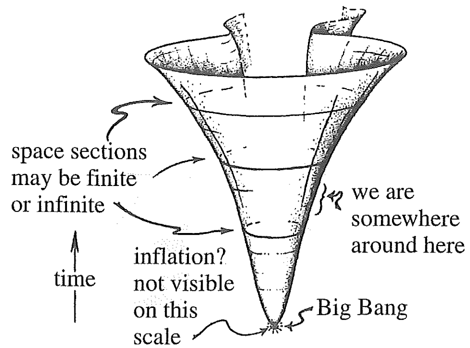


Figure 3-3: Space-time depiction of our universe's expected evolution, as modified to incorporate the observed (sufficiently large) $\Lambda > 0$. The depicted uncertainty in the behaviour at the back of the picture is to reflect an uncertainty in the overall spatial geometry, which has no significant evolutionary role.

universe, where the remote-future energy tensor is expected to be completely dominated by $\Lambda$, giving $\mathbf{G} \approx \Lambda \mathbf{g}$ in the future limit.

This, of course, assumes that Einstein's equations ($\mathbf{G} = 8\pi\gamma\mathbf{T} + \Lambda\mathbf{g}$) continue to hold indefinitely, so that our presently ascertained value of $\Lambda$ remains a constant. In §3.9, we shall see that, according to the exotic ideas of inflationary cosmology, the de Sitter model is also taken to describe the universe during a much earlier time immediately following the Big Bang, though with an enormously larger value of $\Lambda$. These issues will be of considerable importance for us later (especially in §§3.7–3.9 and 4.3), but will not particularly concern us for the moment.

De Sitter space is a highly symmetrical space-time, which can be described as a (pseudo-)sphere in Minkowski 5-space; see figure 3-4(a). Explicitly, it arises as the locus $t^2 - w^2 - x^2 - y^2 - z^2 = -3/\Lambda$, obtaining its local metric structure from that of the ambient Minkowski 5-space with coordinates $(t, w, x, y, t)$. (For those familiar with the standard way of writing metrics using differentials, this Minkowski 5-metric takes the form $\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}w^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$.) The de Sitter space is fully as symmetrical as Minkowski 4-space, each having a 10-parameter symmetry group. We may also recall the hypothetical anti–de Sitter space that was considered in §1.15 (see footnote 9 on p. 113). It is very closely related to de Sitter space, and it also has a symmetry group of this size.

De Sitter space is an empty model, its energy tensor $\mathbf{T}$ being zero, so it has no (idealized) galaxies to define time-lines, whose orthogonal 3-space sections could have been used to determine specific 3-geometries of "simultaneous time". In fact, rather remarkably, it turns out that in de Sitter space we can choose such



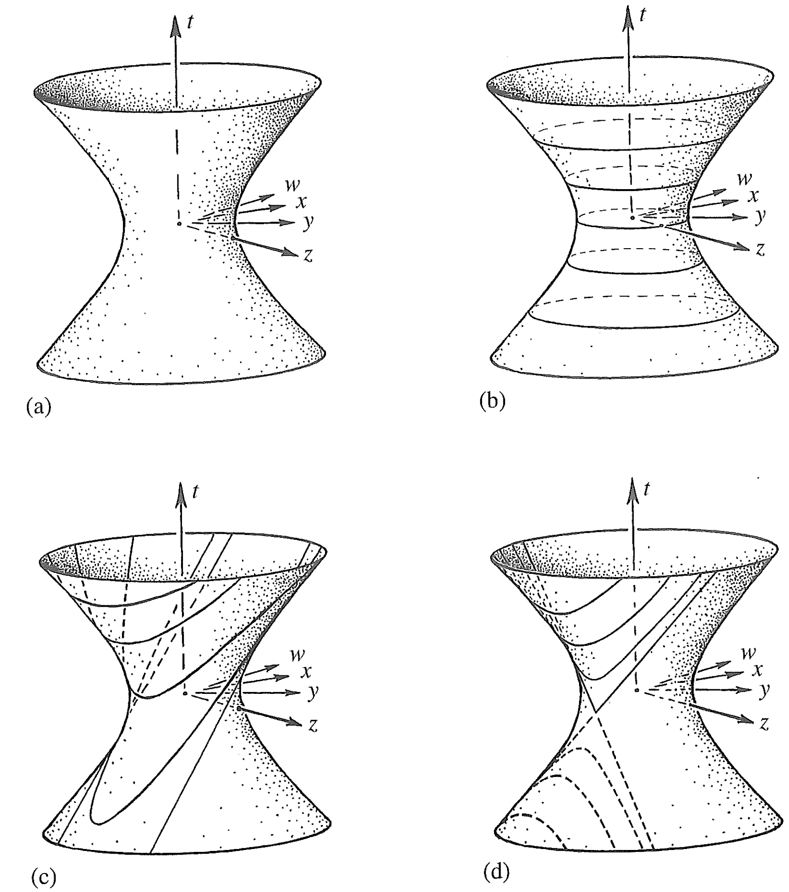


Figure 3-4: (a) De Sitter space, (b) with $K > 0$ time-slicing ($t$ = const.), with (c) $K = 0$ time-slicing ($t - w$ = const.) as in steady-state cosmology, and (d) with $K < 0$ time-slicing ($-w$ = const.).

3-dimensional spatial (simultaneous-time) sections in three essentially different ways, so that de Sitter space can be interpreted as an expanding spatially uniform universe with each of the three alternative types of spatial curvature, depending upon the way that it is sliced with such 3-surfaces, taken as being of constant cosmic time: $K > 0$ (given by $t$ = const.), $K = 0$ (given by $t - w$ = const.), and $K < 0$ (given by $-w$ = const.); see figure 3-4(b)–(d). This was elegantly shown by Erwin Schrödinger in his 1956 book, *Expanding Universes*. The old steady-state model that we shall come to in §3.2 is described by de Sitter space, according to the $K = 0$ slicing shown in figure 3-4(c) (and conformally represented in figure 3-26(b) in §3.5). Most versions of inflationary cosmology (that we shall be coming to in §3.9) also use this $K = 0$ slicing, as this allows the inflation to continue in a uniformly exponential way for an indefinite time.

In fact, with regard to our actual universe on an extremely large scale, present observations do not point unambiguously to which of these spatial geometries might provide the most appropriate picture. But whatever the ultimate answer, it does now appear that the case $K = 0$ is very close to being correct (somewhat remarkably, in view of strong-seeming evidence for $K < 0$ towards the end of the twentieth century). In one sense, this is the least satisfactory observational situation, since if all that can be said is that $K$ is very close to zero, we still cannot be sure that more refined observations (or a more convincing theory) may not later point to one of the other spatial geometries (spherical or hyperbolic, that is) being more appropriate for our universe. If, for example, good evidence for $K > 0$ were finally to emerge, this would have genuine philosophical significance, since it would have the implication that the universe is not spatially infinite. As things stand, however, it is normally simply asserted that the observations tell us that $K = 0$. This may well be a very good close approximation, but in any case we do not know how close to actual spatial homogeneity and isotropy the overall universe might be, particularly in view of certain counter indications in the CMB observations [e.g. Starkman et al. 2012; Gurzadyan and Penrose 2013, 2016].

To complete the picture of the entire space-time, according to Friedmann's models and their generalizations, we need to know how the "size" of the spatial geometry would evolve with time, right from the start. In the standard cosmological models, like Friedmann's – or the generalizations known as *Friedmann–Lemaître–Robertson–Walker* (FLRW) models, where all in this general class have spatial sections that are homogeneous and isotropic, the whole space-time sharing the symmetry of these sections – there is a well-defined notion of a *cosmic time* $t$ to describe the evolution of the universe model. This cosmic time is the time measure, starting with $t = 0$ at the Big Bang, that would be measured by an ideal clock following the world-lines of the idealized galaxies; see figure 3-5 (and figure 1-17 in §1.7). I shall refer to these world-lines as the *time-lines* of the FLRW model (sometimes referred to as the world-lines of the *fundamental observers*, in cosmology texts). The time-lines are the geodesic curves orthogonal to the spatial sections, those being the 3-surfaces of constant $t$.

The case of de Sitter space is somewhat anomalous, in this respect, because, as mentioned earlier, it is empty in the sense that $\mathbf{T} = \mathbf{0}$, in Einstein's $\mathbf{G} = 8\pi\mathbf{T} + \Lambda\mathbf{g}$, so there are no matter world-lines to provide time-lines or, consequently, to define spatial geometries, and consequently we have a choice, locally, as to whether we regard the model as describing a $K > 0$, $K = 0$, or $K < 0$ universe. Nevertheless,



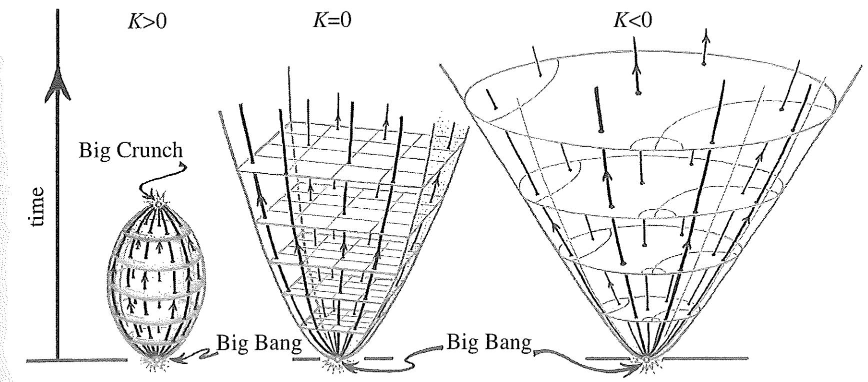


Figure 3-5: The Friedmann models of figure 3-2 with time-lines (idealized galaxy world-lines) drawn in.

globally the three situations are different, as we can see from figure 3-4(b)–(d) that in each case a different portion of the entire de Sitter space is covered by the slicing. In the discussions below, I shall assume that $\mathbb{T}$ is non-zero, providing a positive energy density of matter, so that the time-lines are well defined, and so are the spacelike 3-surfaces of constant time for each $t$ value, as shown in figure 3-2.

In the positive spatial curvature case $K > 0$, for a standard dust-filled Friedmann universe, we can use the radius $R$ of the 3-sphere spatial sections to characterize the "size", and examine this as a function of $t$. When $\Lambda = 0$, we find a function $R(t)$ that describes a *cycloid* in the $(R, t)$-plane (taking the speed of light $c = 1$), this being the curve having the simple geometrical description of being traced out by a point on the circumference of a circular hoop (of fixed diameter equal to the maximum value $R_{\max}$ attained by $R(t)$) that is rolling along the $t$-axis (see figure 3-6(b)). We note that (after a time given by $\pi R_{\max}$) the value of $R$ reaches the value 0, again, as it had at the Big Bang, so the entire universe model (with $0 < t < \pi R_{\max}$) collapses down to a second singular state, often referred to as the *Big Crunch*.

In the remaining cases $K < 0$ and $K = 0$ (and $\Lambda = 0$), the universe model expands indefinitely, and there is no Big Crunch. For $K < 0$ there is an appropriate notion of "radius", analogous to $R$, but for $K = 0$ we can just pick an arbitrary pair of idealized galaxy world-lines and take $R$ to be their spatial separation. In the case $K = 0$, the expansion rate slows down to zero asymptotically, but it reaches a limiting positive value in the case $K < 0$. With present observations leading to the belief that $\Lambda$ is actually positive and is large enough ultimately to dominate

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ROGER PENROSE

# FASHION Faith and FANTASY in the New Physics of the Universe

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