

Calculations for the Relativity of Simultaneity

Following the usual conventions in special relativity, we assign coordinates x, t to a system of coordinates adapted to the resting inertial frame. An inertial frame moves at constant speed v in the $+x$ direction of the resting frame. Coordinates x', t' are assigned in the usual way. If we imagine a long ruler at rest in the moving frame, the coordinate x' will coincide with length measurements on the ruler and the coordinate t' will coincide with the readings of clocks at rest in the moving frame.

The Lorentz transformation is

$$\begin{aligned}x' &= \gamma(x - vt) \\t' &= \gamma\left(t - \left(\frac{v}{c^2}\right)x\right) \\ \gamma &= 1/\sqrt{1 - v^2/c^2}\end{aligned}$$

Consider a set of simultaneous events, all with $t=0$, at locations $x = \dots, -2, -1, 0, 1, 2, \dots$ in the resting frame. They will have times t' associated with those events by clocks at rest in the moving frame according to

$$t' = -\gamma\left(\frac{v}{c^2}\right)x$$

The positions of these events in the moving frame as indicated by the coordinate x' is:

$$x' = \gamma x$$

Combining we have:

$$t' = -\left(\frac{v}{c^2}\right)x'$$

Since t' is the time read by clocks in the moving frame at the spatial location x' where x' is the distance measured by co-moving rod from the moving coordinate system's origin, we have in words:

$$(\text{time on clock}) = -(\text{speed of motion of frame} \times \text{distance from origin})/c^2$$