Eric Anderson

The Reduction of Theories

Ernest Nagel

Ch. 11 of *The Structure of Science: Problems in the Logic of Scientific Explanation*

Theory Reduction: What is it?

"the explanation [or derivation] of a theory or a set of experimental laws established in one area of inquiry, by a theory … formulated for some other domain."

Theory Reduction: What's Nagel's angle?

- Nagel thinks that theory reduction has engendered misunderstandings

- When one theory is reduced to another, the usage of key terms is altered or extended; This can create spurious problems of interpretation if the logic of the reduction is not clearly understood.

Whether reduction succeeds …

"…the claim that the findings of physics are radically incompatible with so-called 'common sense'..."

"…the successful reduction of thermodynamics to statistical mechanics in the nineteenth century was taken to prove … that the diverse qualities of things and events which men encounter in their daily lives are not 'ultimate' traits of the world and are perhaps not even 'real.'"

Or fails …

"… the difficulty in finding consistent visualizable models for the mathematical formalism of quantum mechanics has been taken as evidence for the 'mysterious' character of subatomic processes and for the claim that behind the opaque symbolism of the 'world of physics' there is a pervasive 'spiritual reality' that is not indifferent or alien to human values."

"…the failure to explain electromagnetic phenomena in terms of mechanics… have been construed as evidence for the "bankruptcy" of classical physics … and for a variety of sweeping doctrines concerning levels of being, emergence, and creative novelty."

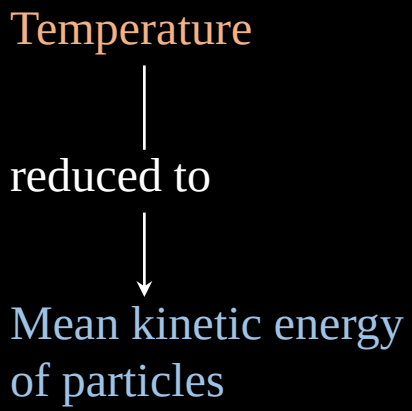
Nagel thinks interpretations such as these are dubious.

He does not critique them in detail but will indicate that they result from a mistaken logical or metaphysical—rather than empirical, historical, and methodological—interpretation of theory reduction.

Thermodynamics and Statistical Mechanics

- The reduction of thermodynamics to statistical mechanics is Nagel's main example
- How does this reduction work?

"By what reasoning is it apparently possible to derive statements containing such terms as 'temperature,' 'heat,' and 'entropy' from a set of theoretical assumptions in which those terms do not appear?"



Deriving the ideal gas law from statistical mechanics;

For a quantity of gas in a container of volume V:

Thermodynamics:

$PV = kT$

- P is pressure
- V is volume
- T is **temperature**
- k is a constant for a given amount of gas

Statistical mechanics:

$PV = 2/3\ E$

- P is pressure
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- E is **mean kinetic energy** of gas molecules

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But what is the nature of this postulate?

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Thermodynamics and Statistical Mechanics

- To clarify and demystify this process, Nagel provides general conditions for theory reduction.

- These come in two types: formal conditions and informal conditions

Formal preliminaries

1. Axioms, specific hypotheses, and experimental laws involved in a reduction must be available as explicitly formulated statements with unambiguous rules of use. (This requirement is only “an ideal demand.”)

2. Every statement of a science involved in a reduction can be analyzed as a linguistic structure, built out of more elementary expressions.

3. The reduced and reducing sciences will share a large number of expressions that have the same meaning in both sciences; but the reduced science will employ expressions that do not occur in the reducing science prior to the reduction (e.g. P and V vs. T).

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But if the reduced science contains terms that do not occur in the reducing science, the logical derivation of the former from the latter is *prima facie* impossible.

So, there are two necessary formal conditions for the reduction to take place.

Formal conditions for reduction

1. Condition of connectability:

   We need to introduce **assumptions** which postulate a connection between the state of affairs signified by '**A**,' which is the term appearing only in the reduced science, and a state of affairs represented by terms already appearing in the reducing science.

   This is the postulate we met earlier, connecting T to E:

   $$kT = 2/3\ E$$

Formal conditions for reduction

2. Condition of derivability:

With the help of these **assumptions**, we need to be able to logically derive the laws of the reduced science, including those containing the term '**A**,' from the theoretical premises of the reducing science.

So, from PV = 2/3 E, with the help of kT = 2/3 E, we may derive PV = kT.

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      But the question remains: what is the **nature** of this assumption?

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2. Conventional: The assumptions *stipulate* a definitional correspondence between '**A**' and '**B**.'

3. Factual / material: The assumptions are physical hypotheses, asserting that the occurrence of the state of affairs signified by '**B**' in the reducing science is a sufficient (or necessary and sufficient) condition for the state of affairs designated by '**A**.'

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Is this postulate

1. Logical
2. Conventional

   or
3. Factual / material

   ?

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3. Factual / material

   But this also seems right: Although E cannot be measured directly, there are ways to ascertain it experimentally that do not rely on measuring T. So, it is possible to determine experimentally whether T is proportional to E.

1. Conventional

   This seems right

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2. Factual / material

   This also seems right

Nagel does not think there is a contradiction here

The nature of the linking assumption depends on the context in which the reduction is exposited.

In some contexts it will be a stipulative convention; in others it will be a factual hypothesis.

“The essential point in this discussion is that in the reduction of thermodynamics to mechanics a postulate connecting temperature and mean kinetic energy of gas molecules must be introduced, and that this postulate cannot be warranted by simply explicating the meanings of the expressions contained in it.”

NOTE WELL:

Nagel argues that we cannot simply redefine “temperature” as molecular energy. This won’t work because that is not what the word means in classical thermodynamics, and it is temperature in this sense that we wish to reduce to molecular energy.

If we simply redefine temperature, we still need to show that the two senses of the word are co-extensive—which means adopting one of the previous strategies.

Informal conditions for theory reduction

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1. First are conditions that distinguish between important and trivial reductions.
    - The theoretical assumptions of the reducing science should be empirically supported.
    - The reduction should be unifying and fertile: it should supply a basis for the unification of disparate laws in the reduced science and should lead to the discovery of new laws.

Informal conditions for theory reduction

2. Whether one science can be reduced to another *depends on the stage of development* of the two sciences.
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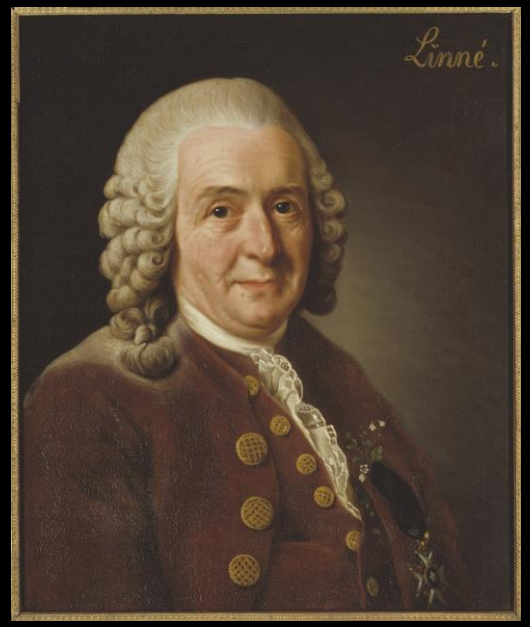

Linnaean botany would not have benefitted from a physio-chemical theory of organisms.

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> "…questions that at bottom relate to the strategy of research, or to the logical relations between sciences as constituted at a certain time, are commonly discussed as if they were about some ultimate and immutable structure of the universe."

Informal conditions for theory reduction

3. Reduction of one science to another is the derivation of one set of empirically verifiable *statements* from another such set; it is not the derivation of the *properties*, or *"natures"* of one domain from those of another.
    - The latter conception of reduction creates spurious problems.
    - The ontological "natures" or "elementary constituents" of things are not accessible to direct observation.
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Reduction is *theory relative*: our particular theories of things may or may not be reducible to other theories, but reduction is not a function of, and does not carry radical implications for, the metaphysical or ontological status of the things themselves.

Critic of reduction:

“‘a headache is not an arrangement or rearrangement of particles in one’s cranium’”

"…the reduction of one science to a second … does not wipe out or transform into something insubstantial or 'merely apparent' the distinctions and types of behavior which the [reduced] discipline recognizes. Thus, if and when the detailed physical, chemical, and physiological conditions for the occurrence of headaches are ascertained, headaches will not thereby be shown to be illusory. …all that will have happened is that an explanation will have been found for the occurrence of headaches. … It will not establish a logically necessary connection between the occurrence of headaches and the occurrence of certain events or processes specified by physics, chemistry, and physiology. Nor will it consist in establishing the synonymy of the term 'headache' with some expression defined by means of the theoretical primitives of these disciplines. It will consist in stating the conditions, formulated by means of these primitives, under which, and as a matter of sheer contingent fact, a determinate psychological phenomenon takes place."

Gems

Demystification of theory reduction

Formal vs. informal conditions

Dense, at times needlessly formalized, presentation