THE EINSTEIN-BESSO MANUSCRIPT ON THE MOTION OF THE PERIHELION OF MERCURY

I

The manuscript, written jointly by Einstein and his lifelong friend Michele Besso, deals with the perihelion motion of Mercury[1] on the basis of the "Entwurf" theory.[2] Internal and external evidence suggest that the two men closely collaborated on this problem during a visit by Besso with Einstein in Zurich in June 1913. The bulk of the manuscript was presumably written during this visit. Einstein may have added some material later in 1913; some of the Besso material is certainly from later, most likely from early 1914.

From early on in his search for a new relativistic theory of gravitation, Einstein was interested in the problem of Mercury's perihelion. In a letter to a friend in 1907,[3] Einstein had already expressed his hope that such a theory would explain the anomalous advance of Mercury's perihelion. After this letter, however, the Mercury anomaly is not mentioned again, neither in Einstein's published papers nor in surviving correspondence, until late 1915.[4] On 18 November 1915 Einstein presented a paper[5] to the Prussian Academy of Sciences showing that his new, still developing general theory of relativity yields a perihelion advance for Mercury of 43″ per century, in striking agreement with observation. The success carried over unproblematically to the final version of the theory published shortly thereafter.[6]

When Einstein subsequently reported on his new theory to Sommerfeld[7] and Lorentz,[8] he mentioned that one of the reasons for abandoning the 1913 "Entwurf" theory was that it failed to explain the Mercury anomaly.[9] In the letter to Sommerfeld, he explicitly mentioned that the "Entwurf" theory predicts a perihelion advance for Mercury of 18″ per century. This is also the number given by Droste in a paper published in December 1914.[10] The manuscript under discussion reveals that Einstein actually did the calculation himself, shortly after he and Grossmann had finished the "Entwurf" paper[11] in early 1913. The expression Einstein and Besso arrived at for

[1]For a discussion of the history of the problem of the anomalous advance of the perihelion of Mercury, see *Roseveare 1982*.

[2]See the editorial note, "Einstein on Gravitation and Relativity: The Collaboration with Marcel Grossmann," pp. xx, for more on this theory.

[3]Einstein to Conrad Habicht, 24 December 1907 (Vol. 5, Doc. 69).

[4]There is a reference to calculations on perihelion motion in a letter to Erwin Freundlich, dated 30 September 1915.

[5]*Einstein 1915d*.

[6]*Einstein 1915e*.

[7]Einstein to Arnold Sommerfeld, 28 November 1915.

[8]Einstein to H.A. Lorentz, 1 January 1916.

[9]See *Norton 1984*, sec. 7.

[10]*Droste 1914*, p. 1010.

[11]*Einstein and Grossmann 1913* (Doc. 13).

the basic effect (i.e., the perihelion motion produced by the sun conceived of as a static mass distribution) is correct, but there is a mistake of a factor 10 in the number that is inserted for the mass of the sun, which yields a mistake of a factor 100 in the final result. Einstein writes on [p. 28]: "1821″ = 30′ independently checked."[12] The mistake is discovered in the following pages (on [p. 30] by Einstein, on [p. 35] by Besso), but there is no clear and unambiguous statement of the correct result in the manuscript. The so-called Scratch Notebook[13] contains an expression for the perihelion advance of Mercury which to a good approximation is equivalent to the expressions given in this manuscript. In the "Scratch Notebook" the correct numbers are inserted and the end result is given as 17″.

More important than the actual numbers is Einstein and Besso's derivation of the expression for the perihelion motion predicted by the "Entwurf" theory. It turns out that the method used in 1913 is virtually identical to the method Einstein used in his November 1915 paper on Mercury.[14] This may help to explain why Einstein was able to write this paper in such a short time.[15]

The remainder of this editorial note is organized as follows. In sec. II a brief outline will be given of the three main derivations in the manuscript, with further details provided in footnotes to the transcription. These three derivations, all in the context of the "Entwurf" theory, concern the motion of perihelia in the metric field of both a static and a rotating sun (see [pp. 1–30] and [pp. 32–35], plus some material on [p. 40]) and the motion of nodes in the field of a rotating sun (see [pp. 45–49], plus some material on [p. 31] and [pp. 41–42]). These three topics occupy 39 of the 53 pages of the manuscript. The remaining 14 pages deal with the following topics: a plan for various corrections to the analysis in *Newcomb 1895* on the basis of the "Entwurf" theory ([p. 31]); the metric field inside a rotating shell and the relativity of inertia ([pp. 36–38]);[16] the perihelion motion in a special relativistic gravitational theory ([p. 39]); an expression for the period of a Newtonian orbit in terms of the orbiting particle's total energy ([p. 40]); the "Entwurf" field equations and Minkowski space-time in a rotating coordinate system ([pp. 41–42]); the "Entwurf" field equations for what is called the "Eulerian case" ("Eulerscher Fall") ([pp. 43–44]); the metric field inside a rotating ring and the

[12]". . . unabhängig geprüft."

[13]Vol. 3, Appendix A, [p. 61].

[14]See *Earman and Janssen 1993*, pp. 142–143, pp. 156–157.

[15]See David Hilbert to Einstein, 19 November 1915: ". . . congratulations on conquering the perihelion motion. If I could calculate as fast as you, the electron would be forced to surrender in the face of my equations and the hydrogen atom would have to present an excuse for the fact that it does not radiate" (". . . herzliche Gratulation zu der Überwältigung der Perihelbewegung. Wenn ich so rasch rechnen könnte, wie Sie, müsste bei meinen Gleichungen entsprechend das Elektron kapituliren und zugleich das Wasserstoffatom seinen Entschuldigungszettel aufzeigen, warum es nicht strahlt").

[16]On these pages Einstein calculates the Machian effects he described in letters to Mach and Lorentz of this period: the gravitational effects at the center of a spherical mass shell when the shell either rotates uniformly or is accelerated uniformly and rectilinearly (see Einstein to Ernst Mach, 25 June 1913 [Vol. 5, Doc. 448], and Einstein to H. A. Lorentz, 14 August 1913 [Vol. 5, Doc. 467]).