H. A. Lorentz, Theory of the (Normal) Zeeman Effect. (Theory of Electrons)

Model of emitting electron in atoms & molecules

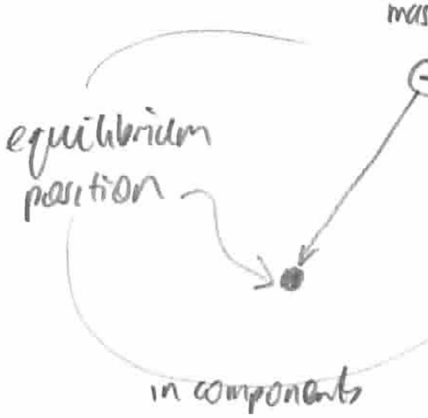
Explain production of a single spectral line

Each atom, molecule "one single electron"

Elastic force on electron "about whose nature we are very much in the dark"

mass m

$\underset{\sim}{r} = (\xi, \eta, \zeta)$



Force proportional to $\underset{\sim}{r}$ directed to equilibrium position

$$m \frac{d^2}{dt^2} \underset{\sim}{r} = -f \underset{\sim}{r}$$

↑ constant

$$n_0 = \sqrt{\frac{f}{m}}$$

$$m \frac{d^2 \xi}{dt^2} = -f \xi \xrightarrow{\text{solve}} \xi = a \cos(n_0 t + p)$$

$$m \frac{d^2 \eta}{dt^2} = -f \eta \Longrightarrow \eta = a' \cos(n_0 t + p')$$

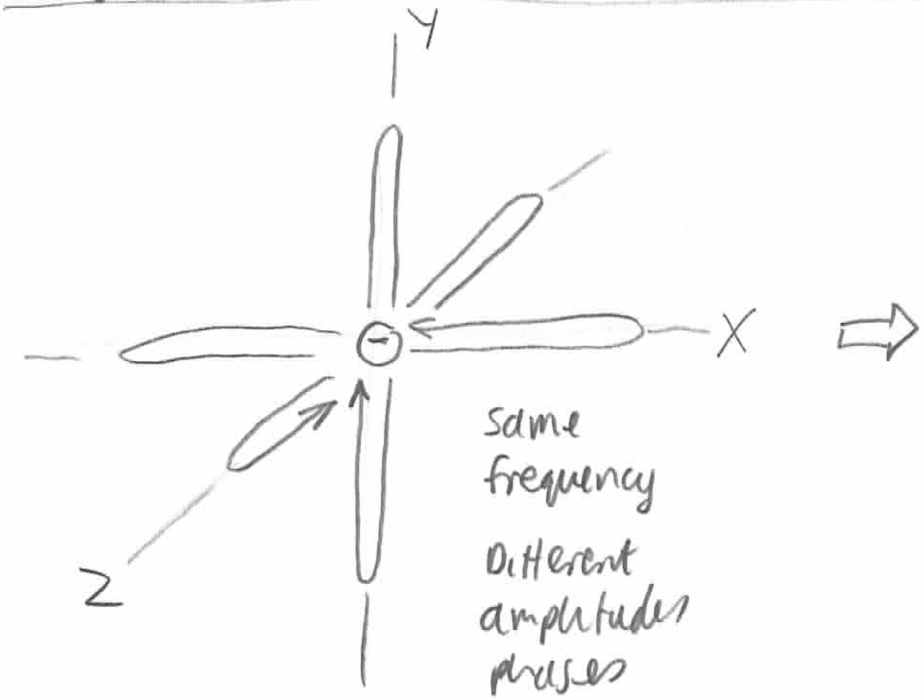
$$m \frac{d^2 \zeta}{dt^2} = -f \zeta \Longrightarrow \zeta = a'' \cos(n_0 t + p'')$$

Different amplitudes — SAME frequency — Different phase

Radiative losses ignored

This is motion of −ve charge inside non-resisting uniform charge distribution Thomson "plum pudding".
LORENTZ DOES NOT SAY THIS!

Motion of electron is resultant of three harmonic oscillators



same frequency

Different amplitudes phases

(visually) very complicated combined motion

3-D Lissajous figures

same phase

phase differs by π

ellipse

etc.

Turn on constant magnetic field $\underset{\sim}{H}$.
Extra force on electron is $\frac{e}{c}(\underset{\sim}{V} \times \underset{\sim}{H})$ (Gaussian units)

Do not assume +ve or −ve → charge

velocity

Assume (1) $\underset{\sim}{H} = (0, 0, H_z)$ directed in z only

(2) $\underset{\sim}{H}$ is small

Equations of motion become

General Solution

General since 6 free parameters $a_1, a_2, p_1, p_2, a''', p'''$

And linear combinations of the two "OR" solutions

$$m\frac{d^2\xi}{dt^2} = -f\xi + \frac{eH_z}{c}\frac{d\eta}{dt} \Rightarrow \xi = a_1 \cos(n_1 t + p_1) \text{ OR } \xi = a_2 \cos(n_2 t + p_2)$$

these two now coupled

$$m\frac{d^2\eta}{dt^2} = -f\eta - \frac{eH_z}{c}\frac{d\xi}{dt} \Rightarrow \eta = -a_1 \sin(n_1 t + p_1) \text{ OR } \eta = a_2 \sin(n_2 t + p_2)$$

$$m\frac{d^2\zeta}{dt^2} = -f\zeta \Longrightarrow \zeta = a''' \cos(n_0 t + p''')$$

unchanged

where
$$n_1^2 - \frac{eH_z}{mc}n_1 = n_0^2$$
$$n_2^2 + \frac{eH_z}{mc}n_2 = n_0^2$$

small $H_z \Rightarrow$

$$n_1 = n_0 + \frac{eH_z}{2mc}$$
$$n_2 = n_0 - \frac{eH_z}{2mc}$$

NB shifts depend on H_z, m ONLY!!

check new solution:

$$\xi = a\cos(nt+p) \qquad \eta = \mp a\sin(nt+p)$$

$$\frac{d\xi}{dt} = -an\sin(nt+p) \qquad \frac{d\eta}{dt} = \mp an\cos(nt+p)$$

$$\frac{d^2\xi}{dt^2} = -an^2\cos(nt+p) \qquad \frac{d^2\eta}{dt^2} = \pm an^2\sin(nt+p)$$

$$\boxed{m\frac{d^2\xi}{dt^2} = -f\xi + \frac{eH_3}{c}\frac{d\eta}{dt}} \qquad \boxed{m\frac{d^2\eta}{dt^2} = -f\eta - \frac{eH_3}{c}\frac{d\xi}{dt}}$$

becomes

$$-man^2\cos(nt+p) = -fa\cos(nt+p) + \frac{eH_3}{c}\cdot\mp an\cos(nt+p)$$

$$-man^2 = -fa \mp \frac{eH_3}{c}an$$

mn_0^2 since $n_0 = \sqrt{\frac{f}{m}}$

becomes

$$\pm man^2\sin(nt+p) = \pm fa\sin(nt+p) - \frac{eH_3}{c}\cdot -an\sin(nt+p)$$

$$\pm man^2 = \pm fa + e\frac{H_3}{c}an$$

mn_0^2

$$\therefore \boxed{n^2 \pm \frac{eH_3}{mc}n = n_0^2} \Leftrightarrow \text{SAME} \Leftrightarrow \boxed{\pm n^2 - \frac{eH_3}{mc}n = \pm n_0^2}$$

← condition for solution to hold

Write as $n^2 \pm \frac{eH_3}{mc}n = n_0^2$ i.e. $n_1^2 - \frac{eH_3}{mc}n_1 = n_0^2$ OR $n_2^2 + \frac{eH_3}{mc}n_2 = n_0^2$

For H_3 small

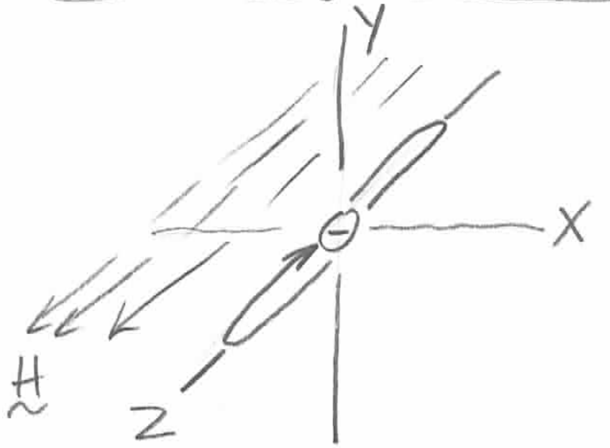
$$n^2\left(1 \pm \frac{eH_3}{mc}\cdot\frac{1}{n}\right) = n_0^2$$

$$n\sqrt{1 \pm \frac{eH_3}{mc}\cdot\frac{1}{n}} = n_0$$

$$n\left(1 \pm \frac{eH_3}{2mc}\cdot\frac{1}{n}\right) \doteq n_0$$

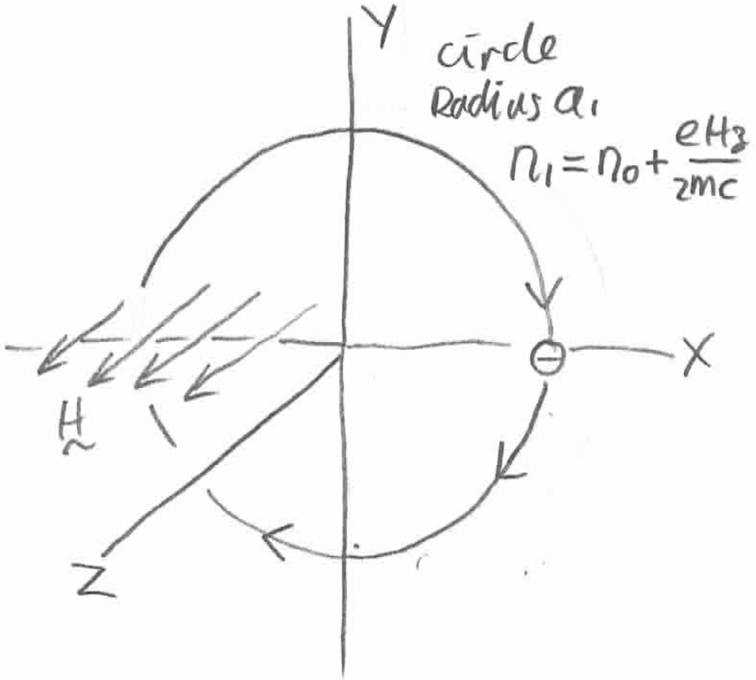
$$n = n_0 \mp \frac{eH_3}{2mc} \qquad n_1 = n_0 + \frac{eH_3}{2mc} \qquad n_2 = n_0 - \frac{eH_3}{2mc}$$

General Solution is a sum of three motions

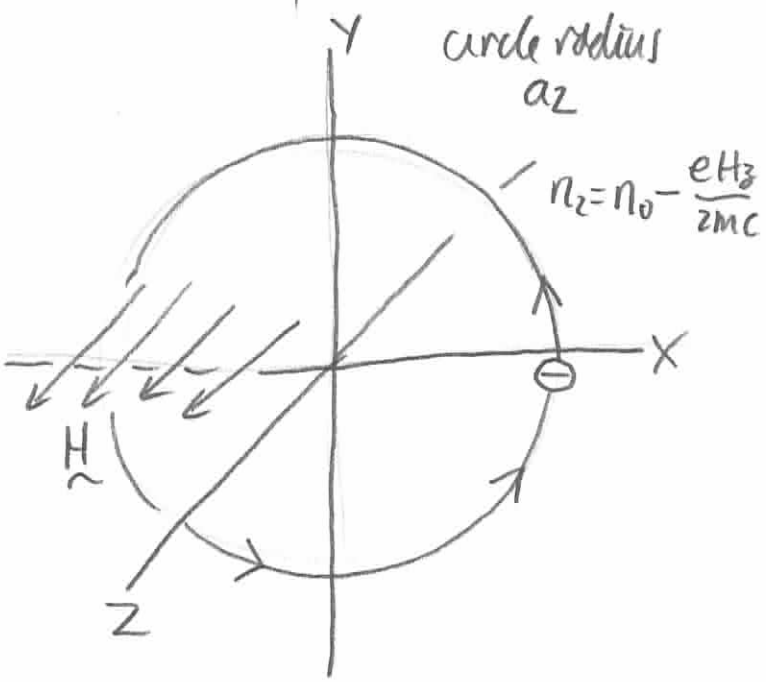


$$\xi = a'' \cos(n_0 t + p'')$$

(as before

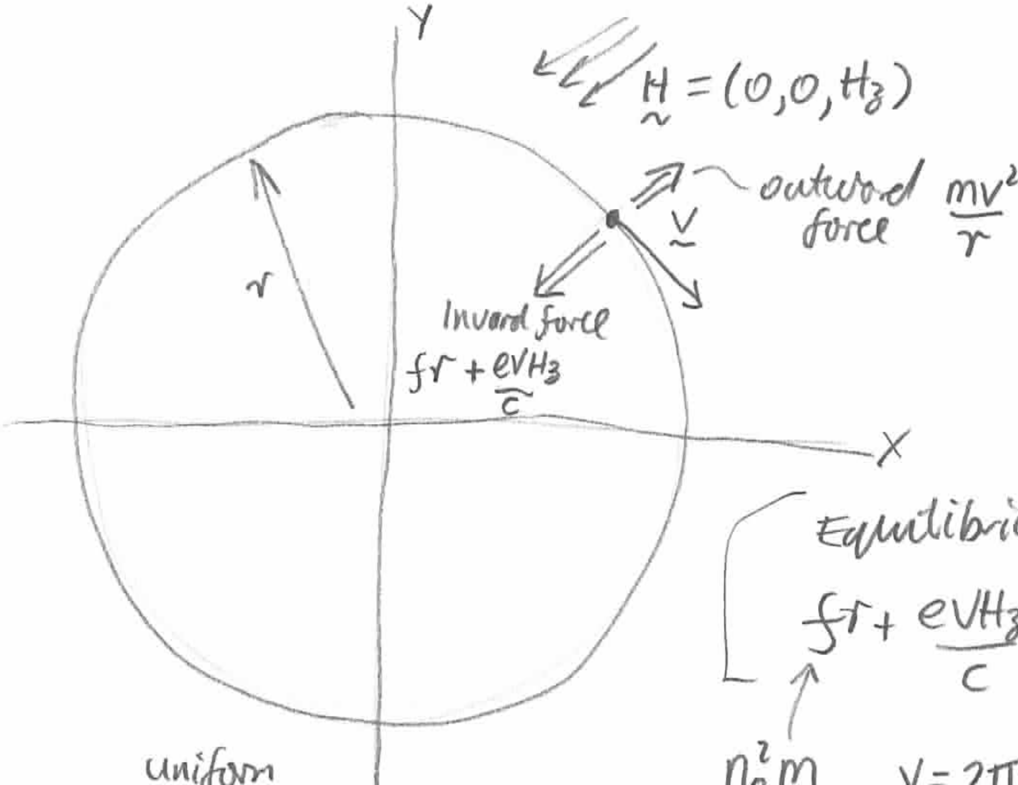


$$\xi = a_1 \cos(n_1 t + p_1)$$
$$\eta = -a_1 \sin(n_1 t + p_1)$$



$$\xi = a_2 \cos(n_2 t + p_2)$$
$$\eta = a_2 \sin(n_2 t + p_2)$$

Lorentz: Plausibility of new circular solutions



uniform circular motion

$n = \frac{2\pi}{T}$ period

Equilibrium

$$\left[\, fr + \frac{eVH_z}{c} = \frac{mV^2}{r} \,\right]$$

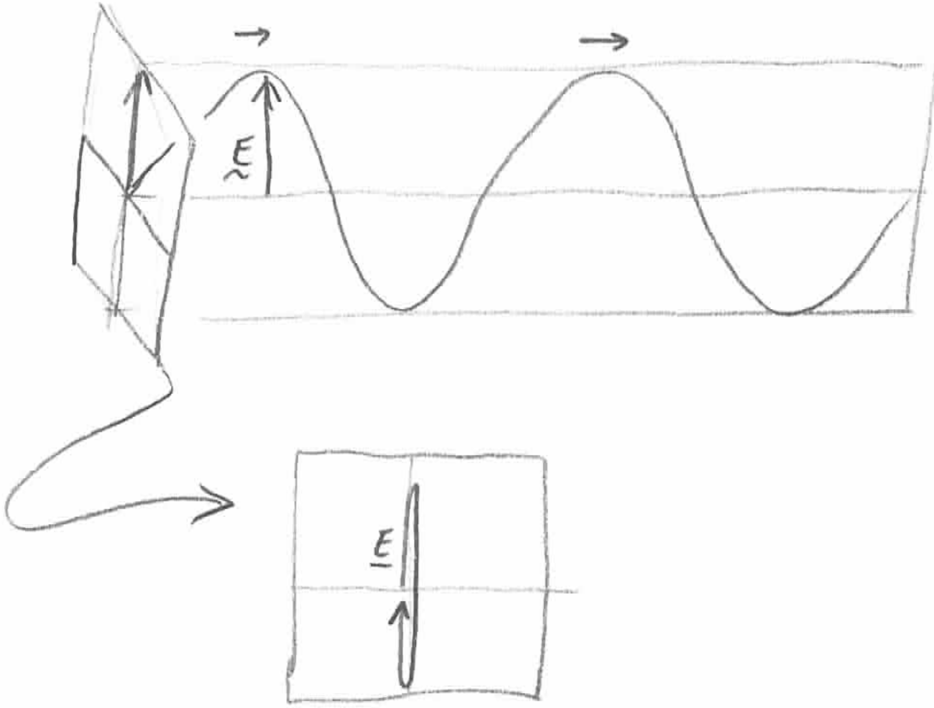
f: $n_0^2 m$ $\qquad V = \frac{2\pi r}{T} = rn$

$$\therefore n_0 m^2 r + \frac{eH_z}{c} \cdot rn = \frac{m}{r} r^2 n^2$$

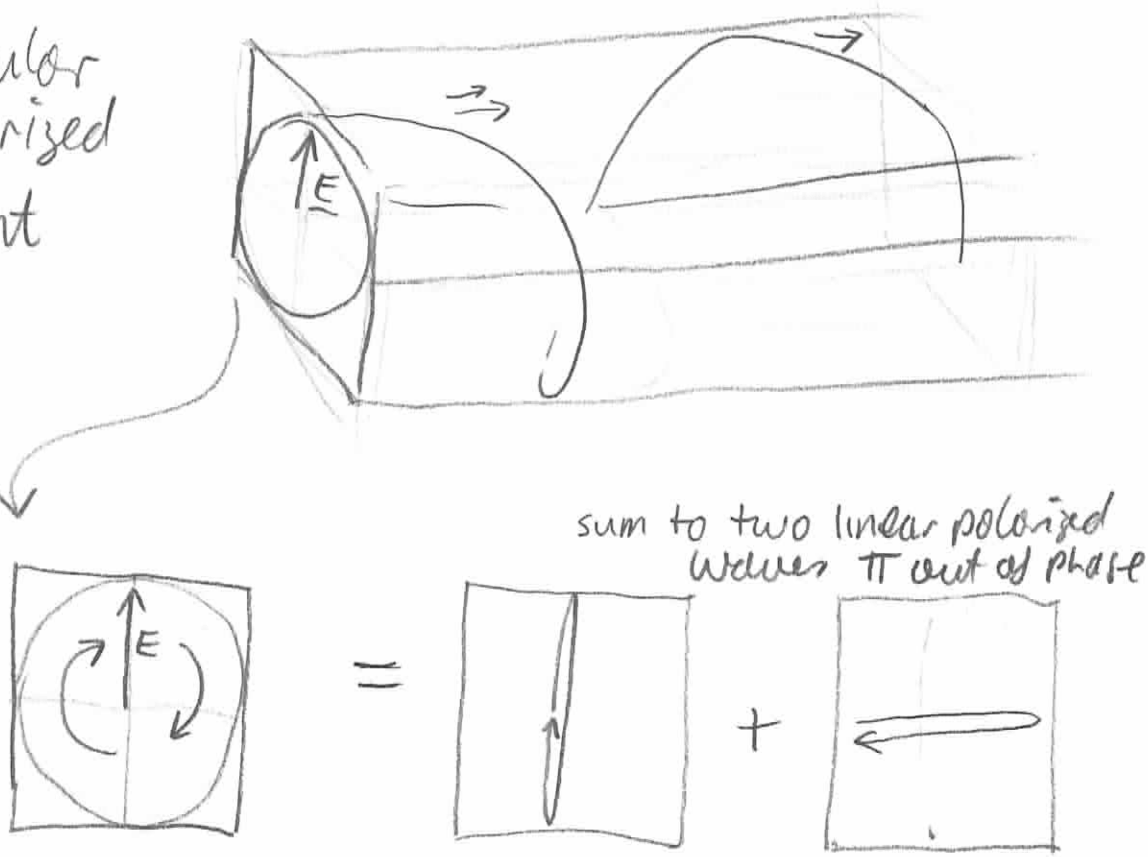
$$\therefore \underbrace{n^2 - \frac{eH_z}{mc} n = n_0^2}_{\text{condition for } n_1}$$

Polarization of Light

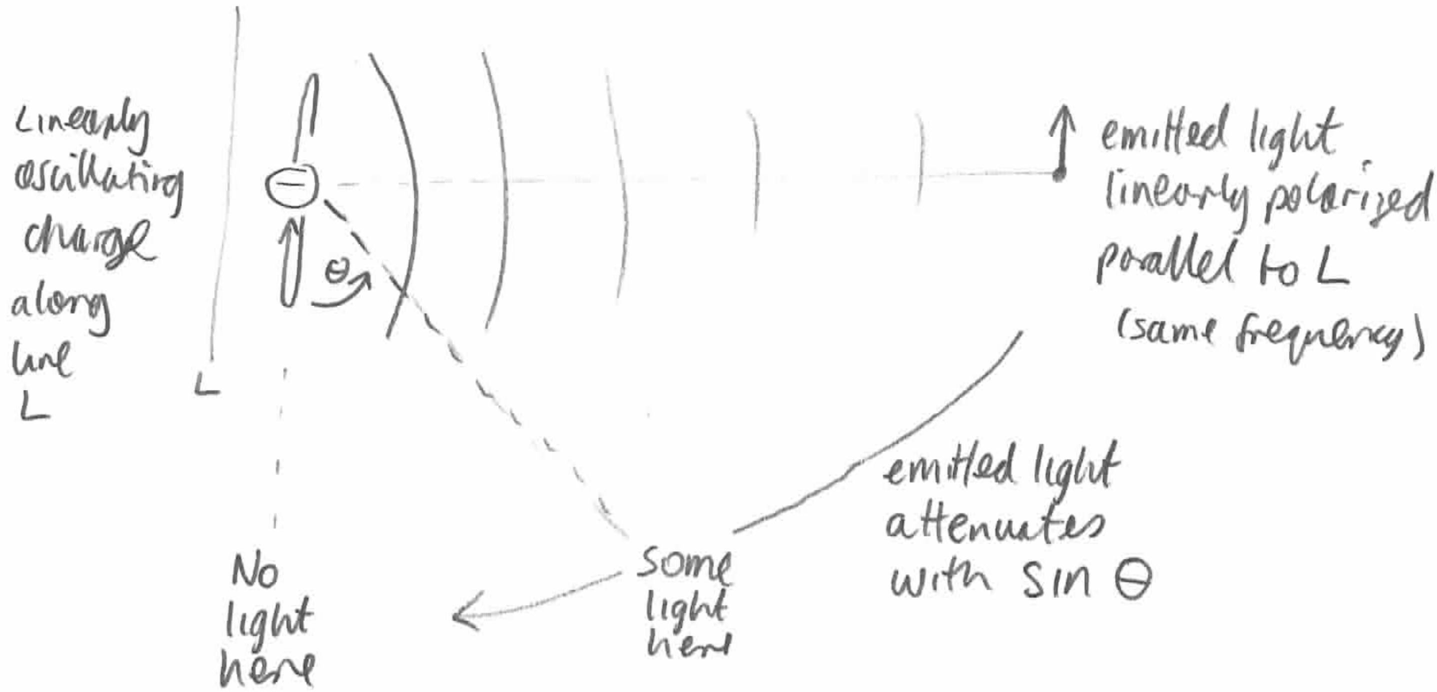
Linear (plane) polarized

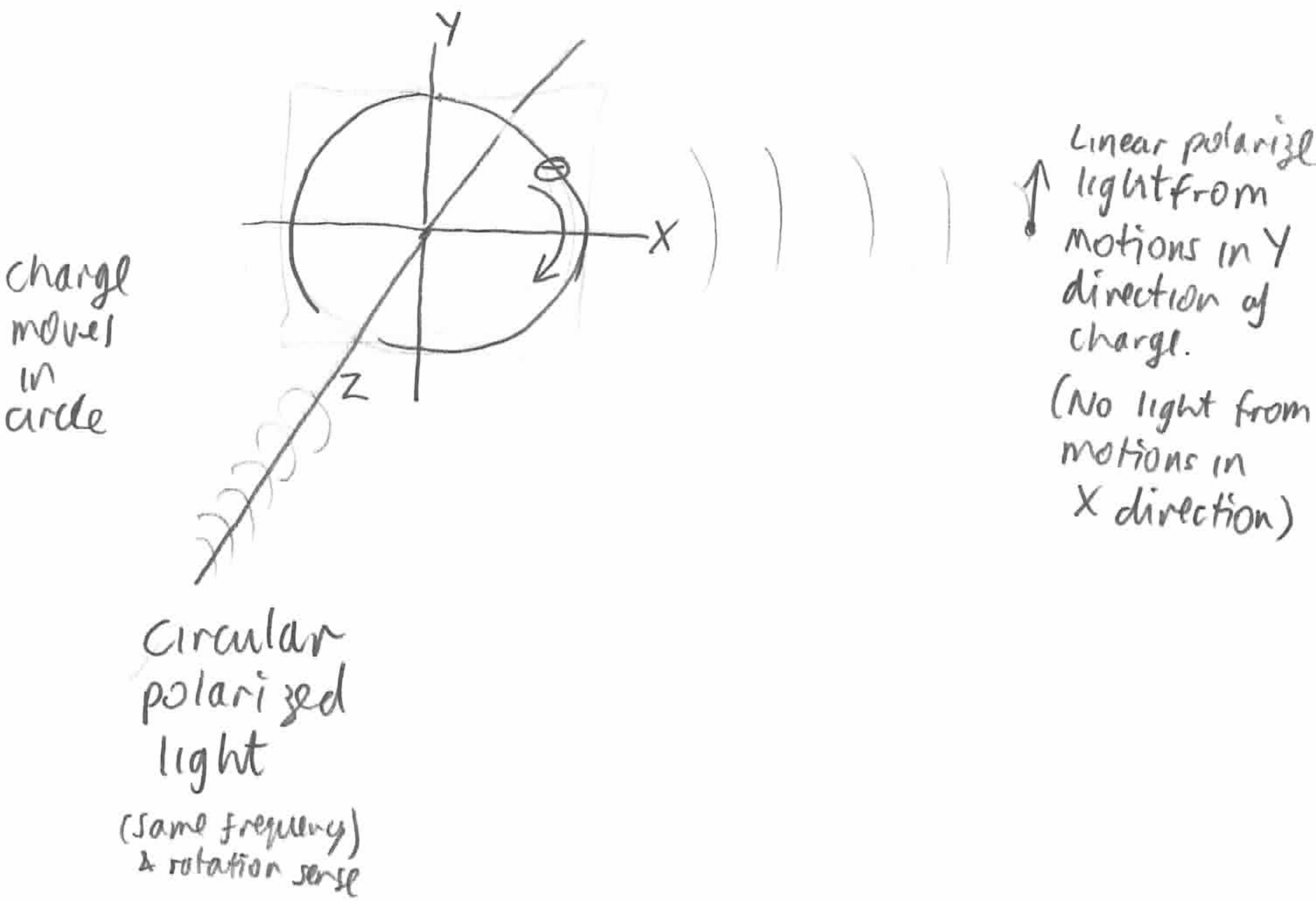


circular polarized light



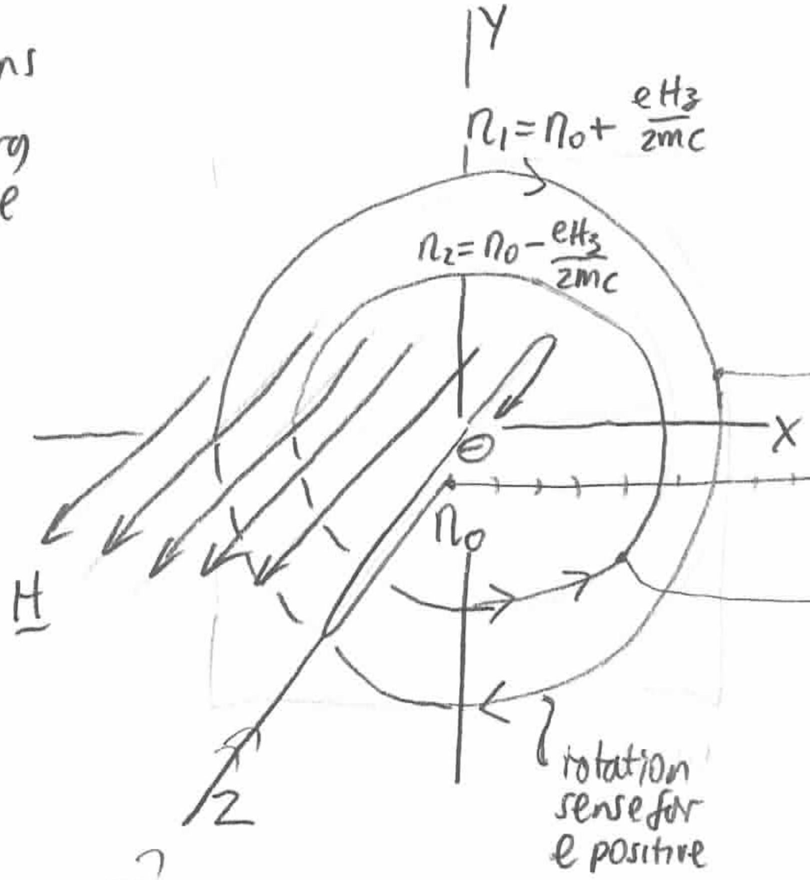
Lorentz on polarization of light emitted by accelerating charges





(Normal) Recover Zeeman Effect

motions of emitting charge



Radiation emitted transverse to magnetic fied

$n_0 + \frac{eH_z}{2mc}$ Polarized in Y direct.

n_0 Polarized in Z direction

$n_0 - \frac{eH_z}{2mc}$ Polarized in Y direct

Frequences

Triple lines (triplet)

$n_0 + \frac{eH_z}{2mc}$ circular polorized (right handed) (i.e e is negative)

$n_0 - \frac{eH_z}{2mc}$ circular polarized (opposite handedness)

Double line (Doublet)

Radiation emitted in direction of magnetic field

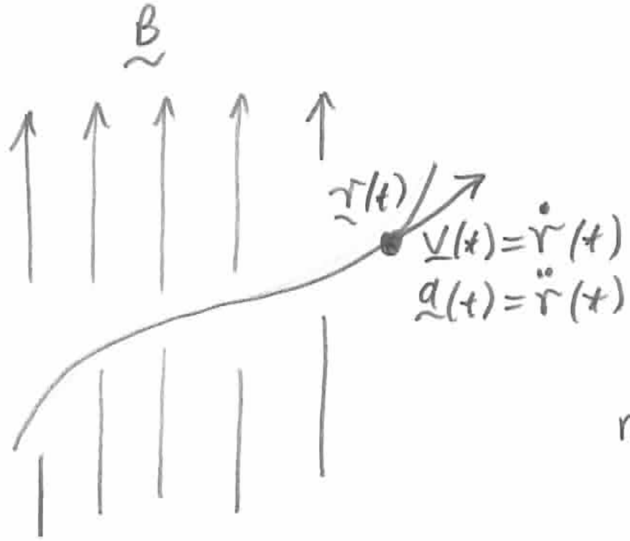
Lorentz reports Zeeman:

1. sense of rotation of lower/higher frequencies such that $\boxed{e \text{ is negative}}$ ← NOT ASSUMED BEFORE

2. Determine e/m

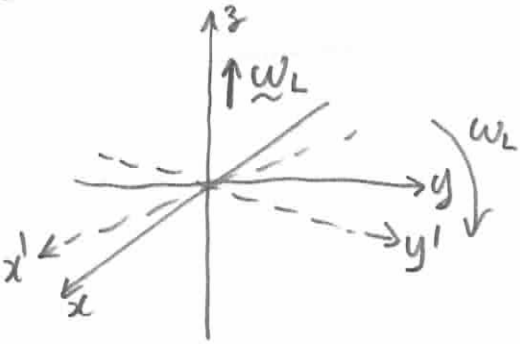
Larmor's Theorem:	Effect weak magnetic field on charged particle motion	$\equiv$	Slow rotation of coordinate system

(Hindmarsh, P. 65)



Charge e moves along trajectory $\underset{\sim}{r}(t)$ in <u>weak</u>, constant magnetic field $\underset{\sim}{B}$

$$m_0 \ddot{\underset{\sim}{r}} = \underbrace{m_0 \underset{\sim}{f_0}}_{\text{all other forces}} - \underbrace{\frac{e}{c}(\dot{\underset{\sim}{r}} \times \underset{\sim}{B})}_{\text{magnetic force}}$$

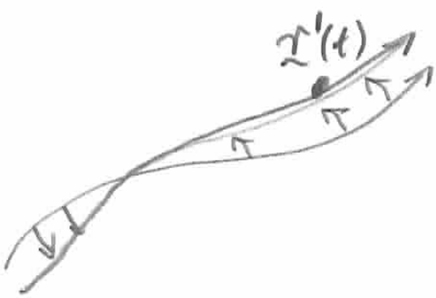


Transform to new coordinate system that rotates <u>slowly</u> around axis of field direction at ω_L

Neglect centrifugal terms in ω_L^2

$$\underset{\sim}{\omega_L} = -\frac{e}{2m_0 c}\underset{\sim}{B}$$

No $\underset{\sim}{B}$



New motion $\underset{\sim}{r}'(t)$ governed by

$$m_0 \ddot{\underset{\sim}{r}}' = m_0 \underset{\sim}{f_0}$$