_n + 4\pi V^2 \left\{\mathfrak{p}_n\mathfrak{d}^2 - \mathfrak{d}_n(\mathfrak{p}_x\mathfrak{d}_x + \mathfrak{p}_y\mathfrak{d}_y + \mathfrak{p}_z\mathfrak{d}_z)\right\} + \\ + \frac{1}{4\pi}\left\{\mathfrak{p}_n\mathfrak{H}^2 - \mathfrak{H}_n(\mathfrak{p}_x\mathfrak{H}_x + \mathfrak{p}_y\mathfrak{H}_y + \mathfrak{p}_z\mathfrak{H}_z)\right\} \tag{107}$$

If we want, on that basis, to calculate the energy which flows more out- than inwards between the times $t_0 - T$ and t_0, and consequently, by remarking the latter, integrate with respect to time. As regards the two latter terms, we can also think of a surface, that progresses with velocity $\mathfrak{p}$.

§ 81. To also arrange the integration of the first term in such a way, that we have to deal with such a movable surface, we at first set for the increase of the integral $V^2 \int [\mathfrak{d}' . \mathfrak{H}']_n d\sigma$ at certain *t*, when we displace the surface σ in the direction of $\mathfrak{p}$ about an infinitely small distance ϵ, the sign

$$\varkappa \, \epsilon,$$

where $\varkappa$ is of course a very special function of *t*. Furthermore, we think of a surface σ_0, which falls into σ at time t_0, yet which is rigidly connected with earth. Then, at time *t* the "distance" of σ and σ_0 has the value $\mathfrak{p}(t_0 - t)$, which is to be considered as infinitely small, and our integral for the fixed surface σ amounts

$$\mathfrak{p}\varkappa(t_0 - t),$$

more than for σ_0. The time integral, about which we speak eventually, is thus about

$$\mathfrak{p} \int_{t_0-T}^{t_0} \varkappa(t_0 - t) dt \tag{108}$$

greater than the time integral taken for σ_0, and, since the latter vanishes by (106), we have only to deal with the value (108).

By the way, in $\varkappa$ we don't have to consider the magnitudes containing $\mathfrak{p}$, and thus we may understand, since with this omission

$$V^2 \int [\mathfrak{d}' . \mathfrak{H}']_n d\sigma,$$

is the energy flux, under

$$\varkappa\,\epsilon$$

the difference calculated for the unit time, and under

$$\varkappa\,\epsilon\,dt$$

the difference calculated for the element *dt*, of the energy fluxes through two fixed surfaces that are mutually distant by the length ε.

Now, let $Q\epsilon$ be the energy, which at time *t* is more surrounded by our surface σ in its fixed location, as when this surface would be displaced by ϵ in the direction of $\mathfrak{p}$; then we immediately see, that

$$\varkappa\epsilon dt = \frac{dQ}{dt}\epsilon dt,$$

$$\varkappa = \frac{dQ}{dt}$$

By that, and furthermore by partial integration, (108) is transformed into

$$\mathfrak{p}\int_{t_0-T}^{t_0}\frac{dQ}{dt}(t_0-t)dt = -\mathfrak{p}TQ_{t=t_0-T} + \mathfrak{p}\int_{t_0-T}^{t_0} Q\,dt,$$

or

$$-\mathfrak{p}TQ_{t=t_0} + \mathfrak{p}\int_{t_0-T}^{t_0} Q\,dt,$$

since, except magnitudes of order $\mathfrak{p}$, *Q* has again the original value after the expiration of time *T*.

§ 82. Until now, we only spoke about the first member in (107). If we denote the two other members by A, then we have

$$-\mathfrak{p}TQ_{t=t_0}+\mathfrak{p}\int_{t_0-T}^{t_0}Q\,dt+\int_{t_0-T}^{t_0}dt\int A\,d\sigma$$

the complete value of the energy, that travelled outwards through σ. If we add the increase of the energy in the interior of σ, and the work of the forces, by which the aether is acting on the ponderable body, then we must, if the energy theorem shall be satisfied, obviously obtain zero.

The increase energy in a full period T would be zero, if the surface σ together with the body K would be displaced over the distance $\mathfrak{p}T$, and at this place would have taken the location σ''; it factually consists of the energy amount, which, at time t_0, is more contained in σ than in σ''. This is now, as it follows form the definition given for $Q\varepsilon$, exactly

$$\mathfrak{p}TQ_{t=t_0}\,.$$

The work mentioned above can be expressed, as we will see soon, by an expression of the form

$$\int_{t_0-T}^{t_0}dt\int S\,d\sigma;$$

thus the energy law requires that

$$\mathfrak{p}\int_{t_0-T}^{t_0}Q\,dt+\int_{t_0-T}^{t_0}dt\int A\,d\sigma+\int_{t_0-T}^{t_0}dt\int Sd\sigma=0$$

If it is additionally achieved, to represent Q as an integral over σ, *e.g.* in the form

$$Q = \int q \, d\sigma$$

and to show, that

$$\mathfrak{p}q + A + S = 0 \tag{109}$$

then we have achieved our goal.

§ 83. From the definition given for $Q\epsilon$ we derive, that by $q\epsilon d\sigma$ we have to understand the energy content of the space, which is traversed by the element during the displacement ε, and namely we have, depending on whether the displacement takes place with respect to the inner- or the outer-side of σ, to apply the positive or the negative sign. Thus we have

$$q\epsilon d\sigma = -\epsilon \cos(\mathfrak{p}, n) \left(2\pi V^2 \mathfrak{d}^2 + \frac{1}{8\pi}\mathfrak{H}^2\right) d\sigma,$$

and

$$\mathfrak{p}q = -\mathfrak{p}_n \left(2\pi V^2 \mathfrak{d}^2 + \frac{1}{8\pi}\mathfrak{H}^2\right).$$

Second, as regards the work, we don't have to care about the last member in equation (15) and the analogous formulas.[21] Only the "tensions" come into account, and

$$S \, d\sigma \, dt$$

is the work of the tension with respect to $d\sigma$. The components of this tension are

$$\left\{2\pi V^2(2\mathfrak{d}_x\mathfrak{d}-\alpha\mathfrak{d}^2)+\frac{1}{8\pi}(2\mathfrak{H}_x\mathfrak{H}_n-\alpha\mathfrak{H}^2\right\}d\sigma, \text{ etc.,}$$

from which it follows

$$S=2\pi V^2\left\{2\mathfrak{d}_n(\mathfrak{p}_x\mathfrak{d}_x+\mathfrak{p}_y\mathfrak{d}_y+\mathfrak{p}_z\mathfrak{d}_z)-\mathfrak{p}_n\mathfrak{d}^2\right\}+$$

$$+\frac{1}{8\pi}\left\{2\mathfrak{H}_n(\mathfrak{p}_x\mathfrak{H}_x+\mathfrak{p}_y\mathfrak{H}_y+\mathfrak{p}_z\mathfrak{H}_z)-\mathfrak{p}_n\mathfrak{H}^2\right\},$$

Eventually, *A* means the sum of the two last members in (107).

The given values now actually satisfy the condition (109).

1. ↑ That this is also the case during the observation by a mirror telescope, would also follow from our theorem, *when the mirror would consist of transparent material*. However, as regards the actual mirrors that are constructed by metal, we can remark, that the direction by which the light rays are reflected, and the *location of the unification point* only depends on the curvature, but not on the material nature of the mirror. For the determination of this location, as it was done by various physicists, also the Principle of HUYGENS can be applied, (see also my treatise in Arch. neerl., T. 21).
2. ↑ ARAGO. OEuvres completes, T. 1, p. 107; BIOT. Traité élémentaire d'astronomie physique, 3e éd., T. 5, p. 364.
3. ↑ AIRY. Proc. Royal Society of London, Vol. 20, p. 35, 1871; Vol. 21, p. 121, 1873; Phil. Mag., 4th Ser., Vol. 43, p. 310, 1872.

4. ↑ We neglect the rotation of the sun and the motions at its surface, from which it is known that they cause a displacement of the spectral lines in accordance with Doppler's law. As regards the experiments that will mentioned soon, light of the *whole disc of the sun* was used.
5. ↑ Mascart. Ann. de l'école normale, 2e ser., T. 1, pp. 166—170, and p. 190, 1872.
6. ↑ Mascart. L. c., pp. 170 and 189.
7. ↑ During the experiments with sun-light, of course, metallic mirrors were used. However, we can easily see, that this changes nothing as regards our considerations (see the note 1 at p. 89)
8. ↑ Respighi. Memor. di Bologna (2), II, p. 279. (Cited in Ketteler. Astronomische Undulationstheorie, p. 66).
9. ↑ Hoek. Astr. Nachr., Bd. 73, p. 193.
10. ↑ Ketteler. Astr. Und.-Theorie, p. 66, 1873; Pogg. Ann., Bd. 144, p. 370,1872.
11. ↑ Mascart. Ann. de l'ecole normale, 2e sér., T. 3, p. 376, 1874.
12. ↑ Hoek. Arch. neerl., T. 3, p. 180, 1868. Ketteler. Astr. Und.-Theorie, p. 67; Pogg. Ann., Bd. 144, p. 372. Mascart. L. c., pp. 390—416.
13. ↑ Ketteler. Astr. Und.-Theorie, pp. 158 and 166; Pogg. Ann., Bd. 147, pp. 410 and 419, 1872.
14. ↑ Mascart. Ann. de l'école normale, 2e sér., T. 1, pp. 191—196, 1873
15. ↑ A derivation of equation (84) from the electromagnetic light theory was published by R. Reiff Wied. Arm., Bd. 50, p. 861, 1893). Long before me, also J. J. Thomson has dealt with this subject (Phil. Mag., 5th. Ser., Vol. 9, p. 284, 1880; Recent Researches in Electricity and Magnetism, p. 543), however, without obtaining Fresnel's coefficient.

16. ↑ Michelson and Morley. American Journal of Science, 3d Ser., Vol. 81, p. 377, 1886.
17. ↑ In the following formulas of this paragraph, $\mathfrak{p}$ simply means the *magnitude* of velocity.
18. ↑ Veltmann. Pogg. Ann., Bd. 150, p. 497, 1873.
19. ↑ Lorentz. Arch. néerl., T. 21.
20. ↑ A derivation of this theorem from formula (87) was published by me in Zittingsverslagen of the Akad. T. Wet. te Amsterdam, 1892—93, p. 149,
21. ↑ Namely, to calculate the work, we can multiply the path $\mathfrak{p}T$ with the average of the force acting in its direction. This average would be zero for the last member in (15), when the surface σ would be displace together with the body, from which it follows, that it is in reality of order $\mathfrak{p}$.

Experiments whose results cannot be explained without further ado.

The rotation of the polarization plane.

§ 84. As the equations of motion of light for an isotropic body that has *not* the same properties as its mirror image, we have to assume by the considerations of the 4th section:

$$Div\,\mathfrak{D} = 0, \tag{I_e}$$

$$Div\,\mathfrak{H} = 0, \tag{II_e}$$

$$Rot\,\mathfrak{H}' = 4\pi\dot{\mathfrak{D}}, \tag{III_e}$$

$$Rot\,\mathfrak{E} = -\dot{\mathfrak{H}}, \tag{IV_e}$$

$$\mathfrak{E} = 4\pi V^2\mathfrak{d} + [\mathfrak{p}.\mathfrak{H}], \tag{V_e}$$

$$\mathfrak{H}' = \mathfrak{H} - 4\pi[\mathfrak{p}.\mathfrak{d}], \tag{VI_e}$$

$$\mathfrak{D} = \mathfrak{d} + \mathfrak{M}, \tag{X}$$

$$\mathfrak{E} = \sigma\mathfrak{M} + j\,Rot\,\mathfrak{M} + k[\dot{\mathfrak{M}}.\mathfrak{p}], \tag{XI}$$

where by $\mathfrak{E}, \mathfrak{d}, \mathfrak{H}$ and $\mathfrak{H}'$ we have to understand averages.

We now want to presuppose, that the velocity $\mathfrak{p}$ would have the direction of the *x*-axis, and to study the propagation of plane waves, whose normal coincides with that axis as well.

§ 85. To find a particular solution of the equations corresponding to such waves, we put

$$\mathfrak{H}_x = 0,\ \mathfrak{H}_y = ae^{nt-mx},\ \mathfrak{H}_z = \nu\,\mathfrak{H}_y$$

where *a*, *v*, *n* and *m* are constants. Already by that, the condition (II_e) is satisfied.

Now, the equation (IV_e) will be satisfied by us, by putting

$$\mathfrak{E}_x = 0,\ \mathfrak{E}_y = \frac{n}{m}\mathfrak{H}_z,\ \mathfrak{E}_z = -\frac{n}{m}\mathfrak{H}_y,$$

and then it follows from (V_e), (VI_e) and (III_e), one after the other,

$$\mathfrak{d}_x = 0,\ 4\pi V^2\mathfrak{d}_y = \left(\frac{n}{m} + \mathfrak{p}_x\right)\mathfrak{H}_z,\ 4\pi V^2\mathfrak{d}_z = -\left(\frac{n}{m} + \mathfrak{p}_x\right)\mathfrak{H}_y,$$

$$\mathfrak{H}'_x = 0,\ \mathfrak{H}'_y = \left(1 - \frac{n}{m}\frac{\mathfrak{p}_x}{V^2}\right)\mathfrak{H}_y,\ \mathfrak{H}'_z = \left(1 - \frac{n}{m}\frac{\mathfrak{p}_x}{V^2}\right)\mathfrak{H}_z.$$

$$\mathfrak{D}_x = 0,\ 4\pi\mathfrak{D}_y = \left(\frac{m}{n} - \frac{\mathfrak{p}_x}{V^2}\right)\mathfrak{H}_z,\ 4\pi\mathfrak{D}_z = -\left(\frac{m}{n} - \frac{\mathfrak{p}_x}{V^2}\right)\mathfrak{H}_y,$$

whose latter values are also in agreement with condition (I_c).

Eventually we derive from (X)

$$\mathfrak{M}_x = 0,\ 4\pi V^2 \mathfrak{M}_y = \left(V^2 \frac{m}{n} - \frac{n}{m} - 2\mathfrak{p}_x\right) \mathfrak{H}_z\,,$$

$$4\pi V^2 \mathfrak{M}_z = -\left(V^2 \frac{m}{n} - \frac{n}{m} - 2\mathfrak{p}_x\right) \mathfrak{H}_y,$$

and then we only have to satisfy condition (XI).

The first one of the herein summarized relation gives nothing new, while the second and third ones read:

$$\mathfrak{E}_y = \sigma \mathfrak{M}_y - j\,\frac{\partial \mathfrak{M}_z}{\partial x} + k\dot{\mathfrak{M}}_z \mathfrak{p}_x, \tag{110}$$

and

$$\mathfrak{E}_z = \sigma \mathfrak{M}_z + j\,\frac{\partial \mathfrak{M}_y}{\partial x} - k\dot{\mathfrak{M}}_y \mathfrak{p}_x. \tag{111}$$

Now, since by the reported formulas

$$\mathfrak{E}_y = -\nu \mathfrak{E}_z \text{ und } \mathfrak{M}_y = -\nu \mathfrak{M}_z$$

it can be written for (110) and (111)

$$\nu(\mathfrak{E}_z - \sigma \mathfrak{M}_z) = j\,\frac{\partial \mathfrak{M}_z}{\partial x} - k\dot{\mathfrak{M}}_z \mathfrak{p}_x,$$

and

$$\mathfrak{E}_z - \sigma \mathfrak{M}_z = -\nu \left(j\,\frac{\partial \mathfrak{M}_z}{\partial x} - k\dot{\mathfrak{M}}_z \mathfrak{p}_x\right),$$

thus at first we find

$$\nu^2 = -1,\ \nu = \pm i,$$

and furthermore

$$4\pi V^2 \frac{n}{m} = \{\sigma \pm i(jm - kn\mathfrak{p}_x)\} \left(V^2 \frac{m}{n} - \frac{n}{m} - 2\mathfrak{p}_x\right). \tag{112}$$

Now, if σ, j, k and n are given, we can determine m from this equation, namely we obtain *two* values, depending on whether we apply the above, or the below sign.

§ 86. We put

$$n = in',\ m = im',$$

by that, equation (112) is transformed into

$$4\pi V^2 \frac{n'}{m'} = \{\sigma \mp (jm' - kn'\mathfrak{p}_x)\} \left(V^2 \frac{m'}{n'} - \frac{n'}{m'} - 2\mathfrak{p}_x\right) \tag{113}$$

.

from which two *real* values are given from m', which we want to denote by m'_1 and m'_2.

For $\nu = +i,\ m' = m'_1$, it becomes now

$$\mathfrak{H}_y = ae^{i(n't - m'_1 x)},\ \mathfrak{H}_z = iae^{i(n't - m'_1 x)},$$

and for $\nu = -i,\ m' = m'_2$,

$$\mathfrak{H}_y = ae^{i(n't - m'_2 x)},\ \mathfrak{H}_z = -iae^{i(n't - m'_2 x)}.$$

If we eventually take the *real* parts, we arrive at the following two particular solutions

$$\mathfrak{H}_y = a\ \cos(n't - m_1'x),\ \mathfrak{H}_z = -a\ \sin(n't - m_1'x), \quad (114)$$

$$\mathfrak{H}_y = a\ \cos(n't - m_2'x),\ \mathfrak{H}_z = a\ \sin(n't - m_2'x), \quad (115)$$

which obviously represent two opposite, circular-polarized light beams of propagation velocities n'/m_1' and n'/m_2'.

The composition of these states of motion leads in a known way to a beam of linear-polarized light, whose oscillation direction gets rotated. Namely, addition of the values (114) and (115) gives the solution

$$\mathfrak{H}_y = 2a\ \cos\frac{1}{2}(m_1' - m_2')x\ \cos\left\{n't - \frac{1}{2}(m_1' + m_2')x\right\},$$

$$\mathfrak{H}_z = 2a\ \sin\frac{1}{2}(m_1' - m_2')x\ \cos\left\{n't - \frac{1}{2}(m_1' + m_2')x\right\}.$$

The rotation ω of the polarisation plane related to unit volume, consequently amounts

$$\omega = \frac{1}{2}(m_1' - m_2').$$

§ 87. If we replace in equation (113), $\mp j$ by α, and $\mp k\mathfrak{p}_x$ by β, it follows

$$4\pi V^2\frac{n'}{m'} = (\sigma + \alpha m' + \beta n')\left(V^2\frac{m'}{n'} - \frac{n'}{m'} - 2\mathfrak{p}_x\right).$$

Since the terms with α, β and $\mathfrak{p}_x$ are in any case very small, the value of *m* following from it, can be represented by a row that progresses with respect to the powers of α, β and $\mathfrak{p}_x$. The first term independent of these magnitudes, has the value

$$m_0' = n' \sqrt{\frac{4\pi}{\sigma} + \frac{1}{V^2}},$$

and then we also find

$$m' = m_0' + \frac{n'}{V^2}\mathfrak{p}_x - \frac{2\pi}{\sigma^2} n'^2 \alpha - \frac{2\pi}{\sigma^2}\frac{n'^3}{m_0'}\beta - \frac{2\pi}{\sigma^2 V^2}\frac{n'^3}{m_0'}\alpha\mathfrak{p}_x +$$

$$+ A\alpha^2 + B\alpha\beta + C\alpha^2\mathfrak{p}_x,$$

where we didn't calculated the three latter terms more closely, and we have neglected all higher powers of α and β, as well as all terms that include $\mathfrak{p}_x^2$. To these latter ones, also the terms with β^2 and $\beta\mathfrak{p}_x$ do belong, since $\beta = \mp k\mathfrak{p}_x$.

Now, we obtain m_1', or m_2', depending on whether we put $\alpha = -j,\ \beta = -k\mathfrak{p}_x$, or $\alpha = +j,\ \beta = +k\mathfrak{p}_x$. The sought rotation of the polarization consequently becomes

$$\omega = \frac{2\pi}{\sigma^2} n'^2 \left(1 + \frac{n'}{m_0'}\frac{\mathfrak{p}_x}{V^2}\right) j + \frac{2\pi}{\sigma^2}\frac{n'^3}{m_0'}\mathfrak{p}_x k,$$

or, when we denote the propagation velocity $\dfrac{n'}{m_0'}$ by *W*,

$$\omega = \frac{2\pi}{\sigma^2} n'^2 \left(1 + \frac{W\mathfrak{p}_x}{V^2}\right) j + \frac{2\pi}{\sigma^2} n'^2 W\mathfrak{p}_x k.$$

The natural rotation of the polarization plane in stationary bodies would consequently be

$$\frac{2\pi}{\sigma^2} n'^2 j; \tag{116}$$

if we were allowed to consider as constant σ and *j*, then it would be, as it follows from the meaning of n', proportional to the square of the oscillation time. It's known that all bodies deviate more or less from this law; but we already know, that σ is changing with the duration of oscillation, and *j* probably might depend on it as well.

The translation has two influences by our equation. First, it changes the already existing rotation in the ratio

$$1 + \frac{W\mathfrak{p}_x}{V^2}, \tag{117}$$

and furthermore it additionally causes a rotation.

$$\frac{2\pi}{\sigma^2} n'^2 W\mathfrak{p}_x k. \tag{118}$$

The theory cannot give a relation between this value and (116); probably such a relation doesn't exist at all, and cases could exist, in which *j* is very small, while *k* nevertheless has a noticeable value.

By the way, it is probably not required to be remarked, that the phenomena represented by (118) are similar to the polarization in so far, as it also only arises by an *exterior* influence, namely by the translation, and most strongly emerges, when this influence has the direction of the light rays.

§ 88. Experiments on the rotation of the polarization plane at different orientation, as far as I know, were only undertaken by MASCART[1] He was unable to conclude a change of rotation with respect to quartz, when the light rays have, on one hand, the direction of Earth's motion, and on the other hand, the opposite direction. For the observation it had to be concluded, that the change in any case didn't amount the 20000th part of the rotation, and as regards a certain direction of the light rays, the rotation was altered by Earth's motion by less than 1/40000.

Due to the lack of a theory applicable for anisotropic bodies, we maybe also apply the above reported formulas to quartz. Now, since the refractive index is 1,55, and $\mathfrak{p}_x/V = 1/10000$, then the value of the second member in (117) becomes 0,000064. The change of rotation caused by that, could not have been overlooked by MASCART, and thus his negative result can only by explained by the assumption, that, in the formula for ω, *k* has a value comparable with j/V^2, and the opposite sign of *j*.

Now, whether (for quartz and other bodies) the two terms containing $\mathfrak{p}_x$ will mutually be cancelled, or whether an observable influence of Earth's motion remains finally, has to be decided by additional investigations.

The interference experiment of Michelson.

§ 89. As it was first noticed by MAXWELL, and which follows from a very simple calculation, the time required by a light ray to travel forth and back between two points *A* and *B* must change, as soon as these points are subject to a common displacement, without dragging the aether. Although the variation is a magnitude of second order, it is nevertheless big enough that it can be demonstrated by means of a sensitive interference method.

The experiment was executed by MICHELSON in the year 1881.[2] His apparatus, a kind of interference-refractor, had two equally long, horizontal, mutually perpendicular arms *P* and *Q*, and from the two mutually interfering light beams, one went forth and back along arm *P* and the other one along arm *Q*. The whole instrument, including the light source and the observation device, could be rotated around a vertical axis, and especially the two locations come into consideration, at which arm *P* or arm *Q* had (so far as possible) the direction of Earth's motion. Now, during the rotation from one "main-position" into the other, a displacement of the interference fringes was expected on the basis of FRESNEL's theory.

However, the change in this displacement caused by the variation of the propagation times — we want to call it MAXWELL's displacement for sake of brevity — was found, and thus MICHELSON thought that he is allowed to conclude that the aether wouldn't remain at rest when the Earth is moving, a conclusion however, whose correctness was soon questioned. By inadvertence, MICHELSON has estimated the change of the phase differences as expected by the theory, to double of the correct value; if we correct this error, we arrive at displacements, which just could be hidden by the observational errors.

Together with Morley, Michelson has started again the investigation,[3] where (to increase the sensitivity) he let reflect every light beam by some mirrors back and forth. This artifice gave the same advantage, as if the arms of the earlier apparatus would have been considerably extended. The mirror was carried by a heavy stone plate, that floated on mercury and thus was easily rotatable. Altogether, every beam had to traverse a path of 22 meters, and by Fresnel's theory, when passing from one main-position to the other, a displacement of 0,4 of the fringe-distance was to be expected. Nevertheless, during the rotation only displacements of at most 0,02 of the fringe-distance were obtained; they probably might stem from observational errors.

Now, is it allowed to assume on the basis of this result, that the aether shares the motion of Earth, and thus Stokes' aberration theory is the correct one? The difficulties, with which this theory is confronted when explaining aberration, seem too great to me as for having that opinion, so I rather should try to remove the contradiction between Fresnel's theory and Michelson's result. Indeed this can be achieved by means of a hypothesis, which I already have spoken out some time ago,[4] and to which, as I found out later, also Fitzgerald arrived.[5] Of which the hypothesis consists, shall be shown in the next §.

§ 90. For simplification we want to assume, that we would work with an instrument as that during the first experiments, and that with respect to one main-position, the arm P coincides exactly with the direction of Earth's motion. Let $\mathfrak{p}$ be the velocity of this motion, and L the length of every arm, thus $2L$ the path of the light rays. Then by the theory [6], the translation causes that the time, in which one light-beam travels forth and back along P, is longer by

$$L \cdot \frac{\mathfrak{p}^2}{V^3}$$

than the time, in which the other beam completes its path. Exactly this difference would also exist, when (without that the translation would have an influence) arm P would be longer by

$$L \cdot \frac{\mathfrak{p}^2}{2V^2}$$

than arm Q. Similar things are true for the second main-position.

Thus we see, that the phase difference expected by the theory could also arise, when (during the rotation of the apparatus) sometimes one, sometimes the other arm would have the greater length. From that if follows, that they can be compensated by opposite variations of the dimensions.

If we assume, that the arm lying in the direction of Earth's motion, is shorter by

$$L \cdot \frac{\mathfrak{p}^2}{2V^2}$$

than the other one, and simultaneously the translation would have an influence which follows from FRESNEL's theory, then the result of Michelson's experiment is fully explained.

Consequently we have to imagine, that the motion of a rigid body, *e.g.* a brass rod or of the stone plate used in later experiments, would have an influence on the dimensions throughout the aether, which, depending on the orientation of the body with respect to the direction of motion, is different. *E.g*, if the dimensions parallel to the direction of motion would be changed in the ratio of 1 to $1+\delta$, and the dimensions perpendicular to them by a ratio of 1 to $1+\epsilon$, than it should be

$$\epsilon - \delta = \frac{\mathfrak{p}^2}{2V^2} \tag{119}$$

Here, the value of one of the magnitudes δ and ϵ would remain undetermined. It could be $\epsilon = 0,\ \delta = -\frac{\mathfrak{p}^2}{2V^2}$, but also $\epsilon = \frac{\mathfrak{p}^2}{2V^2},\ \delta = 0$, or $\epsilon = \frac{\mathfrak{p}^2}{4V^2}$, and $\delta = -\frac{\mathfrak{p}^2}{4V^2}$.

§ 91. As strange as this hypothesis would appear at first sight, nevertheless one must admit that it's not so far off, as soon as we assume that also the molecular forces, similarly as we now definitely can say it of the electrical and magnetic forces, are transmitted through the aether. If this is so, then the translation will change the action between two molecules or atoms most likely in a similar way, as the attraction or repulsion between charged particles. Now, since the shape and the dimensions of a fixed body are, in the last instance, determined by the intensity of the molecular effects, then also a change of the dimensions is inevitable.

Thus from a theoretical perspective there is no objection to the hypothesis. As regards the experimental confirmation, it is to be noticed at first, that the relevant elongations and contractions are extremely small. We have $\mathfrak{p}^2/V^2 = 10^{-8}$, and thus (in case we put $\epsilon = 0$) the contraction of one diameter of Earth would amount ca. 6,5 c.M. The length of a meter rod, however, changes by $^1/_{200}$ Micron (when we bring it from one main-position into the other). If we would like to observe magnitudes so small, then we probably can hope to succeed only by an interference method. Thus we would have to work with two mutually perpendicular rods, and of two mutually interfering light beams, let one travel back and forth with respect to the first rod, and the other with respect to the second rod. By that we come again, however, to MICHELSON's experiment, and we wouldn't observe any displacement of the fringes during the rotation. In reverse as we have expressed it earlier, we could say now, that the displacement stemming from the changes of length, is compensated by MAXWELL's displacement.

§ 92. It is noteworthy, the we are led exactly to the above presupposed changes of dimensions, when we *first* (without consideration of the molecular motion) assume, that in a rigid body which remains at its own, the forces, attractions or repulsions which act on an arbitrary molecule, are mutually in equilibrium, and *second* — for which, however, there is no reason — when we apply to these molecular forces the law which we have derived in § 23 for the electrostatic actions. If we understand by S_1 and S_2, not two systems of charged particles as in that paragraph, but two systems of molecules, — the second at rest and the first with the velocity $\mathfrak{p}$ in the direction of the x-axis —, between whose dimensions the relation given early exists, and if we assume, that in both systems the x-components of the forces are the same, but the y- and z-components are mutually different by the factors given in § 23, then it is clear, that the forces in S_1 will be mutually canceled, as soon as this happens in S_2. Consequently, if S_2 is the state of equilibrium of a stationary, rigid body, then in S_1 the molecules have exactly those positions, in which they can remain under the influence of translation. The displacement would of course cause this configuration by itself, and thus by (24) it would cause a contraction in the direction of motion in the ratio of 1 to $\sqrt{1-\frac{\mathfrak{p}^2}{V^2}}$. This leads to the values

$$\delta=-\frac{\mathfrak{p}^2}{2V^2},\ \epsilon=0,$$

which is in agreement with (119).

In reality the molecules of a body are not at rest, but there exists a stationary motion in every "equilibrium state". As to how this condition is of influence as regards the considered phenomenon, may remain undecided; in any case, due to inevitable observational errors, the experiments of Michelson and Morley let remain a considerable wide margin for the values of δ and ϵ.

The polarization experiments of Fizeau.

§ 93. In the oblique passage of a polarized light beam through a glass plate, the azimuth of the polarization changes in general, namely this phenomenon is depending on the nature of the plate, so that the increase or decrease of its refractive index is followed by a rotation of the polarization plane of the emanating light. This fact was the starting point for the experiments with glass columns, executed by Fizeau[7], whose results deserve our attention to a high degree. The apparatus employed, consisted of a polarized prism, a number of glass columns located after one another, and an analyzer. At the time of solstice, mostly at noon, the devise was turned at first with the polarizator into the east, and with the analyzer into the west, then they were brought into the opposite direction, while in the whole time, a beam of light rays was sent through by means of appropriately located mirrors. Although some irregularities showed up in the settings of the analyzer, yet altogether, a constant difference between the obtained readings for both locations seemed to exist.

When I developed the present theory, I hoped at first to be able to explain this difference, but soon I found myself disappointed in my expectation. If the equations developed by me are correct, then an influence, as the one expected by FIZEAU, cannot exist. The prove for that should be given by the next paragraph.

§ 94. Since we were working with white light, and the rotation of the polarization plane in the glass columns is not the same for all colors, so it was necessary to compensate the dispersion that arose from it. For that, circular-polarizing fluids were used, *e.g.* lemon oil or turpentine, and sometimes thin quartz plates that were cut perpendicular to the axis. For simplicity, we want to assume however, that light is homogeneous and therefore that no such substances are available in the apparatus. The theorem derived in § 59, is then readily applicable as it applies to an arbitrary system of refractive or birefringent bodies.

Now, an ideal experiment with respect to a stationary earth shall be compared with a real experiment, in which the apparatus in relation to Earth's motion is oriented in an arbitrary way. In the first case, the polarizer shall receive rays from the direction s and the oscillation period T; here, we imagine the analyzer thus placed that it does not transmit light. In the latter case the "corresponding" state of motion (§ 59) shall exist. For that, the incident light must have the relative oscillation period T(§ 60 a), and still have the ray-direction s (§ 60, b). Behind the analyzer, it will be dark again (§ 60, *b*), and we may therefore conclude:

Which direction Earth's motion may have, whether from the polarizer to the analyzer, or vice versa, light will always be erased at the presupposed position of the analyzer, as long as nothing is changed as regards the relative period of oscillation and the direction of the rays in relation to the apparatus.

Obviously, these conditions would have been met by the experiments, when the sun would have emitted white light. The relative oscillation period would thus have been as it is required by DOPPLER's law, and namely at each position of the apparatus. As for the direction of the beams in relation to the glass columns, it has probably not been exactly the same with respect to the various readings; however, this has not caused an error, since an influence of a small directional change of the incident light would hardly have been overlooked by the observer.

§ 95. The phenomenon that was expected by FIZEAU and what he really believed to have observed, would have to occur even when using homogeneous light. Thus, here we come to a contradiction, that I can not solve. A source of error, of which one could say that it would have caused the differences in the analyzer locations, I could not discover. The activated circular-polarizing substances were probably a little too thick to allow a prominent influence of Earth's motion considered in § 87. Nor is it possible to think of an effect of terrestrial magnetism. The only thing might yet be, that the two mirrors located east and west of the apparatus, have not always received light of the same nature. Namely, to reflect rays of the sun, sometimes by one, and sometimes by the other mirror, the heliostat had to have different positions; between the angles at which he threw back light in both cases, there was a difference depending on the position of the sun, and we know that light reflected by a metal surface, has not the same composition in all directions of incidence. . Since the mutual position of the mirror was not known to me, I was unable to calculate the influence of this error, and it was only possible to estimate it only superficially, by making an appropriate assumption on that location, and by applying the usual formulas for the metal reflection. In this way, the calculation, however, led to a difference in the analyzer positions with respect to the two locations of the system, but it was clearly smaller than the differences observed by FIZEAU. It should be noted, incidentally, that by one of the experimental series, the heliostat mirror was replaced by a totally reflecting prism and that this seems to have been without influence on the results.

Everything taken together, the question is forced upon us, whether it might be possible to adapt the theory to observations, without ceasing to explain the other phenomena discussed in this work. I haven't succeeded in this, and I must therefore leave the whole question open, in the hope that others might overcome the difficulties that still exist.

That the improvement of the theory will not be so easy, and that the phenomena in the experiments of FIZEAU in any case did not happen in the way, as they were interpreted by him in his introductory observations, this is what I finally want to show.

It will suffice to consider for this purpose, a single glass plate. If we decompose the velocity of translation in two component that are perpendicular to the plate, or parallel, then, if we neglect magnitudes of second order, the effects of those components remain side by side. The problem can thus be reduced to two simpler cases. It is now possible, without making special assumptions about the nature of light oscillations, that a translation perpendicular to the plate, can *not* have the expected influence of FIZEAU; we will derive some general considerations. As for the other direction of translation, we can not speak so determined; it can only be shown, that the moving plate certainly not behaves like a stationary one of somewhat different refractive index.

§ 96. We consider two isotropic media (separated from each other by a plane) whose ponderable parts are either at rest, or move with a common velocity $\mathfrak{p}$ in a direction perpendicular to the marginal surface. If one part of this surface, whose dimensions are considerably larger than the wavelength, is hit by plane waves, which are laterally limited by a cylinder that shares the translation, then the reflection and refraction give rise to two similar light beams. Any theory of aberration has to assume now that, independent of translation, the describing lines of the cylindrical marginal-surfaces, the *relative* light rays, are subject to the ordinary laws of reflection and refraction.

Accordingly, we can once and for all imagine *four* cylinder: 1, 2, 3, 4, as those mentioned above, — we want to say "four *paths of light*" —, of which 1 and 2 are in the first, 3 and 4 in the second medium, and which belong together in the following way. From an incident motion in 1, a reflected one shall emerge in 2, and a transmitted one in 4, while also an incident beam in 3 gives rise to motions in 2 and 4. In reverse, incident oscillations in 2 or 4 will excite motions in paths 1 and 3.

For simplicity, we also assume [8] that the part of the marginal surface that was hit by light, has two symmetry axes that are mutually perpendicular, one of which lies in the plane of incidence of the rays. The figure consisting of four light paths, thus has two symmetry planes which go through one of these axes and the normal to the marginal surface. That one, which coincides with the plane of incidence, may be called the *first*, the other one be called the *second* symmetry plane.

§ 97. Of the deviations from equilibrium that constitutes light, it shall be assumed that they belong to *vector quantities*. If several such variables come into account, such as in the electro-magnetic theory of light: the dielectric polarization, the electric force, the magnetic force, or even the earlier vectors $\mathfrak{D}'$ and $\mathfrak{H}'$, then we have to imagine that for a given body (at a given beam direction, relative oscillation period and translation) these vectors were all determined by one of them. Thus it will be sufficient, to choose *one* of the vectors for consideration. This we call the *light vector* and introduce the following presuppositions, which partly includes a hypothesis about the nature of bodies and light, and partly a limitation in the choice of the light vector.

1°. If a state of motion exists in a system of bodies, in which the components of the light vector are certain functions of relative coordinates and time *t*, thus also the functions that arise when we replace *t* by -*t*, represents the values of the components that correspond to a possible motion. But, in the course of this reversal, we also have to reverse the motion and the velocity $\mathfrak{p}$.

2°. We also arrive at a possible motion, when we take the mirror image of an arbitrary, given motion in relation to a stationary plane, namely in such a way that both the translational velocity, as well as all light-vectors are replaced by the mirror images.

If we are dealing with the pure aether, then we satisfy these conditions if we choose the dielectric displacement as the light vector.

§ 98. In a *polarized* light beam, the light vector is parallel at all points to a certain line; it can be decomposed into three mutually perpendicular components, the first having the direction of the beam, while the second lies in the plane of incidence and the third is perpendicular to it. Now, since the properties of a polarized beam, except the intensity and period of oscillation, only depends on *one* magnitude — such as the azimuths of the polarizer —, then the ratios between the mentioned components must have specific values, as soon as the ratio between the second and third is given; yet this *single* ratio must be allowed to have any arbitrary value. This can also be expressed as: If we decompose the light vector into two components, of which one has the direction of the beam, while the other is perpendicular to it, then the latter can be arbitrarily rotated around the beam, and in every direction, the ratio is determined between the two.

The state of motion is thus completely known, once the nature of the body, the translation, the relative period, the ray direction and finally the direction and magnitude of the "transverse" component of the light vector, are given. At the places where we will later speak of the light vector, we will only think of that transverse component.

Now, if this vector in the incident light is perpendicular to the plane of incidence, it must also have the same direction in the reflected and transmitted beams; in the same way, also the light vector in these beams must be parallel to the plane of incidence, as soon as the light vector of the incident light lies in that plane. To justify these theorems, we only have to consider the mirror image of the entire state of motion in relation to the first plane of symmetry. For example, the light vector of the incident light might have the first of those directions. In the transition to the mirror image, this vector gets the opposite direction, or, as it can also be said, the opposite phase; the light vector of the other two light beams now must be changed in the same way, hence the accuracy of the above claim follows immediately.

The problem is now reduced to the two main cases, *i.e.* that the light vectors are everywhere perpendicular to the plane of incidence, or are everywhere located in its interior. In the course of the further investigation, we always have to think of one of these cases; however, it applies to one case as well as to the others.

As regards each light path, we call a certain direction of the light vector positive, and namely, this direction shall be the same for all the light paths in the first main-case, while in the second main-case the positive directions chosen for 2 and 4 are mirror images of those adopted for 1 and 3 with respect to the second plane of symmetry.

Eventually, in order to represent the vibrations conveniently, we look at two points*P* and *Q*, which on both sides of the border area, lie in a fixed distance from it, at the intersection of two planes of symmetry.

Let *P* belong to the space, in which 1 and 2 are overlapping. Similarly, let *Q* simultaneously lie in 3 and 4. Only values of the light vectors in *P* and *Q* shall be given.

§ 99. If the light vector as regards incident motion has the value

$$q\ \cos\left(2\pi\frac{t}{T}+r\right),$$

it can be represented (for a reflected or transmitted beam that emerges from it) by

$$a\,q\,\cos\left(2\pi\frac{t}{T}+r-b\right),$$

where *a* and *b* are certain constants. In order to mutually distinguish the various cases, we want to append two indices on any of these magnitudes, the first of them is related to the path of the incident light, and the second is related to the beam that arose from it; additionally, also those *a* and *b* which remained without prime, are related to the case, when the translation is directed into the side of the incident light, while the primed letters apply to an equal and opposite displacement.

Let in light path 1 be an incident motion (while the system is progressing into the side of the first medium), at which the light vector has the value

$$\cos\ 2\pi\frac{t}{T}$$

From that, in 2 and 4 the light beams emerge which are represented by

$$a_{1.2}\ \cos\left(2\pi\frac{t}{T}-b_{1.2}\right),$$

and

$$a_{1.4}\ \cos\left(2\pi\frac{t}{T}-b_{1.4}\right),$$

Afterwards, we imagine this state of motion as reversed. First, we thus assume, that the translation is turned away from the first medium, and second, we replace *t* by -*t*. Then we find, that in 1 the light emerges

$$\cos\ 2\pi\frac{t}{T}$$

when in the paths 2 and 4 the incident motions

$$a_{1.2}\ \cos\left(2\pi\frac{t}{T}+b_{1.2}\right) \tag{120}$$

and

$$a_{1.4}\ \cos\left(2\pi\frac{t}{T}+b_{1.4}\right) \tag{121}$$

exist.

However, since the light vector, which is generated by the motion (120) in the first path, has the value

$$a_{1.2}a'_{2.1}\cos\left(2\pi\frac{t}{T}+b_{1.2}-b'_{2.1}\right)$$

and also the light vector emerging form (121), is to be replaced by

$$a_{1.4}a'_{4.1}\cos\left(2\pi\frac{t}{T}+b_{1.4}-b_{4.1}\right),$$

then it is given

$$a_{1.2}a'_{2.1}\cos\left(2\pi\frac{t}{T}+b_{1.2}-b'_{2.1}\right)+a_{1.4}a'_{4.1}\cos\left(2\pi\frac{t}{T}+b_{1.4}-b_{4.1}\right)=$$

$$=\cos\ 2\pi\frac{t}{T}$$

From that if follows

$$a_{1.2}a'_{2.1}\cos(b_{1.2}-b'_{2.1})+a_{1.4}a_{4.1}\cos(b_{1.4}-b_{4.1})=1\,, \quad (122)$$

and

$$a_{1.2}a'_{2.1}\sin(b_{1.2}-b'_{2.1})+a_{1.4}a_{4.1}\sin(b_{1.4}-b_{4.1})=0\,, \quad (123)$$

§ 100. The following remark leads to a simple relation. If we start by a condition, at which the incident light follows path 1, and if we take the mirror image with respect to the second plane of symmetry (§ 96), then we arrive at a condition, at which the light is incident in 2. Consequently it has to be

$$a_{2.1}=a_{1.2}\,,\ b_{2.1}=b_{1.2} \quad (124)$$

and in the same way

$$a'_{2.1}=a'_{1.2}\,,\ b'_{2.1}=b'_{1.2}\,. \quad (125)$$

For the difference $b_{1.2}-b'_{2.1}$ which comes into (123), we may put $b_{1.2}-b'_{1.2}$, which is evidently of order $\mathfrak{p}/V$, since the magnitudes $b_{1.2}$ and $b'_{1.2}$ are only different form one another by having different directions of translation.

By (123), also $\sin(b_{1.4} - b_{4.1})$ must now be of order $\mathfrak{p}/V$. Since we additionally (without changing anything of the matter) can increase or decrease $b_{4.1}$ by a multiple of π, and also an uneven multiple of π as long as the sign of $a_{4.1}$ is reversed, then we may assume, that also the angle $b_{1.4} - b_{4.1}$ itself is of order $\mathfrak{p}/V$. The two cosines in (122) thus differ from unity only by magnitudes of second order, so that we may put

$$a_{1.2}\, a'_{2.1} + a_{1.4}\, a_{4.1} = 1.$$

In the same way

$$a'_{1.2}\, a_{2.1} + a'_{1.4}\, a'_{4.1} = 1,$$

and under consideration of (124) and (125) we thus find

$$a_{1.4}\, a_{4.1} = a'_{1.4}\, a'_{4.1}.$$

Now suppose, similarly to the experiment of FIZEAU, a plan-parallel glass plate (at whose two sides the aether is located) will be hit in oblique direction by a light beam, whose light vector has one of the directions previously distinguished, *i.e.* that it is polarized either in the plane of incidence, or perpendicular to it. The relation, be which the amplitude is diminished during the entrance, can thus be (depending in the direction of translation) represented by $a_{1.4}$ or $a'_{1.4}$, and also, as we can easily see, by the corresponding relation when leaving the plate by $a_{4.1}$ or $a'_{4.1}$. Altogether, the amplitude is thus altered in the ratio of 1 to $a_{1.4}\, a_{4.1}$ or $a'_{1.4}\, a'_{4.1}$. Now, since these products have the same value, the reversal of the translation changes nothing of the intensity of the leaving light, which consequently must be (except magnitudes of second order) the same, as if the plate would stand still: This is true for both main-positions of the polarization plane; consequently, when the incident rays are linearly polarized in an arbitrary way, the oscillation direction of the transmitted light is independent of the translation.

Here, it is to be noticed, that for the plane of incidence, as well as for the component polarized perpendicularly to the plane of incidence, we have to assume the dragging coefficient of FRESNEL. Thus both are propagating with the same velocity, by which a phase difference between them and an elliptic polarization of the transmitted light is excluded.

§ 101. If the direction of translation is, as it was assumed in the last paragraph, not parallel to the marginal surface, but parallel to it, thus it must be distinguished, whether it lies in the plane of incidence, or perpendicular to it. We only want to consider the first case, and additionally restrict ourselves to the plane of incidence of polarized light.

At first it should be remembered, as to how we arrive to the value of the reflected amplitude for such light. If we choose the marginal surface with respect to *y z*-, and the plane of incidence with respect to the *x z*-plane, and we argue on the basis of the electromagnetic theory, then we have to put $\mathfrak{E}_x = \mathfrak{E}_z = 0$, and also $\mathfrak{H}_y = 0$, while the marginal conditions consist of the continuity $\mathfrak{E}_y$, $\mathfrak{H}_x$ and $\mathfrak{H}_z$. Since in every of both media it is given by equation (IV_c) (§ 52)

$$\frac{\partial \mathfrak{H}_x}{\partial t} = \frac{\partial \mathfrak{E}_y}{\partial z}, \text{ und } \frac{\partial \mathfrak{H}_z}{\partial t} = -\frac{\partial \mathfrak{E}_y}{\partial x},$$

the the continuity of $\mathfrak{H}_x$ and $\mathfrak{H}_z$ has the same meaning as the continuity of $\partial \mathfrak{E}_y / \partial z$ and $\partial \mathfrak{E}_y / \partial x$. The first of those derivatives, however, will be steady, as soon as $\mathfrak{E}_y$ has this property itself, and at the end we are only dealing with $\mathfrak{E}_y$ and $\dfrac{\partial \mathfrak{E}_y}{\partial x}$.

Indeed — and this remark is true for every light theory — the known formula of FRESNEL is given, when we assume, that this or that magnitude that come into consideration as regards to oscillations, and simultaneously its derivative with respect to the normal of the marginal surface, is steady.

As regards plane waves, the differentiation with respect to *x* amounts to the same, as if we would differentiate with respect to *t*, and then multiply by a factor *m* dependent on the direction and velocity of the waves. If we denote (for the incident, reflected and transmitted light) the values of the magnitude just mentioned in the immediate vicinity of the marginal surface by

$$\varphi_1(t), \varphi_1'(t) \text{ and } \varphi_2(t),$$

and the values of *m* by

$$m_1, m_1' \text{ und } m_2$$

then we obtain as marginal conditions

$$\varphi_1(t) + \varphi_1'(t) = \varphi_2(t)$$

and

$$m_1 \frac{\partial \varphi_1(t)}{\partial t} + m_1' \frac{\partial \varphi_1'(t)}{\partial t} = m_2 \frac{\partial \varphi_2(t)}{\partial t}.$$

The last formula leads — when we neglect additive constants — to

$$m_1\varphi_1(t) + m_1'\varphi_1'(t) = m_2\varphi_2(t),$$

and it is further given by elimination of $\varphi_2(t)$

$$\varphi_1'(t) = \frac{m_1 - m_2}{m_2 - m_1'}\varphi_1(t).$$

Now, that the amplitude of the reflected beam (at constant direction of the incident light) depends on the refractive index of the second body, stems from the fact, that, as it can easily be seen, m_2 changes with this exponent.

Now, in the next paragraph it should be demonstrated, that this m_2 (as long as the direction of the incident relative ray remains the same) is not affected by a translation in the direction of the *z*-axis. If it would be allowed to assume, that also with respect to a moving plate, the marginal conditions consist of the continuity of a certain magnitude φ and its derivatives, then, at least for light polarized in the plane of incidence, we would have demonstrated the impossibility of the phenomenon sought by FIZEAU. However, in reality the assumption on the marginal conditions is not allowed without closer investigation; the things said show at least, however, that the moving plate in no ways acts as a stationary one of somewhat different refractive index.

§ 102. Let, with respect to the previously introduced axes,

$$\cos\alpha,\ 0,\ \sin\alpha$$

be the direction constants of the rays incident on the plate. Neglecting magnitudes of second order, we consequently obtain the direction of the wave normal by application of the fundamental law of aberration; namely we have to compose a velocity V in the direction of the rays with a translational velocity $\mathfrak{p}$. Now, if the latter is parallel to the z-axis, then the direction constants of the wave normal become,

$$\cos\alpha',\ 0,\ \sin\alpha'$$

where

$$\alpha' = \alpha + \frac{\mathfrak{p}_z}{V}\cos\alpha$$

The absolute velocity of the waves is V; however, the relative velocity V' will be found, when we diminish V by the component of $\mathfrak{p}$ with respect to the wave normal. If we understand by x, y, z relative coordinates, then for the incident light, expressions of the form

$$A\ \cos\frac{2\pi}{T}\left(t-\frac{x\ \cos\alpha' + z\ \sin\alpha'}{V'}+B\right)$$

apply, or

$$A\ \cos\frac{2\pi}{T}\left(t-\frac{x\ \cos\alpha + z\ \sin\alpha}{V}-\frac{\mathfrak{p}_z z}{V^2}+B\right) \qquad (126)$$

On the other hand, for glass we have to assume FRESNEL's dragging coefficient. Consequently, when we denote the propagation velocity in stationary glass by W, and the directions constants of the wave normal in the plate by

$$\cos\beta,\ 0,\ \sin\beta$$

we have to put for the relative velocity of the waves with respect to glass, by (82),

$$W' = W - \mathfrak{p}_x \sin\beta \frac{W^2}{V^2} \tag{127}$$

To light in the plate an expression applies now, which has the form:

$$A' \ \cos\frac{2\pi}{T}\left(t - \frac{x\ \cos\beta + z\ \sin\beta}{W'} + B'\right) \tag{128}$$

and those will follow the incident oscillations in all points of the marginal surface, when the coefficient of *z* is the same as in formula (126).

Therefore we have

$$\sin\beta = \left(W - \mathfrak{p}_z \sin\beta \frac{W^2}{V^2}\right)\left(\frac{\sin\alpha}{V} + \frac{\mathfrak{p}_z}{V^2}\right),$$

or we denote the refraction angle in the stationary plate by β_0, so that

$$\sin\beta_0 = \frac{W}{V}\sin\alpha$$
$$\sin\beta = \sin\beta_0 + \frac{W\mathfrak{p}_z}{V^2}\cos^2\beta_0.$$

From that it follows

$$\cos\beta = \cos\beta_0 - \frac{W\mathfrak{p}_z}{V^2}\sin\beta_0\cos\beta_0 \tag{129}$$

However, for the factor which we above have called m_2, the value is given by (128)

$$-\frac{\cos\beta}{W'},$$

and this one, in consequence of (127) and (129), actually is independent of the translation.

1. ↑ Mascart. Ann. de l'école normale, 2e sér., T. 1, pp. 210—214, 1872.
2. ↑ Michelson. American Journal of Science, 3d Ser., Vol. 22, p. 120, 1881.
3. ↑ Michelson and Morley. American Journal of Science, 3d Ser., Vol. 34, p. 333, 1887; Phil. Mag. 5th Ser., Vol 24, p. 449, 1887.
4. ↑ Lorentz. Zittingsverslagen der Akad. v Wet. te Amsterdam, 1893—93, p. 74.
5. ↑ As Fitzgerald was so friendly to tell me, that he dealt with this hypothesis already for a longer time in his lectures. In the literature, I only found it mentioned by Lodge, in the treatise „Aberration problems" (London Phil. Trans, Vol. 184, A, p. 727, 1893). I allow myself, to also add at his place, that this treatise, besides some theoretical considerations, also contains the description of very interesting experiments, in which two discs of metal (Diameter 1 Yard) perpendicularly fixed on that axis, were rotated with great velocity. By means of a certain interference method it was investigated, whether the aether that was present between the discs, was co-rotating; the result

was negative, even though the number of rotations in a second was increased up to 20 or more. LODGE concludes, that the discs haven't communicated to the aether the 800th part of their velocity.

6. ↑ see LORENTZ, Arch. néerl., T. 21, pp. 168—176, 1887.
7. ↑ FIZEAU. Ann. de chim. et de phys., 3e sér., T. 58, p. 129. 1860; Pogg. Ann., Vol. 114, p. 554, 1861.
8. ↑ This assumption can be dropped later, since the ratio of the intensities of the light beams is independent of the size and shape of the cross sections.

Collection of the most important expressions

ρ	Charge of an ion.
m	Mass of an ion.
V	Velocity of light in the aether.
t	Time.
t'	Local time (§ 31).
T	Oscillation period.
$\mathfrak{v}$	Velocity of an ion.
$\mathfrak{p}$	Translational velocity of ponderable matter.
$\mathfrak{q}$	Displacement of an ion from its equilibrium position.
$\mathfrak{m}$	Electric moment of a molecule.
$\mathfrak{M}$	Electric moment of the unit volume of ponderable matter.
$\mathfrak{d}$	Dielectric displacement in the aether.
$\mathfrak{D}$	Dielectric polarization in a ponderable body.
$\mathfrak{S}$	Electric current.
$\mathfrak{E}$	Electric force.
$\mathfrak{F}$	Electric force for stationary ions.
$\mathfrak{H}$	Magnetic force.

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