A translation dictionary: from *On the verge..* to Kramers-Heisenberg

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The *Fraktur* script symbols used by Kramers and Heisenberg can be somewhat off-putting to modern eyes. Herewith, a brief translation dictionary for connecting the notation K&H use to the more modern style employed in the *On the verge..* paper (part 2). The *On the verge..* notation is on the left, K&H on the right. Equation numbers in round brackets, when given, refer to the corresponding reference.

The applied electric field (due to the incoming light):

$$\vec{E}(t) = \mathrm{Re}(E\exp(2\pi i\nu t)\hat{x}) \quad \leftrightarrow \quad \mathfrak{E}(t) = \mathrm{R}(\mathfrak{E}e^{2\pi i\nu t}) \ \ (1)$$

Note that $\mathfrak{E}$ is a general three-vector in K&H, while we have chosen to consider only plane polarized (in the x-direction) light in *On the verge*. Consequently, only the displacement (or polarization) in the x-direction $\Delta x(t)$ need be considered. Moreover, only scattered light with the same polarization direction is considered in the latter. In K&H the fields and amplitudes are left as general three-vectors to allow the consideration of arbitrary polarizations, although this generality is of hardly any import in the final results.

Next, the classical dipole moment (K&H-"electrical moment") of the radiating electron (where $-e$ is the electron charge)

$$-ex(t) = -e\sum_{\vec{\tau}} A_{\vec{\tau}} e^{2\pi i\vec{\tau}\cdot\vec{w}} \ \ (88) \quad \leftrightarrow \quad \mathfrak{M}(t) = \sum_{\tau_1..\tau_s} \mathfrak{C}_{\tau_1..\tau_s} e^{2\pi i(\tau_1 w_1 + ..\tau_s w_s)} \ \ (7)$$

Modern notation uses Greek nu (resp. omega) for cyclic (resp. angular) frequencies, so

$$\nu_k = \frac{\partial H_0}{\partial J_k} \quad \leftrightarrow \quad \omega_k = \frac{\partial H}{\partial J_k} \ \ (9)$$

$$\vec{\tau}\cdot\vec{\nu} \quad \leftrightarrow \quad \omega = \tau_1\omega_1 + ... + \tau_s\omega_s \equiv \frac{\partial H}{\partial J} \ \ (10, 11)$$

(From this point, the equation numbers refer to the present discussion). In comparing the non-transient terms in *On the verge* (100) (multiplied by $-e$ to convert the displacement to the dipole moment) with K&H (15), we have the following correspondences:

$$-e^2 E\tau_l \frac{\partial A_{\vec{\tau}'}}{\partial J_l} A_{\vec{\tau}} \frac{-e^{2\pi i(\vec{\tau}\cdot\vec{\nu}+\vec{\tau}'\cdot\vec{\nu}+\nu)t}}{\vec{\tau}\cdot\vec{\nu}+\nu} \quad \leftrightarrow \quad \frac{\partial \mathfrak{C}'}{\partial J}(\mathfrak{E}\mathfrak{C}) \frac{e^{2\pi i(\omega+\omega'+\nu)t}}{\omega+\nu} \tag{1}$$

$$-e^2 E\tau_l' \frac{\partial A_{\vec{\tau}}}{\partial J_l} A_{\vec{\tau}'} \frac{e^{2\pi i(\vec{\tau}\cdot\vec{\nu}+\vec{\tau}'\cdot\vec{\nu}+\nu)t}}{\vec{\tau}\cdot\vec{\nu}+\nu} \quad \leftrightarrow \quad -\mathfrak{C}' \frac{\partial \mathfrak{C}}{\partial J'} \mathfrak{E} \frac{e^{2\pi i(\omega+\omega'+\nu)t}}{\omega+\nu} \tag{2}$$

$$e^2 E A_{\vec{\tau}} A_{\vec{\tau}'} \tau_k \frac{\partial \nu_l}{\partial J_k} \tau_l' \frac{e^{2\pi i(\vec{\tau}\cdot\vec{\nu}+\vec{\tau}'\cdot\vec{\nu}+\nu)t}}{(\vec{\tau}\cdot\vec{\nu}+\nu)^2} \quad \leftrightarrow \quad \mathfrak{C}'(\mathfrak{E}\mathfrak{C}) \frac{\partial \omega'}{\partial J} \frac{e^{2\pi i(\omega+\omega'+\nu)t}}{(\omega+\nu)^2} \tag{3}$$

The right-hand-side of (3) may be re-expressed as follows

$$\mathfrak{C}'(\mathfrak{E}\mathfrak{C}) \frac{\partial \omega'}{\partial J} \frac{e^{2\pi i(\omega+\omega'+\nu)t}}{(\omega+\nu)^2} = \mathfrak{C}'(\mathfrak{E}\mathfrak{C}) \frac{\partial \omega}{\partial J'} \frac{e^{2\pi i(\omega+\omega'+\nu)t}}{(\omega+\nu)^2} \tag{4}$$

$$= -\mathfrak{C}'(\mathfrak{E}\mathfrak{C}) (\frac{\partial}{\partial J'} \frac{1}{\omega+\nu}) e^{2\pi i(\omega+\omega'+\nu)t} \tag{5}$$

Adding the right-hand-sides of (1), (2) and (5), and relabelling the summation indices $\tau, \tau' \to \tau', \tau$, we obtain (15) in K&H. The overall factor of 4 arises from a difference of 2 in the definition of the amplitudes $A_{\vec{\tau}}$ and $\mathfrak{C}$.