# Contraction of the length of a curve under uniform contraction of the enclosing space

Consider a curve in a space with Cartesian coordinates (*x,y*), where the curve is represented by the functional dependence

*y* = *y*(*x*)

The length of the curve between limits *x* = *a* and *x* = *b* is

We form a new curve

*y'* = *y’*(*x’*)

 contracted vertically by the constant factor *k* and horizontally by the same factor *k*. The new variables *y’* and *x’* related to the old variables *y* and *x* according to:

*y’* = *ky* and *x’* = *kx*

(Informally, the value of the new curve *y’* at a point *x’* reduced by a factor of *k* from *x* is equal to the value of the original function *y*(*x*) at *x*, but now also reduced by a factor of *k*.) The new curve extends between the limits *x’* = *ka* and *x’* = *kb*. We have for the length of the new curve:

Substituting unprimed variables for primed variables, this length becomes

That is, the length of the curve is also contracted by a factor *k.*

Inspection of the calculation shows that the result depends on both variables *x* and *y* being contracted by the same factor *k*. Otherwise, we have a failure of the cancellations of factors: