Electrodynamics 001

Notes by John D Norton

Electric Field Quantities and Units

Coulomb's Law

Force between 2 charges $F = \dfrac{q_1 q_2}{r^2}$

define: unit charge produces force 1 dyne at 1cm

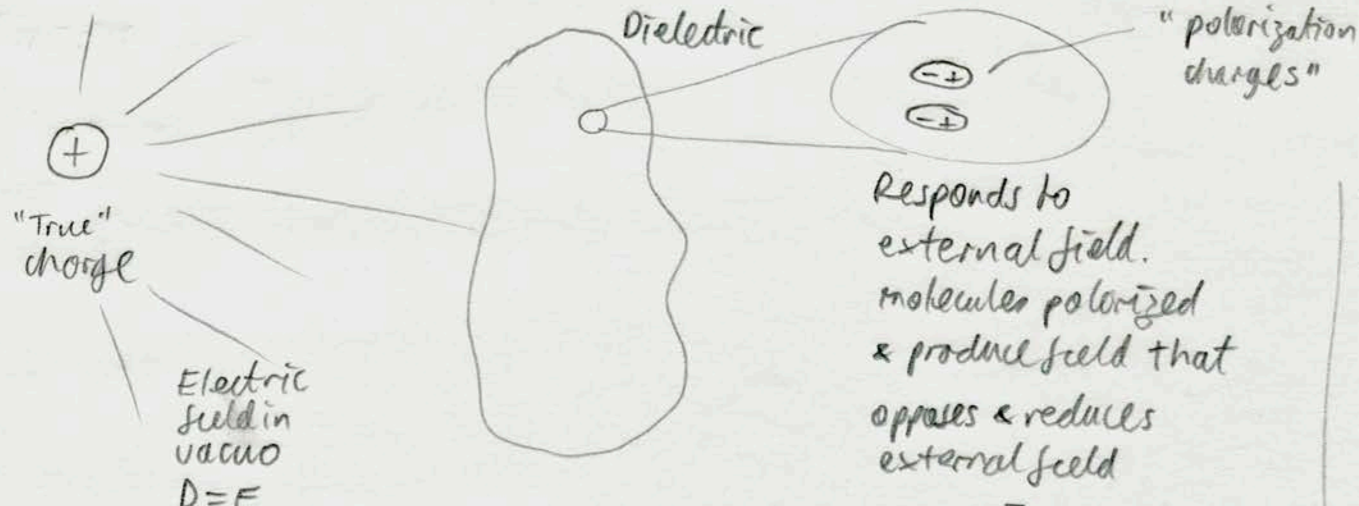
r: distance

Gaussian (CGS) units

units: Coulomb defined by forces between current carrying conductors

$F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{r^2}$

MKSA (Giorgi) units



Responds to external field. molecules polarized & produce field that opposes & reduces external field

⇩

	Electric Displacement		Electric Field		Dielectric Polarization
Gaussian:	$\underset{\sim}{D}$	$=$	$\underset{\sim}{E}$	$+$	$4\pi \underset{\sim}{P}$
MKSA:	$\underset{\sim}{D}$	$=$	$\varepsilon_0 \underset{\sim}{E}$	$+$	$\underset{\sim}{P}$
	Due to true charges		Due to all charges		Due to polarisation charges

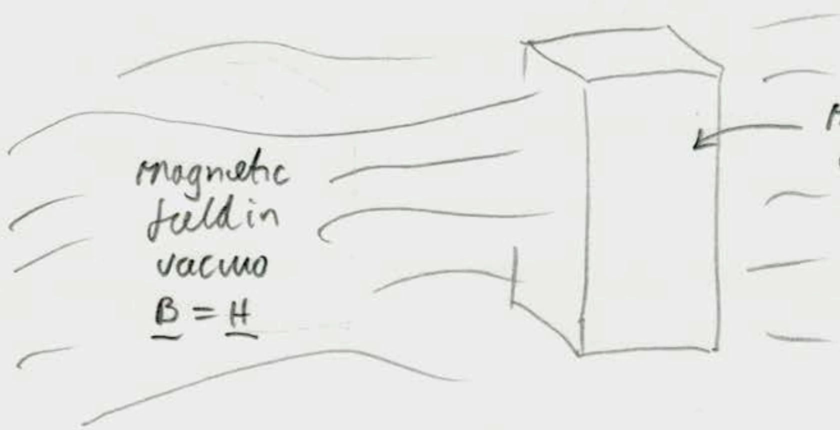
Magnetic Field Quantities and Units

Lorentz Force Law:

Force on charge q moving at $\mathbf{v}$ due to fields $\mathbf{E}$, $\mathbf{B}$

Implicitly sets units for $\mathbf{B}$

Gaussian	MKSA
$\mathbf{F} = q\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B}\right)$	$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$



medium generates own field in response to external field or independently of it (e.g. permanent magnet)

	magnetic induction		magnetic Field		magnetization
Gaussian	$\mathbf{B}$	$=$	$\mathbf{H}$	$+$	$4\pi\mathbf{M}$
MKSA	$\mathbf{B}$	$=$	$\mu_0\mathbf{H}$	$+$	$\mathbf{M}$
	Field due to true magnetic sources		measured field		Due to field of medium

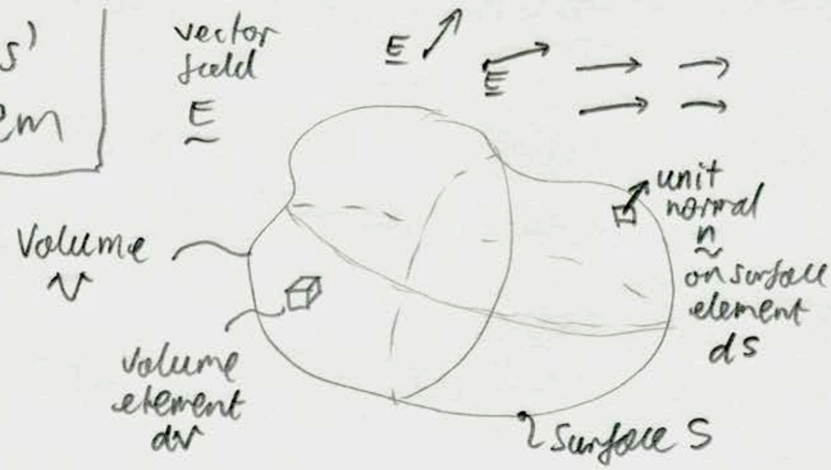
There are none so $\nabla \cdot \mathbf{B} = 0$

Why $\mathbf{B}$ and not $\mathbf{H}$ in Lorentz Force Law?

Becker, p.162 : Expt. shows $\mathbf{B}$ is the one to use!?

Useful Differential-Vector theorems

Gauss' Theorem



$\underset{\sim}{E} \cdot \underset{\sim}{n} = E_n$
is component of $\underset{\sim}{E}$ orthogonal to dS

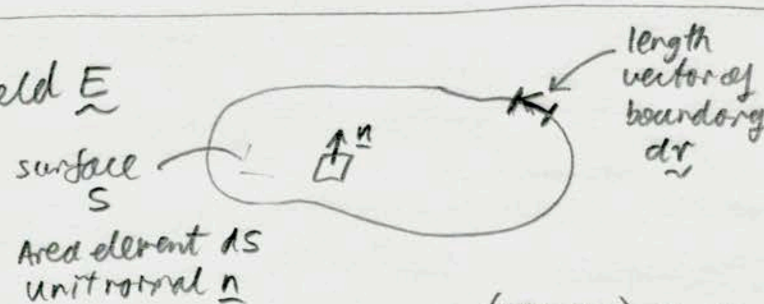
$\underset{\sim}{\nabla} \cdot \underset{\sim}{E} = \underline{\text{div } E}$

$= \dfrac{\partial E_x}{\partial x} + \dfrac{\partial E_y}{\partial y} + \dfrac{\partial E_z}{\partial z}$

$$\iint_S \underset{\sim}{E} \cdot \underset{\sim}{n} \, ds = \iiint_V \underset{\sim}{\nabla} \cdot \underset{\sim}{E} \, dv$$

Stokes' Theorem

Vector field $\underset{\sim}{E}$

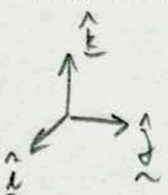


$$\oint_S \underset{\sim}{E} \cdot d\underset{\sim}{r} = \iint_S (\underset{\sim}{\nabla} \times \underset{\sim}{E}) \cdot \underset{\sim}{n} \, ds$$

$(\underline{\nabla} \times \underline{E}) \cdot \underline{n}$ is component of $\underline{\nabla} \times \underline{E}$ normal to S

$\underline{\nabla} \times \underline{E} = \underline{\text{Curl } E}$

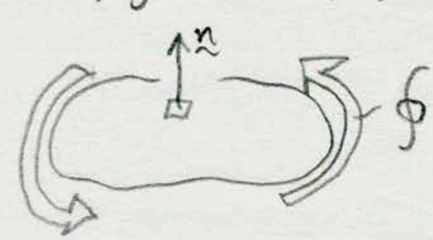
Unit vectors



$$\underset{\sim}{\nabla} = \hat{\underset{\sim}{i}} \frac{\partial}{\partial x} + \hat{\underset{\sim}{j}} \frac{\partial}{\partial y} + \hat{\underset{\sim}{k}} \frac{\partial}{\partial z}$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{E} = \begin{vmatrix} \hat{\underset{\sim}{i}} & \hat{\underset{\sim}{j}} & \hat{\underset{\sim}{k}} \\ \partial_x & \partial_y & \partial_z \\ E_x & E_y & E_z \end{vmatrix}$$

Direction of path of integration given by right hand screw rule



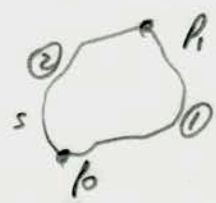
Vector field $\underline{E}$ irrotational $\underline{\nabla}\times\underline{E}=0$	$\Longleftrightarrow$	Field $\underline{E}$ can be generated as gradient of a potential field φ $\underline{E} = -\underline{\nabla}\varphi = -\underline{\text{grad}}\,\varphi$

Proof $\Longleftarrow$ If $\underline{E} = -\underline{\nabla}\varphi$

then $\underline{\nabla}\times\underline{E} = -\underline{\nabla}\times\underline{\nabla}\varphi = 0$ by a standard vector identity

$\Longrightarrow$ Assume $\underline{\nabla}\times\underline{E}=0$ everywhere

Pick P_0 and P_1 arbitrarily and any simple closed path on which they lie

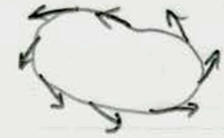


From Stokes theorem $\oint_{①②} \underline{E}\cdot d\underline{r} = \iint_S \underline{\nabla}\times\underline{E} = 0$

$\therefore \int_{\substack{P_0\to P_1\\ ①}} \underline{E}\cdot d\underline{r} = \int_{\substack{P_0\to P_1\\ ②}} \underline{E}\cdot d\underline{r}$ i.e. The integral is path independent set $\varphi(P_0)=0$ and define $\varphi(P_1) = -\int_{P_0\to P_1} \underline{E}\cdot d\underline{r}$ on any path

$\varphi(P)$ so defined satisfies $-\underline{\nabla}\varphi = -\underline{\nabla}\int \underline{E}\cdot d\underline{r} = \underline{E}$

Spin offs | If $\underline{\nabla}\times\underline{E}\neq 0$, then $\underline{E}$ admits circulation. i.e. closed paths along which $\oint \underline{E}\cdot d\underline{r} \neq 0$



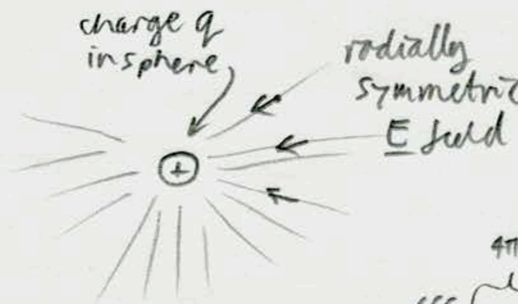
$\underline{E}$ is a force field, $\varphi(P_1)-\varphi(P_1)$ = Potential energy needed to move body from P_1 to P_2

Electrostatic Field Equations

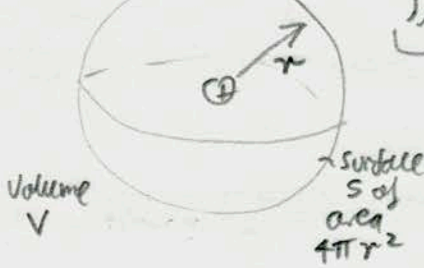
Distribution of charge density ρ produces irrotational $\underline{E}$ field (in vacuo)

$$\underset{\sim}{\nabla}\cdot \underline{E} = 4\pi\rho \qquad \underline{\nabla}\times\underline{E} = 0$$

Recover Coulomb's law via Gauss' theorem



Apply Gauss' theorem to large sphere enclosing charge



$$\underbrace{\iiint \overbrace{\underline{\nabla}\cdot\underline{E}}^{4\pi\ \text{positive charge is}}\, dv}_{4\pi q} = \iint_S \underset{\sim}{E}\cdot\underline{n}\, ds$$

$$4\pi q = \left(\begin{array}{c}\text{component}\\ \text{of } E\\ \text{directed}\\ \text{to center}\end{array}\right)\cdot\left(4\pi r^2\right)$$

$\therefore$ Force unit test charge $= q/r^2$

Delivers one of Maxwell's equations

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{D} = 4\pi\rho \qquad\qquad \underset{\sim}{\nabla}\cdot\underset{\sim}{D} = \rho$$

Gaussian MKSA

in media, $\underline{D}$ is field due to true charges ρ

Magnetostatic Field Equations

No true magnetic monopoles $\therefore$ magnetic induction $\underset{\sim}{B}$ satisfies } Field due to true sources

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0 \quad \text{and} \quad \underset{\sim}{\nabla} \times \underset{\sim}{B} = 0$$

no sources

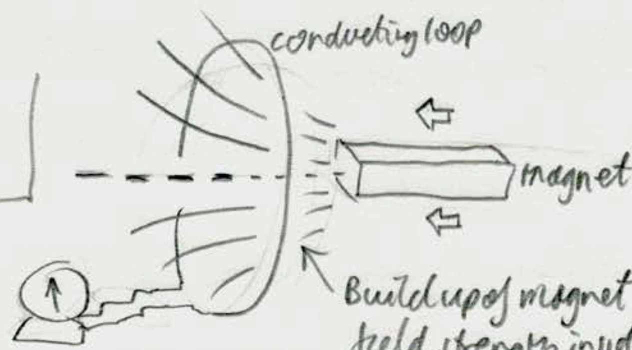
Delivers one of Maxwell's Equations

$$\boxed{\underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0}$$

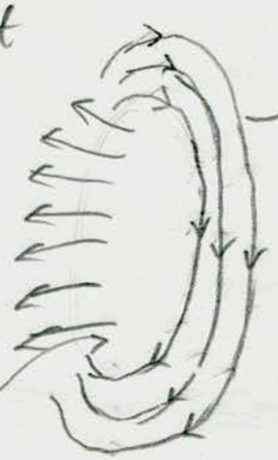
Gaussian and MKSA

move to dynamical fields :

Faraday Induction

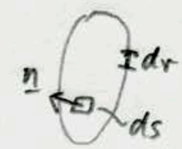


Abstract



$\cdots \quad \underline{\nabla} \times \underline{E} \neq 0$

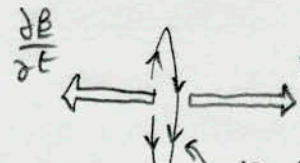
current produced $\propto$ rate growth total magnetic flux in loop



$$\oint \underset{\sim}{E} \cdot d\underset{\sim}{r} = -\frac{1}{c} \frac{d}{dt} \int (\underset{\sim}{B} \cdot \underset{\sim}{n}) \, ds$$

True for loops of any size in field
$\therefore$ we can use Stokes' theorem in reverse order to recover a Maxwell equation

Direction of E field:



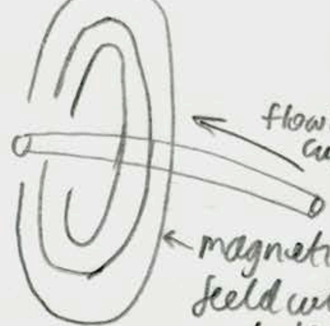
$\underline{\nabla} \times \underline{E} = -\frac{\partial B}{\partial t}$

$$\boxed{\underline{\nabla} \times \underline{E} = -\frac{1}{c} \frac{\partial B}{dt} \qquad \underline{\nabla} \times \underline{E} = -\frac{\partial B}{\partial t}}$$

Gaussian $\qquad$ MKSA

Why $\underline{E}$ and not $\underline{D}$?
Faraday induction con field not $\underline{P}$?

Oersted/ Ampere

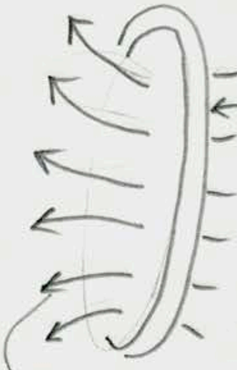


For loop enclosing current flow

$$\oint \underline{H} \cdot d\underline{r} = \frac{4\pi}{c} \underset{\sim}{i}$$

from current density $\underset{\sim}{j}$
via $\underset{\sim}{i} = \iint_S \underset{\sim}{j}\, ds$

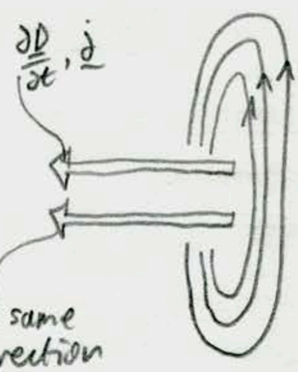
Maxwell's Displacement current (Analog of Faraday induction)



$$\oint \underline{H} \cdot d\underline{r} = \frac{1}{c} \iint_S \frac{\partial D}{dt}\, ds$$

True for loops of all sizes. combine effects

Direction $\underline{H}$



$$\underline{\nabla} \times \underline{H} = \frac{1}{c}\frac{\partial \underline{D}}{dt} + \frac{4\pi}{c}\underset{\sim}{j}$$

Gaussian

$$\underline{\nabla} \times \underline{H} = \frac{\partial D}{dt} + \underset{\sim}{j}$$

MKSA

Why $\underline{H}, \underline{D}$ and not $\underline{E}, \underline{B}$?
Conjecture: mostly to do with conventions in defining $\underline{H}, \underline{D}, \underline{E}, \underline{B}$.

Maxwell's Equations in vacuo (Gaussian)

$\underline{\rho}=0 \quad \underline{m}=0 \quad \therefore \underline{D}=\underline{E} \quad \underline{B}=\underline{H}$

$$\underline{\nabla}\cdot\underline{E} = 4\pi\rho \qquad \underline{\nabla}\cdot\underline{H}=0$$

$$\underline{\nabla}\times\underline{E} = -\frac{1}{c}\frac{\partial H}{\partial t} \qquad \underline{\nabla}\times\underline{H} = \frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c}\underline{j}$$

Einstein 1905

(1) Write in components $\underset{\sim}{E} = (X, Y, Z)$

$\underline{H} = (L, M, N)$

(2) Redefine $\rho \longrightarrow 4\pi\rho$

(Said explicitly in 1905 (as definition?)

" $\rho = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z}$ = 4π-fold density of electricity "

Einstein, Relativitätsprinzip, 1907, p.427

$\underline{\nabla}\cdot\underline{E} = \rho$ is $\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = \rho$

$\underline{\nabla}\cdot\underline{H} = 0$ is $\frac{\partial L}{\partial x} + \frac{\partial M}{\partial y} + \frac{\partial N}{\partial z} = 0$

$\underline{\nabla}\times\underline{E} = -\frac{1}{c}\frac{\partial H}{\partial t}$ is $\begin{vmatrix} \hat{\underline{i}} & \hat{\underline{j}} & \hat{\underline{k}} \\ \partial_x & \partial_y & \partial_z \\ X & Y & Z \end{vmatrix} = -\frac{1}{c}\left(\partial_t L\,\hat{\underline{i}} + \partial_t M\,\hat{\underline{j}} + \partial_t N\,\hat{\underline{k}}\right)$

i.e. $\partial_y Z - \partial_z Y = -\frac{1}{c}\partial_t L$

$\partial_z X - \partial_x Z = -\frac{1}{c}\partial_t M$

$\partial_x Y - \partial_y X = -\frac{1}{c}\partial_t N$

$4\pi \times$ charge density

velocity of charges

so $\rho \underline{u} = 4\pi \underline{j}$

$$\underline{\nabla} \times \underline{H} = \frac{1}{c}\left(\frac{\partial E}{\partial t} + \rho \underset{\sim}{\underline{u}}\right) \quad \text{is}$$

$$\begin{vmatrix} \underline{\hat{i}} & \underline{\hat{j}} & \underline{\hat{k}} \\ \partial_x & \partial_y & \partial_z \\ L & M & N \end{vmatrix} = \frac{1}{c}\left(\left(\partial_t X + \rho u_x\right)\underset{\sim}{\hat{i}} + \left(\partial_t Y + \rho u_y\right) + \cdots\right)$$

i.e.
$$\partial_y N - \partial_z M = \frac{1}{c}\left(\partial_t X + \rho u_x\right)$$
$$\partial_z L - \partial_x N = \frac{1}{c}\left(\partial_t Y + \rho u_y\right)$$
$$\partial_x N - \partial_z L = \frac{1}{c}\left(\partial_t Z + \rho u_z\right)$$

Sample Calculation of Lorentz Covariance

Some field $\varphi(x,y,z,t)$ satisfies

$$\Box^2\varphi = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right)\varphi = 0$$

↑solutions are waves the propagate at c

e.g. Let $\varphi(x,y,z,t) = f(x-ct)$

↑any function

$$\left[\begin{array}{l} \frac{1}{c^2}\partial_t\partial_t f(x-ct) = \frac{1}{c^2}c^2 f''(x-ct) = f''(x-ct) \\ \partial_{xx} f(x-ct) = f''(x-ct) \end{array}\right]$$

$\therefore$ This solves $\Box^2\varphi = 0$

Show $\Box^2\varphi = 0$ is preserved under change of reference frame:

$$\begin{array}{lcl} x' = \gamma(x - vt) & & x = \gamma(x' + vt') \\ y' = y \quad z' = z & \text{with inverse} & y = y' \quad z = z' \\ t' = \gamma(t - \frac{v}{c^2}x) & & t = \gamma(t' + \frac{v}{c^2}x') \end{array} \qquad \gamma = 1/\sqrt{1 - v^2/c^2}$$

From transformation, recover:

$$\frac{\partial}{\partial x} = \underbrace{\frac{\partial x'}{\partial x}}_{\gamma}\cdot\frac{\partial}{\partial x'} + \underbrace{\cancel{\frac{\partial y'}{\partial x}}}_{0}\cdot\frac{\partial}{\partial y'} + \underbrace{\cancel{\frac{\partial z'}{\partial x}}}_{0}\cdot\frac{\partial}{\partial z'} + \underbrace{\frac{\partial t'}{\partial x}}_{-\frac{v}{c^2}\gamma}\cdot\frac{\partial}{\partial t'} = \gamma\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right)$$

$$\frac{\partial}{\partial t} = \underbrace{\frac{\partial x'}{\partial t}}_{-\gamma v}\cdot\frac{\partial}{\partial x'} + \underbrace{\cancel{\frac{\partial y'}{\partial t}}}_{0}\cdot\frac{\partial}{\partial y'} + \underbrace{\cancel{\frac{\partial z'}{\partial t}}}_{0}\cdot\frac{\partial}{\partial z'} + \underbrace{\frac{\partial t'}{\partial t}}_{\gamma}\cdot\frac{\partial}{\partial t'} = \gamma\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right)$$

similarly $\frac{\partial}{\partial y'} = \frac{\partial}{\partial y} \qquad \frac{\partial}{\partial z'} = \frac{\partial}{\partial z}$

Abbreviating $\frac{\partial}{\partial x} = \partial_x$ etc:

$$\Box^2 \varphi = 0$$

is $$\frac{1}{c^2}\partial_t\partial_t\varphi - \partial_x\partial_x\varphi - \partial_y\partial_y\varphi - \partial_z\partial_z\varphi = 0$$

$\Big\Downarrow$ substitute for new coordinates in differential operators

$$\frac{1}{c}\gamma^2\left(\partial_{t'} - v\partial_{x'}\right)\left(\partial_{t'} - v\partial_{x'}\right)\varphi - \gamma^2\left(\partial_{x'} - \frac{v}{c^2}\partial_{t'}\right)\left(\partial_{x'} - \frac{v}{c^2}\partial_{t'}\right)\varphi - \partial_{y'}\partial_{y'}\varphi - \partial_{z'}\partial_{z'}\varphi = 0$$

$$\frac{1}{c^2}\gamma^2\left(\partial_{t'}\partial_{t'} - 2v\partial_{x'}\partial_{t'} + v^2\partial_{x'}\partial_{x'}\right) \qquad \gamma^2\left(\partial_{x'}\partial_{x'} - 2\frac{v}{c^2}\partial_{x'}\partial_{t'} + \frac{v^2}{c^4}\partial_{t'}\partial_{t'}\right)$$

$$\gamma^2\left(\frac{\left(1-\frac{v^2}{c^2}\right)\partial_{t'}\partial_{t'}}{c^2} - \cancel{2\frac{v}{c^2}\partial_{x'}\partial_{t'}} + \cancel{2\frac{v}{c^2}\partial_{x'}\partial_{t'}} - \left(1-\frac{v^2}{c^2}\right)\partial_{x'}\partial_{x'}\right)\varphi$$

$\uparrow$

$1/\left(1-\frac{v^2}{c^2}\right)$

$$\left(\frac{1}{c^2}\partial_{t'}\partial_{t'} - \partial_{x'}\partial_{x'}\right)\varphi$$

combining:

$$\left(\frac{1}{c^2}\partial_{t'}\partial_{t'} - \partial_{x'}\partial_{x'} - \partial_{y'}\partial_{y'} - \partial_{z'}\partial_{z'}\right)\varphi = 0$$

Posit that φ in new coordinate system has same value at point-event with coords (x, y, z, t) / (x', y', z', t') $\Big\}\ \varphi' = \varphi$

$$\therefore \left(\frac{1}{c^2}\partial_{t'}\partial_{t'} - \partial_{x'}\partial_{x'} - \partial_{y'}\partial_{y'} - \partial_{z'}\partial_{z'}\right)\varphi' = 0$$

The eigenvalue equation

$$\sum T_{ik} a_k = \lambda a_i \quad \text{for} \quad i = 1, 2, 3$$

gives with

$$\begin{vmatrix} T_{11}-\lambda & T_{12} & T_{13} \\ T_{21} & T_{22}-\lambda & T_{23} \\ T_{31} & T_{32} & T_{33}-\lambda \end{vmatrix} = 0$$

the three eigenvalues, λ^{I}, λ^{II}, λ^{III} of the tensor and the associated principal-axis directions $\mathbf{a}^{\mathrm{I}}$, $\mathbf{a}^{\mathrm{II}}$, $\mathbf{a}^{\mathrm{III}}$. Referred to these three mutually perpendicular directions as coordinate axes, it follows that

$$T_{11} = \lambda^{\mathrm{I}} \qquad T_{22} = \lambda^{\mathrm{II}} \qquad T_{33} = \lambda^{\mathrm{III}} \qquad T_{ik} = 0 \quad \text{for} \quad i \neq k$$

A tensor with spur zero is designated as the deviator: here the spur, being invariant against rotations of coordinates, is defined by $Sp(\mathbf{T}) = T_{11}+T_{22}+T_{33}$. The associated deviator is obtained from a tensor **T** by subtracting one-third of the spur from all components T_{ii}.

CHAPTER G II

Electrodynamics

The Gaussian CGS system and the Giorgi MKSA system are employed. If a given formula is different in these two systems, then, in the following, both formulae will be given side by side.

1. The field equations and the constitutive equations

	CGS System	MKSA System
Maxwell equations:	$\operatorname{curl}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{g}$	$\operatorname{curl}\mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{g}$
	$\operatorname{curl}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$	$\operatorname{curl}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
	$\operatorname{div}\mathbf{D} = 4\pi\rho$	$\operatorname{div}\mathbf{D} = \rho$
	$\operatorname{div}\mathbf{B} = 0$	$\operatorname{div}\mathbf{B} = 0$
Constitutive equations:	$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$	$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$
	$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$	$\mathbf{B} = \mu_0\mathbf{H} + \mathbf{M}$
Equation of continuity:	$\operatorname{div}\mathbf{g} + \frac{\partial\rho}{\partial t} = 0$	$\operatorname{div}\mathbf{g} + \frac{\partial\rho}{\partial t} = 0$

If the quantities **D** and **H**, considered in electron theory to be secondary quantities, are eliminated from the Maxwell equations, we obtain the new equations:

CGS	MKSA
$\operatorname{curl}\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}(\mathbf{g}+\mathbf{g}_P+\mathbf{g}_M)$	$\frac{1}{\mu_0}\operatorname{curl}\mathbf{B} = \varepsilon_0\frac{\partial \mathbf{E}}{\partial t} + (\mathbf{g}+\mathbf{g}_P+\mathbf{g}_M)$!
$\operatorname{div}\mathbf{E} = 4\pi(\rho+\rho_P)$	$\varepsilon_0\operatorname{div}\mathbf{E} = \rho+\rho_P$

The meanings here are:

$\rho_P = -\operatorname{div}\mathbf{P}$, the density of polarization charge

$\mathbf{g}_P = \frac{\partial \mathbf{P}}{\partial t}$, the density of polarization current

$\mathbf{g}_M = c\,\mathrm{curl}\,\mathbf{M}$	$\mathbf{g}_M = \frac{1}{\mu_0}\mathrm{curl}\,\mathbf{M}$, the density of magnetization current

2. The material constants

Normal isotropic substances (i.e. non-ferroelectric non-ferromagnetic substances) are, in their relations to static fields, characterized by special material constants:

Electrical conductivity:	$\mathbf{g} = \sigma(\mathbf{E}+\mathbf{E}^{(e)})$	$\mathbf{g} = \sigma(\mathbf{E}+\mathbf{E}^{(e)})$
Permittivity (dielectric constant)	$\mathbf{D} = \varepsilon\mathbf{E}$	$\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}$
or electric susceptibility:	$\mathbf{P} = \chi\mathbf{E}$	$\mathbf{P} = \chi\varepsilon_0\mathbf{E}$
	$\varepsilon = 1+4\pi\chi$	$\varepsilon = 1+\chi$
Permeability	$\mathbf{B} = \mu\mathbf{H}$	$\mathbf{B} = \mu\mu_0\mathbf{H}$
or magnetic susceptibility:	$\mathbf{M} = \kappa\mathbf{H}$	$\mathbf{M} = \kappa\mu_0\mathbf{H}$
	$\mu = 1+4\pi\kappa$	$\mu = 1+\kappa$

In fields changing with time, σ, ε, and μ are in general frequency-dependent; the foregoing relationships then only hold between the Fourier components of the field strengths.

3. Energy and force expressions

Energy theorem: $\dfrac{\partial u}{\partial t}+\mathrm{div}\,\mathbf{S} = -\mathbf{g}.\mathbf{E} = -\dfrac{\mathbf{g}^2}{\sigma}+\mathbf{g}.\mathbf{E}^{(e)}$

Energy density of the field:	$du = \frac{1}{4\pi}(\mathbf{E}.d\mathbf{D}+\mathbf{H}.d\mathbf{B})$	$du = \mathbf{E}.d\mathbf{D}+\mathbf{H}.d\mathbf{B}$
specially, for normal substances:	$u = \frac{1}{8\pi}(\mathbf{E}.\mathbf{D}+\mathbf{H}.\mathbf{B})$	$u = \frac{1}{2}(\mathbf{E}.\mathbf{D}+\mathbf{H}.\mathbf{B})$
Poynting vector:	$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{H}$	$\mathbf{S} = \mathbf{E}\times\mathbf{H}$
Force on moving charge:	$\mathbf{F} = e\left(\mathbf{E}+\frac{\mathbf{v}}{c}\times\mathbf{B}\right)$	$\mathbf{F} = e(\mathbf{E}+\mathbf{v}\times\mathbf{B})$
Force density in matter with $\varepsilon = \mu = 1$:	$\mathbf{f} = \rho\mathbf{E}+\frac{\mathbf{g}}{c}\times\mathbf{B}$	$\mathbf{f} = \rho\mathbf{E}+\mathbf{g}\times\mathbf{B}$

Force density in matter with $\mu = 1$, $\sigma = 0$ (conductivity):

$$\mathbf{f} = \rho\mathbf{E}-\frac{1}{8\pi}\mathbf{E}^2\,\mathrm{grad}\,\varepsilon+\frac{1}{8\pi}\mathrm{grad}\left(\mathbf{E}^2\frac{d\varepsilon}{d\sigma}\sigma\right)$$

(The σ in this formula is the matter density.)

The force density can always be represented as the divergence of the Maxwell stress tensor.

4. Wave propagation

For normal homogeneous uncharged media (ε, μ, σ constant in space, $\mathbf{E}^{(e)} = 0$, $\rho = 0$) every field component F satisfies the wave equation

$\nabla^2F = \frac{\varepsilon\mu}{c^2}\frac{\partial^2F}{\partial t^2}+\frac{4\pi\sigma\mu}{c^2}\frac{\partial F}{\partial t}$	$\nabla^2F = \mu\mu_0\left(\varepsilon\varepsilon_0\frac{\partial^2F}{\partial t^2}+\sigma\frac{\partial F}{\partial t}\right)$

For material constants that are frequency-dependent this equation has meaning only for the individual temporal Fourier components of F.

From this, the wave velocity *in vacuo*	$c = 1/\sqrt{(\varepsilon_0\mu_0)}$
Wave velocity in insulators	$c/n = c/\sqrt{(\varepsilon\mu)}$
Index of refraction	$n = \sqrt{(\varepsilon\mu)}$

The calculation of fields *in vacuo* is facilitated by going over to the potentials **A** and ϕ by means of

	$\left\{\begin{aligned}&\mathbf{B} = \mathrm{curl}\,\mathbf{A}\\&\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}-\mathrm{grad}\,\phi\end{aligned}\right.$	$\begin{aligned}&\mathbf{B} = \mathrm{curl}\,\mathbf{A}\\&\mathbf{E} = -\frac{\partial\mathbf{A}}{\partial t}-\mathrm{grad}\,\phi\end{aligned}$
Potential equations:	$\left\{\begin{aligned}&\nabla^2\mathbf{A}-\frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mathbf{g}}{c}\\&\nabla^2\phi-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -4\pi\rho\end{aligned}\right.$	$\begin{aligned}&\nabla^2\mathbf{A}-\varepsilon_0\mu_0\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{g}\\&\nabla^2\phi-\varepsilon_0\mu_0\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}\end{aligned}$
Lorentz convention	$\mathrm{div}\,\mathbf{A}+\frac{1}{c}\frac{\partial\phi}{\partial t} = 0$	$\mathrm{div}\,\mathbf{A}+\varepsilon_0\mu_0\frac{\partial\phi}{\partial t} = 0$

General integral of the ϕ-equation in the CGS system:

$$\phi(\mathbf{r},t) = \int\frac{\rho\left(\mathbf{r}',t-\frac{|\mathbf{r}-\mathbf{r}'|}{c}\right)dv'}{|\mathbf{r}-\mathbf{r}'|}$$