Reminder: Relativity of Simultaneity in Special Relativity

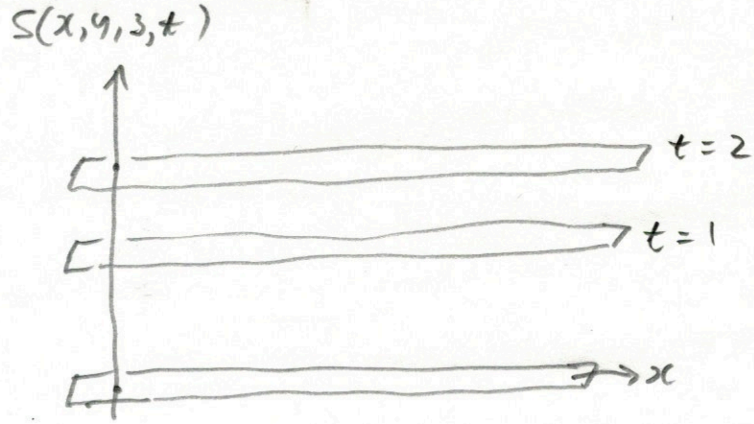
Einstein 1907: "On the Principle of Relativity ..."

$t' = \gamma(t - \frac{v}{c^2}x)$

$x' = \gamma(x - vt)$

$y' = y \quad z' = z$

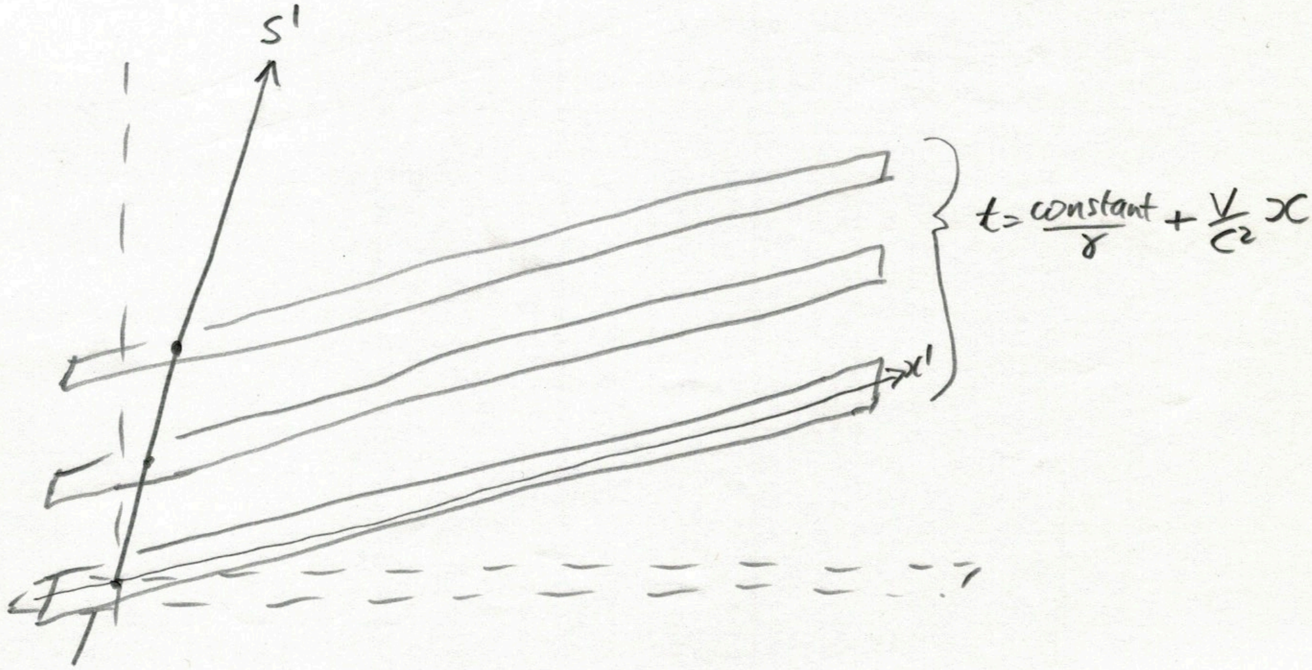
$\gamma = 1/\sqrt{1 - v^2/c^2}$



Hypersurfaces of simultaneous events in $S'(x', y', z', t')$ satisfy $t' = \text{constant}$

i.e. $\gamma(t - \frac{v}{c^2}x) = \text{constant}$

i.e. $t = \frac{\text{constant}}{\gamma} + \frac{v}{c^2}x$



§18 ①–② $\boxed{\text{Uniform acceleration } \gamma \Rightarrow \text{No influence to first order on shape of body}}$

Hence use Cartesian coordinates as spatial coordinates of a uniformly accelerated body

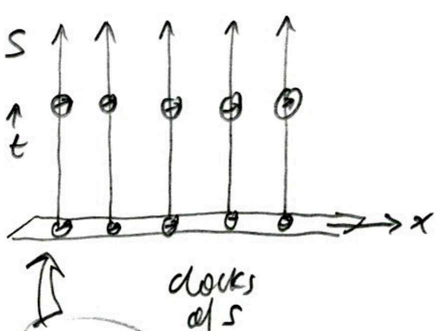
since Possible effect (uniform dilation) same for $+\gamma$, $-\gamma$

acc. in $+x$ direction — accn. in $-x$ direction

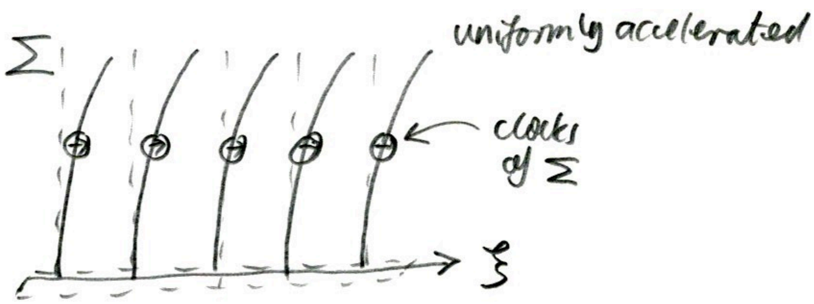
i.e. Effect(γ) = Effect($-\gamma$)

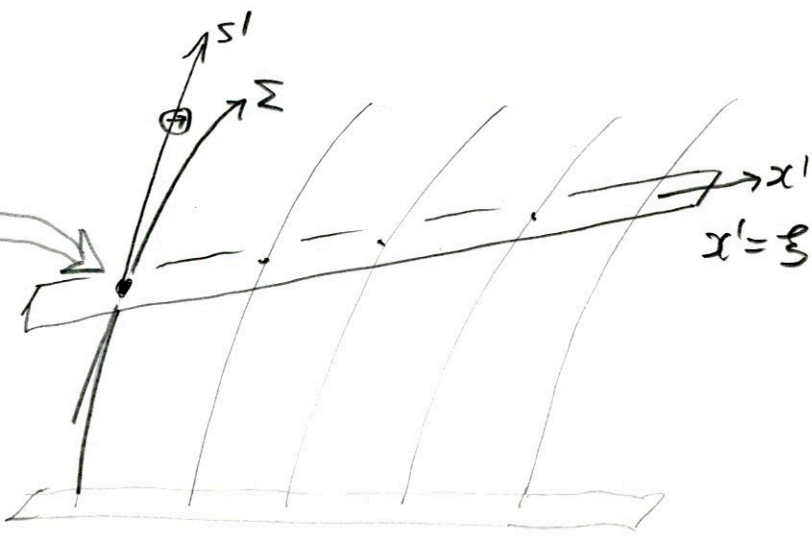
$$\text{Effect} = \underset{\substack{\uparrow\\ \text{No effect if } \gamma=0}}{1} + \underbrace{A\gamma}_{\text{vanishes}} + \underbrace{B\gamma^2}_{\text{2nd order small term is largest}} + C\gamma^3 + \cdots$$

§18 ③ $\boxed{\text{coordinate systems } \Sigma,\ S,\ S' \text{ defined}}$



ordinary inertial system



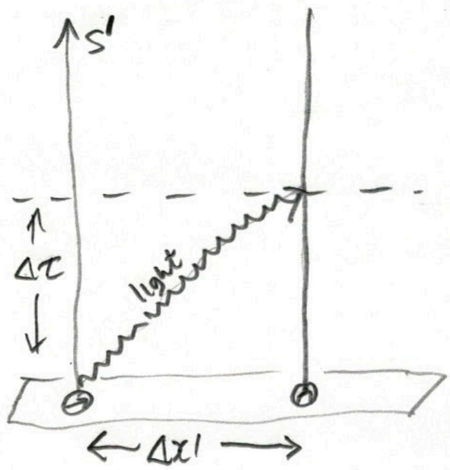


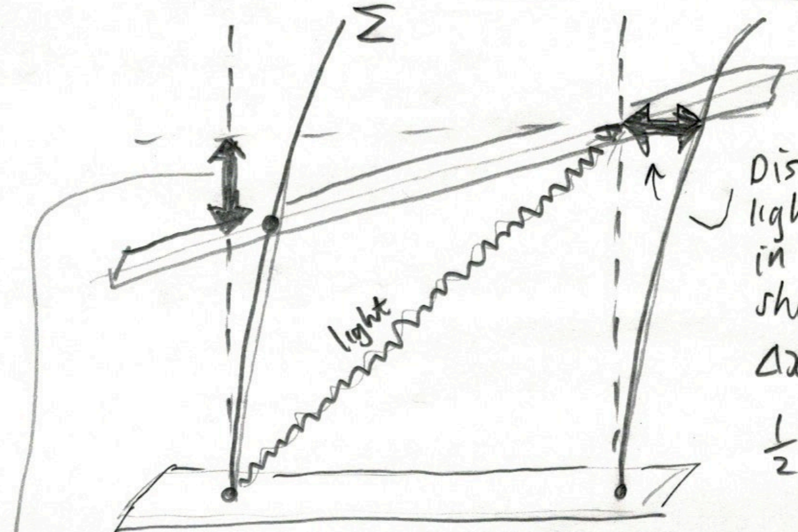
inertial system S' momentarily agrees at spatial origin

18.④,⑤

Σ agrees with S' for short time intervals $\Rightarrow$ speed of light in Σ is universal constant c – if measured over short time intervals

over short time $\Delta\tau$



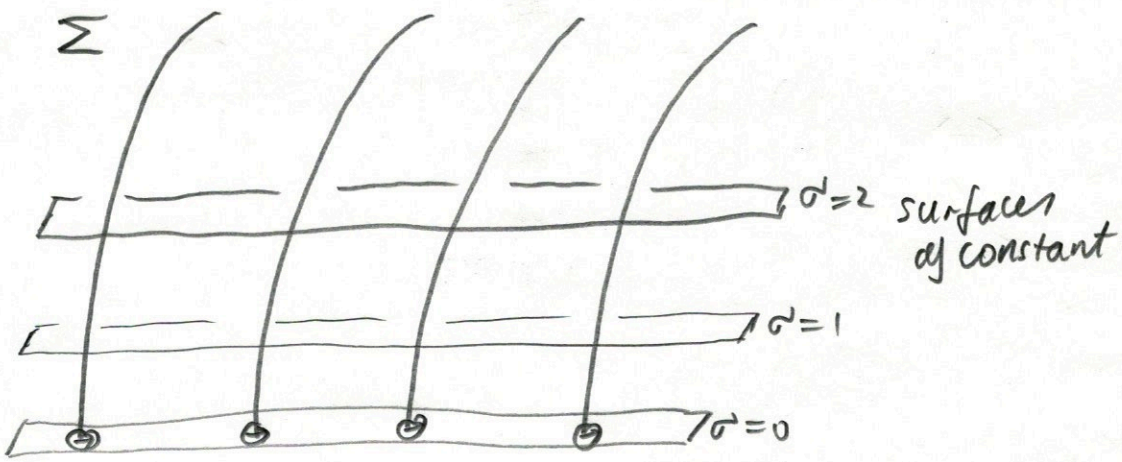


Distance light travels in $\Delta\tau$ falls short of $\Delta x'$ by $\frac{1}{2}\gamma(\Delta\tau)^2$

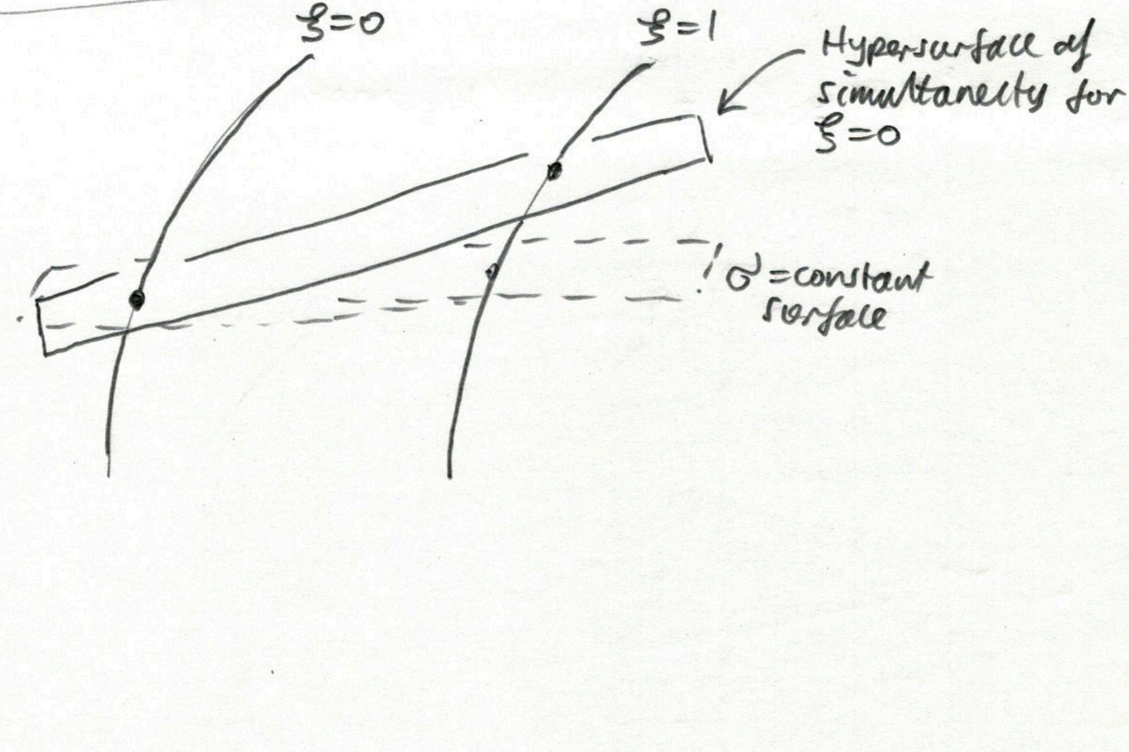
Light arrives early in Σ by a relativity of simultaneity term $\frac{\Delta x' \text{(velocity)}}{c} = \frac{\Delta x'}{c} \cdot \frac{1}{2}\gamma(\Delta\tau)^2$

Both second order small in $\Delta\tau$

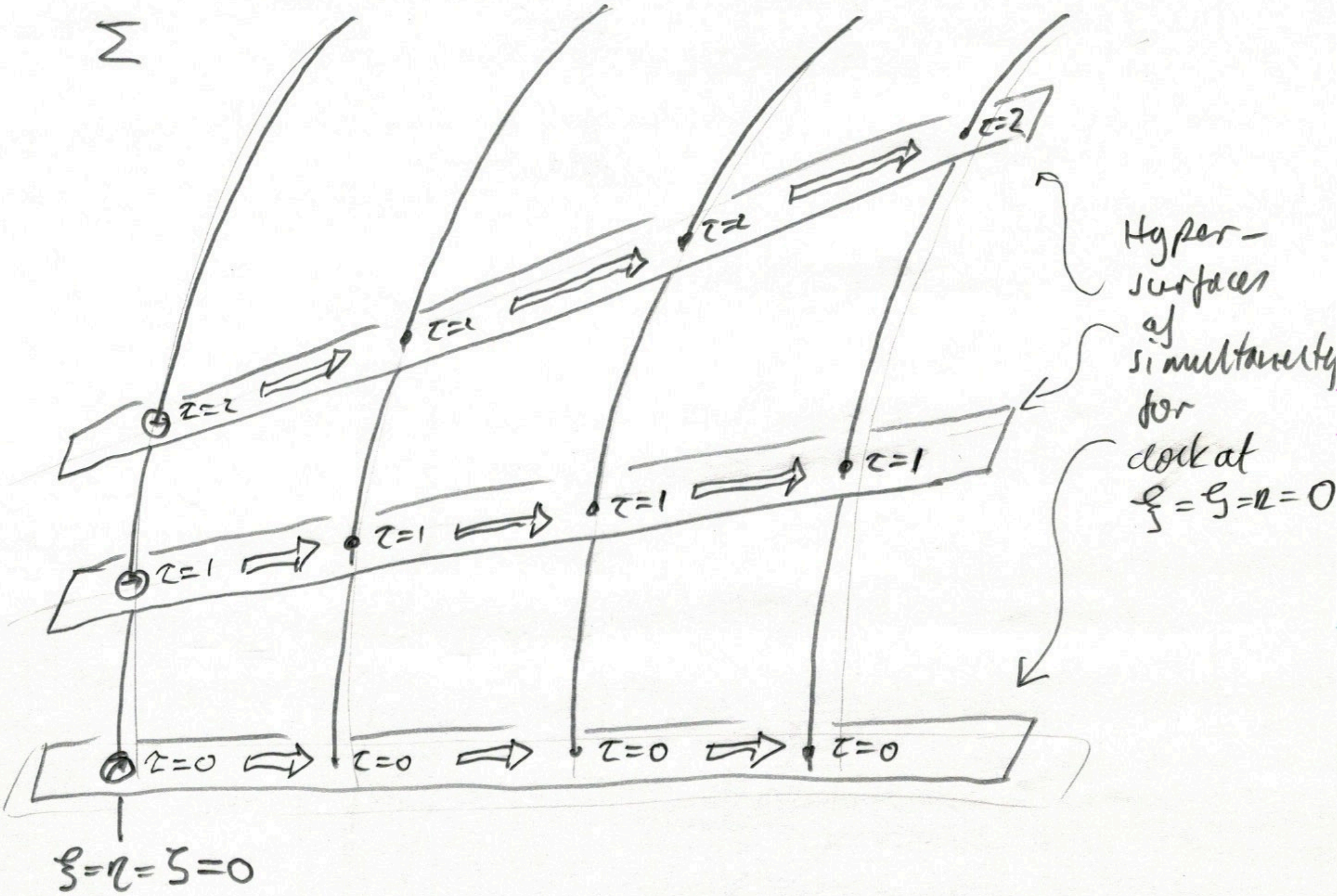
18·⑥ "Local time" = time read by clocks in Σ
σ (initially set to 0 at $t=0$ of S)



18·⑦ Local time does not respect simultaneity



18 · 8 THE time τ of the system = Time read by clock at origin of Σ propagated to all events by Einstein synchrony

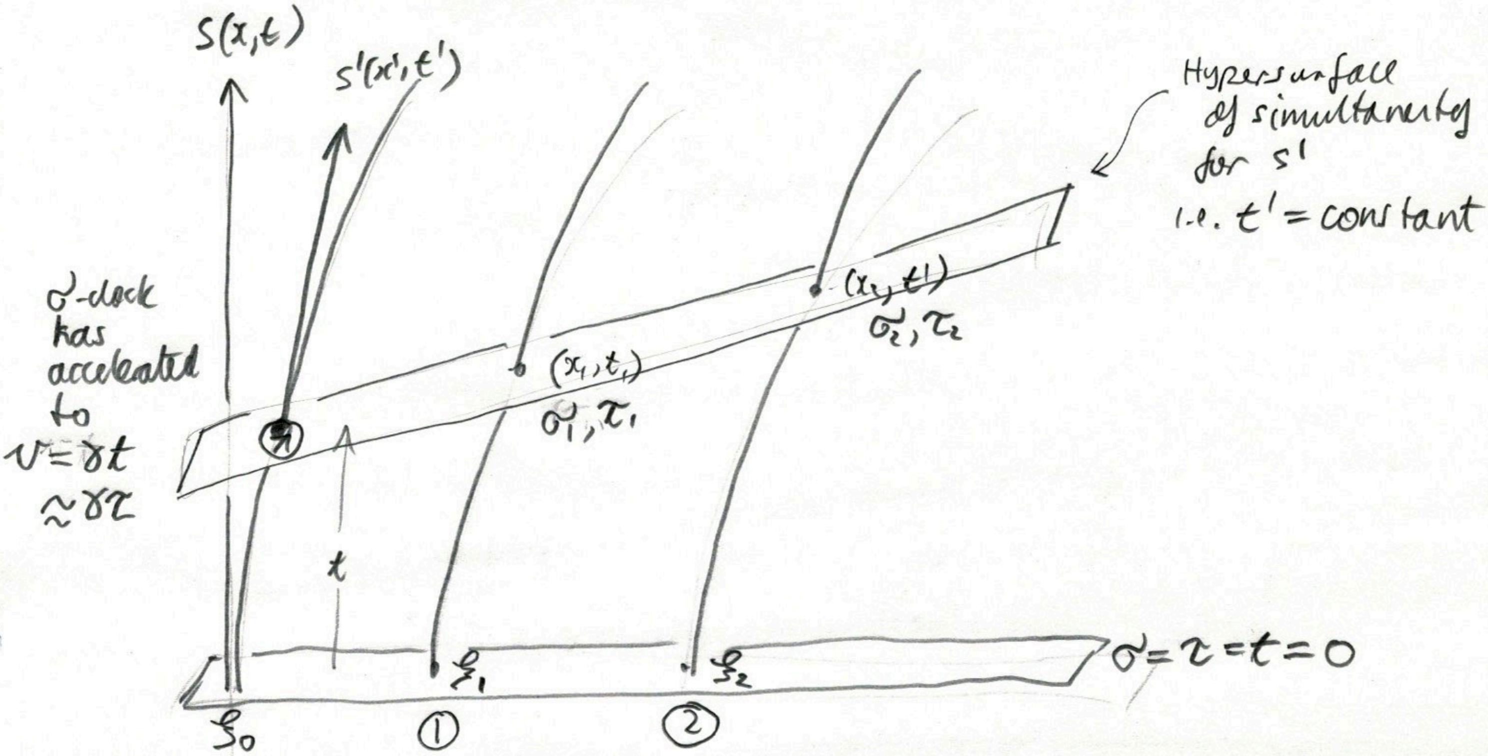


18 ⑨ Relation between local time σ and THE time τ

$$\boxed{\sigma = \tau\left(1 + \frac{\gamma\xi}{c^2}\right)}$$

for small times after clocks set

(i.e. drop all terms in $(\text{time})^2$



condition for hypersurface: $t' = \gamma\left(t - \frac{v}{c^2}x\right) = \text{constant}$

i.e. $t_1' = t_2'$ $\quad t_1 - \frac{v}{c^2}x_1 = t_2 - \frac{v}{c^2}x_2$

$$\therefore\ t_2 - t_1 = \frac{v}{c^2}(x_2 - x_1)$$

$\Downarrow$ $\quad v \approx \gamma\tau \quad t_1 \approx \sigma_1 \quad t_2 \approx \sigma_2$

$\quad x_2 - x_1 \approx x_2' - x_1' \approx \xi_2' - \xi_1'$

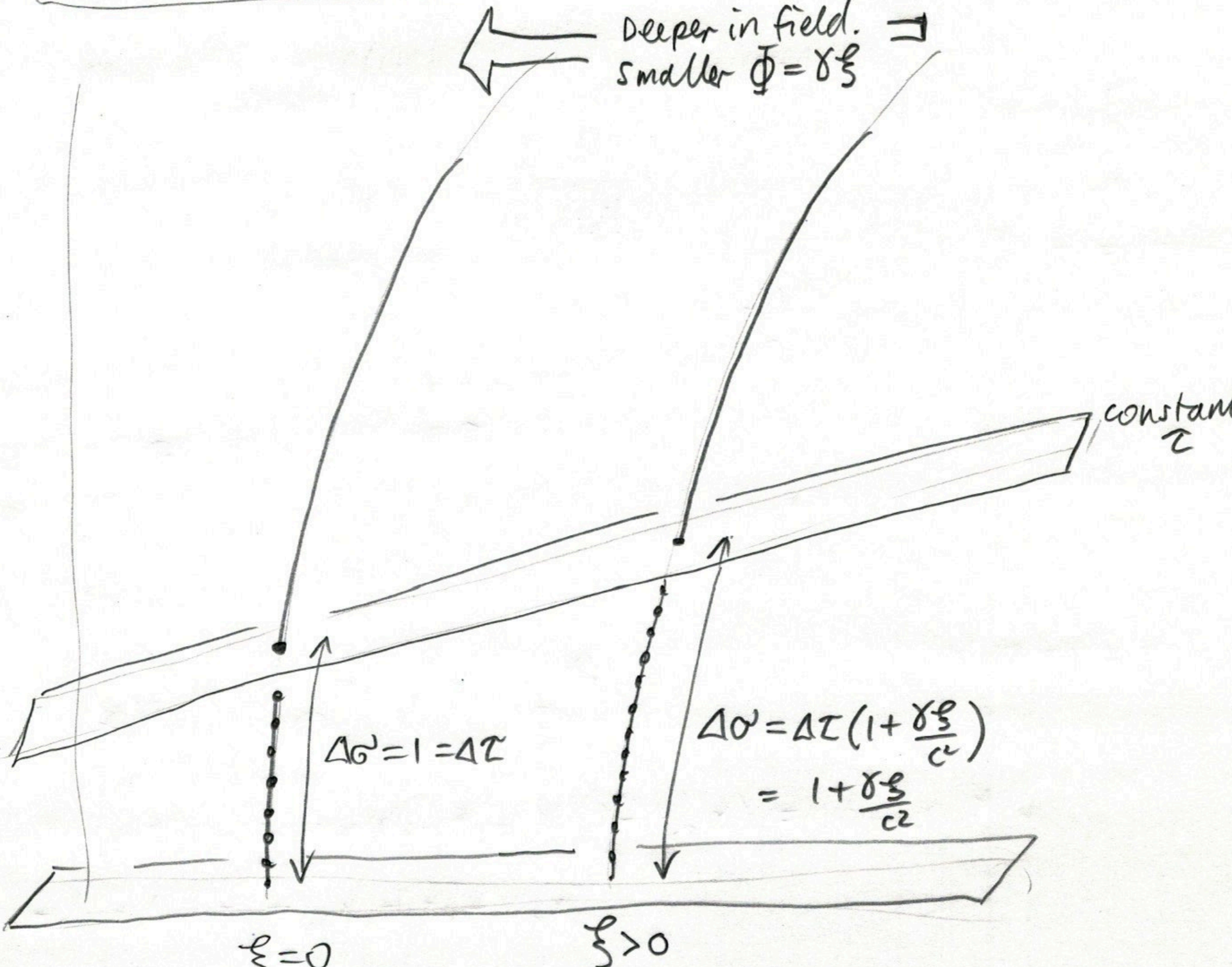
$$\sigma_2 - \sigma_1 = \frac{\gamma\tau}{c^2}(\xi_2 - \xi_1)$$

$\Downarrow$ set point ① → origin: $\xi_1 = 0$, $\xi_2 = \xi$ etc

$\sigma_1 = \tau$

$$\boxed{\sigma = \tau\left[1 + \frac{\gamma\xi}{c^2}\right]}$$

19. Slowing of clocks in uniformly accelerating frame Σ / Homogeneous gravitational field



§20 Synopsis of Results

Maxwell's Equations in $S(x,y,z,t)$

$$\boxed{\frac{1}{c}\left(\rho \underline{u} + \frac{\partial \underline{E}}{\partial t}\right) = \underline{\nabla} \times \underline{H} \qquad \frac{1}{c}\frac{\partial \underline{H}}{\partial t} = \underline{\nabla} \times \underline{E}}$$

$$\Downarrow \quad \underset{\sim}{E}^* = \underset{\sim}{E}\left(1+\frac{\gamma\xi}{c^2}\right) \quad \underline{H}^* = \underline{H}\left(1+\frac{\gamma\xi}{c^2}\right) \quad \rho^* = \left(1+\frac{\gamma\xi}{c^2}\right)$$

Maxwell's equations in $\Sigma(\xi,\eta,\zeta,\sigma)$ (σ: Local time)

$$\boxed{\frac{1}{c}\left(\rho^* \underset{\sim}{u}_\Sigma + \frac{\partial \underline{E}^*}{\partial \sigma}\right) = \underline{\nabla}_\Sigma \times \underline{H}^* \qquad \frac{1}{c}\frac{\partial \underline{H}^*}{\partial \sigma} = \underline{\nabla}_\Sigma \times \underline{E}^*}$$

"for static & stationary phenomena"

$$\Downarrow \quad \underset{\uparrow\, \frac{dx}{d\tau}}{\underline{\omega}} = \left(1+\frac{\gamma\xi}{c^2}\right)\underset{\uparrow\, \frac{dx}{d\sigma}}{\underline{u}_\Sigma}$$

Maxwell's Equations in $\Sigma(\xi,\eta,\zeta,\tau)$ (τ: THE time)

$$\boxed{\frac{1}{c\left(1+\frac{\gamma\xi}{c^2}\right)}\left(\rho^* \underline{\omega} + \frac{\partial \underline{E}^*}{\partial \tau}\right) = \underline{\nabla}_\Sigma \times \underline{H} \qquad \frac{1}{c\left(1+\frac{\gamma\xi}{c^2}\right)}\frac{\partial \underline{H}^*}{\partial \tau} = \underline{\nabla}_\Sigma \times \underline{E}^*}$$

"follow development of non-stationary states"

speed of light $= c\left(1+\frac{\gamma\xi}{c^2}\right)$

$\therefore$ Light bent by gravity

$\Downarrow$ contract with $\underline{E}^*, \underline{H}^*$. Integrate over all space

Energy theorem

$$\boxed{\underbrace{\int\left(1+\frac{\gamma\xi}{c^2}\right)\eta_\tau\, d\omega}_{\text{Rate work done on matter}} + \frac{d}{d\tau}\underbrace{\int\left(1+\frac{\gamma\xi}{c^2}\right)\mathcal{E}\, d\omega}_{\text{Field energy}} = 0}$$

Term $1+\frac{\gamma\xi}{c^2}$ $\Rightarrow$ must adjust locally measured quantities $\eta_\tau, \mathcal{E}$ by $\frac{\gamma\xi}{c^2} = \frac{\Phi}{c^2}$ $\Rightarrow$ Adjust locally measured energy E by $\frac{E}{c^2}\Phi$ $\Rightarrow$ Locally measured energy E has gravitational mass $E/c^2 = m$ which has gravitational energy $m\Phi$