CHRISTIANI HUGENII LIBELLUS DE RATIOCINIIS IN *LUDO ALEÆ.*

OR,

The VALUE of all

CHANCES

IN

Games of Fortune;

CARDS, DICE, WAGERS, LOTTERIES, *&c.*

Mathematically Demonstrated.

LONDON:
Printed by S. KEIMER, for T. WOODWARD, near the *Inner Temple-Gate* in *Fleetstreet*. 1714.

TO

Dr. *RICHARD MEAD*,

Physician to St. *Thomas*'s Hospital, and Fellow of the ROYAL SOCIETY.

HONOUR'D SIR,

WHEN I consider the Subject of the following Papers, I can no more forbear dedicating them to Your Name, than I can refuse giving my assent to any one Proposition in these Sciences, which I have already seen clearly demonstrated. The Reason is plain, for as You have contributed the greatest Lustre and Glory to a very considerable part of the Mathematicks, by introducing them into their noblest Province, the Theory of Physick; the Publisher of any Truths of that Nature, who is desirous of seeing them come to their utmost Perfection, must of course beg Your Patronage and Application of them. By so prudent a Course as this, he may perhaps see those Propositions which

it was his utmoſt Ambition to make capable only of directing Men in the Management of their Purſes, and inſtructing them to what Chances and Hazards they might ſafely commit their Money; turn'd ſome time or other to a much more glorious End, and made inſtrumental likewiſe towards the ſecuring their Bodies from the Tricks of that too ſucceſsful Sharper, Death, *and countermining the underhand Dealings of ſecret and over-reaching* Diſtempers.

THE moſt celebrated Endeavours of the greateſt GENIUS *that ever appear'd before in* Mathematical Learning, *have been able to carry it no farther, than to calculate by its means the Motions of the* Heavenly Bodies *more exactly, explain the Myſtery of* Tides, *the Doctrine of* Sounds *and* Light, *and other curious* Phœnomena *in the Works of Nature, which are without us: But You,* Great Sir, *have made this wonderful Clew conduct us in Paths much more intricate and obſcure, and guide us thro' the far more curious and puzzling Labyrinths of* Human Body; *and have as far out-done thoſe nobleſt Improvements, as the glorious Frame of Man exceeds all the reſt of Nature's Works, and as the Knowledge of ourſelves is preferable to all other parts of* Science *whatever. All our Hopes that this Sort of Learning will meet with a warmer and more welcome Reception from the preſent*

and

and future Ages, than it has from thoſe paſt, are entirely owing to Your inſtructing us in the true Value of Mathematical Diſquiſitions, *and diſcovering to us their moſt excellent Uſes which our Anceſtors were perfectly ignorant of. The* Mathematicks *being formerly believ'd of no further Service, than as they aſſiſted us in the Buſineſs of our Hands, by directing our Mechanical Operations, or diverted our Heads with pleaſant and amuſing* Problems, *were then but particular Studies, and cultivated only by ſome few as a neceſſary help to their Employments, and by fewer as a Diverſion; but ſince You, to our extreme Surprize and Satisfaction, have ſhown their ſcarce dream'd of Excellencies in promoting the* Profeſſion of Phyſick *almoſt to a pitch of Certainty, and demonſtrated them capable, by a due Application, of adding Days and Years to the Length of our Lives, they muſt now neceſſarily become the general Concern of all Mankind, and the greateſt Subject of their Admiration and Eſteem.*

PERHAPS the few following Propoſitions, *may to the World appear ſo foreign to this Purpoſe, as to be thought perfectly incapable of being directed to ſuch a mighty End. For what can there be in common between the Value of a* Chance *in a* Game, *and the* Knowledge *and* Cure *of a* Diſtemper? *And how can the niceſt Determination of the former, any way influence*

fluence or illustrate the latter? But these apparent Inconsistencies, Sir, *are not such as can impose upon Your clearer Judgment, at the Light of which, they vanish like Phantoms at the break of Day. You have reconciled Truths that have seem'd to us much more inconsistent than these, and even demonstrated what the wiser part of Mankind before, had always look'd upon as absurd and impossible, That the State of our Bodies on Earth, are subject to surprizing Alterations and Changes from the various* Positions *of those two heavenly ones, the* SUN *and* MOON. *So that we can now talk of their Conjunctions, Oppositions, and Quadratures, in relation to Human Diseases, without being any longer liable to the usual Reproaches of Ignorance and Infatuation, justly chargeable upon the old judicial* Astrology. *Your late learned Friend, Dr.* PITCAIRNE, *made no difficulty of listing this Subject into the Service of* Physick, *when he gave our* Third Propposition *a very honourable Post in his* Dissertat. de Circulatione sanguinis, *and found it of Force enough to overthrow and defeat for ever, that erroneous and long standing Doctrine of* Secretion, *which supposes the Orifice of each Secretory Duct to be of a different Figure, and as admitting only those Particles of the Blood that are of a similar Figure and Magnitude. Yes, there is but too much of Chance and*

Uncertainty

Uncertainty in Human Conſtitutions, and the Duration of this mortal Life; and if a proper Application of this Doctrine might aſſiſt us to ſet a true and exact Value upon that Uncertainty, perhaps the Phyſicians *will not be the only Perſons that are like to benefit by it; but the* Divines *likewiſe may be furniſh'd from thence with the moſt convincing and, I think, the only Arguments they have left to make uſe of; for that eminent Body have ſo far improv'd in their Admonitions and Reaſonings with Mankind, that nothing but Mathematical Demonſtration ſeems able to do them any farther Service. Such an Application as this, together with all the Advantages the other Parts of the* Mathematicks *can confer upon the Science of* Phyſick, *are what we muſt only expect from Your Pen; whenever Your Hurry of Buſineſs will allow You a few leiſure Intervals, and a convenient Opportunity to reſume it.*

YOU have propheſy'd, Sir, in the Preface to Your excellent Piece upon Poiſons, *that the true* Phyſician *ſhall hereafter be diſtinguiſh'd from the* Quack, *not only by a perfect acquaintance with the learned Languages, but a conſiderable Proficiency likewiſe in the* Mathematicks: *and the Prognoſtick is pretty ſurely grounded; for that happy Diſtinction is not only foretold, but eſtabliſh'd by your Writings, the Pleaſure of reading which, will moſt certainly compleat that*

Pre-

Prediction. *For the Veneration the World pays to Your Works, is so equal to their Value, that we find none desirous of being accounted* Physicians, *who have not first made themselves thoroughly acquainted with them; and consequently none are like to be so, without the Qualifications by You prescribed.*

I have made no mention, Sir, of the many Personal Obligations I lie under to You, and the singular Favours and Encouragement I have met with at Your Hands, because they cannot be imagin'd to have been any Motives to this Address; which is so far from being able to discharge the smallest portion of that great Debt, that on the part of Mr. HUYGENS, *it is only my Duty, and but Justice done You, and on my own, a very considerable Addition to what I already owe Your unlimited Goodness and Generosity, and too great an Honour done to,*

SIR,

Your most Obedient,

Obliged, and humble Servant,

W. Browne.

ADVERTISEMENT

TO THE

READER.

AS all Mathematical Studies in general are unaccountably bewitching and delightful to those that are once happily engaged in them; so that part which considers and estimates our *Expectations* of *Events* that are in themselves *uncertain*, and depend entirely upon *Chance* and *Hazard*, cannot fail of giving a particular Pleasure and Satisfaction. To reduce the inconstant and irregular Proceedings of blind *Fortune* to certain Rules and Limits, and to set a definite *Value* upon her capricious *Favours* and *Smiles*, seem to be Undertakings of so chimærical a Nature, that there is no Body but must be delightfully surprized with that Art which discovers them both really possible, and with a little Application easily practicable.

But I think there is little need of saying in Commendation either of the Subject, any thing or of the Manner in which M. Huygens has consider'd it; the great scarcity of his lit-

tle Treatiſe upon it, and the general want of a new Edition making that altogether ſuperfluous. The juſt value every one has for his Performance this Way, as well as for his other admirable Pieces, abundantly ſhow that they are ſufficiently ſenſible of its Excellencies, and is the greateſt Commendation it can poſſibly receive. Beſides the Latin Editions it has paſs'd thro', the learned Dr. ARBUTHNOTT publiſh'd an *Engliſh* one, together with an Application of the General Doctrine to ſome particular *Games* then moſt in uſe; which is ſo intirely diſpers'd Abroad, that an Account of it is all we can now meet with. A late *French Author* has indeed illuſtrated this Work, with a plentiful Number of Inſtances where it may be ſerviceable; but all that he has done ſerves only to demonſtrate what an inexhauſtible Treaſure this little Book of M. HUYGENS contains, and how eaſily it will ſatisfy every ones Occaſions, that will but be at the Pains to make uſe of it. Our excellent Analyſt M. DE MOIVRE likewiſe has wonderfully improv'd the Subject, and beſides taking the different Dexterities of the Gameſters into the Account, has much ſhorten'd the Calculation, and made the whole more general and compleat. But nothing can reflect more Honour upon M. HUYGENS than this laſt admirable Performance, which muſt needs ſhow us what an excellent Foundation that learned Man laid, that cou'd ſupport and give Strength to ſuch a glorious Structure as M. DE MOIVRE has rais'd upon it. M. DE MOIVRE's Piece therefore will be very far from leſſening the Worth of M. HUYGENS's; and

and the ſuperficial Mathematicians will ſtill be glad to ſatisfy their Enquiries by this laſt Author's eaſy, tho' more tedious Method, as not being able to underſtand the other's more comprehenſive and general one; while thoſe of a greater Depth, will with no leſs delight firſt read M. HUYGENS's Treatiſe, in order to proceed with ſo much the greater Pleaſure afterwards to peruſe M. DE MOIVRE's Additions, and to perceive by what juſt Steps and Degrees the Subject has from theſe two Great Men receiv'd both its firſt Conſideration and laſt Perfection.

MY Deſign in publiſhing this Edition, was to have made it as Uſeful as poſſible, by an Addition of a very large *Appendix* to it, containing a Solution of ſome of the moſt ſerviceable and intricate Problems I cou'd think of, and ſuch as have not as yet, that I know of, met with a particular Conſideration: But an Information I have within theſe few Days receiv'd, that M. MONTMORT's *French* Piece is juſt newly reprinted at *Paris*, with very conſiderable Additions, has made me put a Stop to the *Appendix*, till I can procure a Sight of what has been added anew, for fear ſome part of it may poſſibly have been honour'd with the Notice and Conſideration of that ingenious Author. If the Reader is of Opinion that I ſhou'd likewiſe till then, have deferr'd the Publication of theſe few Pages, tho' printed off; and thinks much that he has not either all that was deſign'd him or none at all; or if he has any Exceptions to their appearing in the *Engliſh* Language, the Bookſeller is the only Perſon that is to anſwer for his not receiving Satisfaction in both thoſe Points.

Juſt Publiſhed the following BOOKS, *Printed for and Sold by* THO. WOODWARD, *next the* Inner Temple-Gate *in* Fleetſtreet.

A Short and eaſy Method to underſtand GEOGRAPHY, wherein are deſcribed the Form of Government of each Country, its Qualities, the Manners of its Inhabitants, and whatſoever is moſt Remarkable in it. To which are added, Obſervations upon thoſe Things of Importance that have happen'd in each State: With an ABRIDGMENT of the *Sphere*, and the Deſcription and Uſe of the *Globe*, *Geographical Maps* and *Sea-Charts*. *Engliſhed* by a Gentleman of *Cambridge*, from the *French* of Monſieur *A. D. Fer*, Geographer to the *French* King.

The Seventh Edition of an Help and Exhortation to *Worthy Communicating*; or, a Treatiſe deſcribing the Meaning, worthy Reception, Duty and Benefits of the *Holy Sacrament*; And anſwering the Doubs of Conſcience, and other Reaſons, which moſt generally detain Men from it. Together with ſuitable Devotions added. By JOHN KETTLEWELL, late Vicar of *Coles-hill* in *Warwickſhire*.

The Pifth Edition of a SERMON preach'd to the *Proteſtants of Ireland*, now in *London*, at the Pariſh-Church of St. *Mary-le-Bow*, *Oct.* 23. 1714. Being the Day appointed by Act of Parliament in *Ireland*, for an *Anniverſary Thankſgiving* for the Deliverance of the *Proteſtants* of that Kingdom from the BLOODY MASSACRE begun by the *Iriſh Papiſts*, on the 23*d* of *October*, 1641. By JOHN RAMSEY, Rector of *Langdon* in *Kent*.

And ſhortly will be publiſh'd,

A large *Appendix* to this Piece of Mr. HUYGENS on the *Value of Chances*; to be printed and ſold ſingle, in order to be bound up with it.

THE VALUE OF CHANCES.

ALTHOUGH in Games depending entirely upon Fortune, the Succeſs is always uncertain; yet it may be exactly determin'd at the ſame time, how much more likely one is to win than loſe. As, if any one ſhou'd lay that he wou'd throw the Number *Six* with a ſingle Die the firſt throw, it is indeed uncertain whether he will win or loſe; but how much more probability there is that he ſhou'd loſe than win, is preſently determin'd, and eaſily calculated. So likewiſe, if I agree with another to play the firſt Three Games for a certain Stake, and I have won one of my Three, it is yet uncertain which of us ſhall firſt get his third Game; but the Value of my Expectation, and his likewiſe, may be exactly diſcover'd; and conſequently it may be determin'd, if we ſhou'd both agree to give over play, and leave the remaining Games unfiniſh'd, how much more of the Stake comes to my Share than his;

or, if another desired to purchase my Place and Chance, how much I might justly sell it for. And from hence an infinite Number of Questions may arise between two, three, four, or more Gamesters: The satisfying of which being a thing neither vulgar nor useless, I shall here demonstrate in few Words, the Method of doing it; and then likewise explain particularly the Chances that belong more properly to Dice.

POSTULAT.

As a Foundation to the following *Propositions*, I shall take Leave to lay down this Self-evident Truth; *That my Chance or Expectation to win any thing, is worth just such a Sum, as wou'd again procure me the same Chance and Expectation at a fair Lay.* As for Example, if any one shou'd put 3 Shillings in one Hand, without letting me know which, and 7 in the other, and give me my Choice of either of them; I say, it is the same thing as if he shou'd give me 5 Shillings; because with 5 Shillings I can, at a fair Lay, procure the same even Chance or Expectation to win 3 or 7 Shillings.

PROP. I.

If I expect a *or* b, *and have an equal Chance of gaining either of them, my Expectation is worth* $\frac{a+b}{2}$.

THAT

To trace this Rule from its firſt Foundation, as well as demonſtrate it, having put x for the value of my Expectation, I muſt with x be able to procure the ſame Expectation at a fair Lay. Suppoſe then that I play with another upon this Condition, That each ſhall ſtake x, and he that wins give the Loſer a. 'Tis plain, the Play is fair, and that I have upon this Agreement an even Chance to gain a, if I loſe the Game; or $2x - a$, if I win it; for then I have the whole Stake $2x$, out of which I am to pay my Adverſary a. And if $2x - a$ be ſuppoſed equal to b, then I have an even Chance to gain either a or b. Therefore putting $2x - a = b$, we have $x = \frac{a+b}{2}$, for the Value of my Expectation. *Q. E. I.*

The Demonſtration of which is very eaſy: For having $\frac{a+b}{2}$, I can play with another, who ſhall likewiſe ſtake $\frac{a+b}{2}$, upon Condition that the Winner ſhall pay the Loſer a. By which means I muſt neceſſarily have an equal Expectation to gain a, if I am Loſer, or b, if I am Winner; for then I win $a + b$, the whole Stake, out of which I am to pay the Loſer a. *Q. E. D.*

In Numbers. If I have an equal Chance to 3 or 7, then my Expectation is, by this *Propoſition*, worth 5; and it is certain I can with 5, again procure the ſame Expectation: For if Two of us ſtake 5 a piece upon this Condition, That he that wins pay the other 3, 'tis plain the Lay is juſt

and that I have an even Chance to come off with 3, if I loſe, or 7 if I win; for then I gain 10, and pay my Adverſary 3 out of it. *Q. E. D.*

PROP. II.

If I expect a, b, *or* c, *and each of them be equally likely to fall to my Share, my Expectation is worth* $\frac{a+b+c}{3}$.

To calculate which, I again put x for the value of my Expectation: Therefore having x, I muſt be able, by fair Gaming, to procure the ſame Expectation. Suppoſing then I play with two others upon this Condition, That every one of us ſtake x; and I agree with one of them, that which ſoever of us Two wins, ſhall give the Loſer b; and with the other, that which ſoever of us Two wins, ſhall give the Loſer c. It appears evidently, that the Lay is very fair, and that I have by this means an equal Chance to gain b, if the firſt wins; or c if the ſecond wins; or $3x-b-c$, if I win my ſelf; for then I have the whole Stake $3x$, out of which I give b to one, and c to the other. But if $3x-b-c$ be ſuppoſed equal to a, then I have an equal Expectation of a, b, or c. Therefore putting $3x-b-c=a$, we ſhall find $x=\frac{a+b+c}{3}$, for the value of my Expectation. *Q. E. I.*

After the ſame Manner, an even Chance to a, b, c, or d, will be found worth $\frac{a+b+c+d}{4}$. And ſo on.

PROP.

PROP. III.

If the number of Chances I have to gain a, *be* p, *and the number of Chances I have to gain* b, *be* q, *ſuppoſing the Chances equal; my Expectation will then be worth* $\frac{ap + bq}{p + q}$.

To inveſtigate this Rule, I again put x for the value of my Expectation, which muſt conſequently procure me the ſame Expectation in fair Gaming. I take therefore ſuch a Number of Gameſters, as may, including my ſelf, be equal to $p + q$, every one of which ſtakes x; ſo that the whole Stake is $px + qx$, and all play with an equal Expectation of winning. With ſo many Gameſters as are expreſs'd by the Number q, I agree ſingly, that whoever of them wins, ſhall give me b; and if I win, he ſhall have b of me: And with the reſt, expreſs'd by $p-1$, I ſingly make this Agreement, That whoever of them wins, ſhall give me a; and if I win, he ſhall receive a of me. It is evident, our playing upon this Condition is fair, no Body having any injury done him; and that my Expectation of b, is q; my Expectation of a, is $p - 1$; and my Expectation of $px + qx - bq - ap + a$ (*i.e.* of winning) is 1; for then I gain the whole Stake $px + qx$, out of which I muſt pay b, to every one of the Gameſters q, and a to every one of the Gameſters $p - 1$, which together makes $bq + ap - a$. If therefore $px + qx - bq - ap + a$ be equal to a, I ſhou'd have p Expectations of a, (for I had $p-1$ Expectations of a, and 1 Ex-

1 Expectation of $px + qx - bq - ap + a$, which is now ſuppoſed equal to a,) and q Expectations of b; and conſequently am again come to my firſt Expectation. Therefore $px + qx - bq - ap + a = a$, and conſequently $x = \frac{ap + bq}{p + q}$, is the value of my Expectation. *Q. E. I.*

In Numbers. If I have 3 Expectations of 13, and 2 Expectations of 8, the value of my Expectations wou'd by this Rule be 11. And it is eaſy to ſhow, that having 11, I cou'd again come to the ſame Expectations. For playing againſt Four more, and every one of us ſtaking 11; with Two of them I agree ſingly, that he that wins ſhall give me 8; or to give him 8, if I win: And with the other Two in like manner, that which ſoever wins, ſhall give me 13; or to give him ſo much if I win. The Play is manifeſtly fair, and I have juſt 2 Expectations of 8, if either of the Two that promis'd me 8 ſhou'd win; and 3 Expectations of 13, if either of the Two that are to pay me 13 ſhou'd win, or if I win my ſelf, for then I gain the whole Stake, which is 55; from which, deducting 13 a piece for the laſt Two I bargain'd with, and 8 a piece for the other Two, there remains 13 for my ſelf. *Q. E. D.*

PROP.

PROP. IV.

To come to the Queſtion firſt propos'd, How to make a fair Diſtribution of the Stake among the ſeveral Gameſters, whoſe Chances are unequal? The beſt way will be to begin with the moſt eaſy Caſes of that Kind.

SUPPOSING therefore that I play with another upon this Condition, That he who gets the firſt three Games ſhall have the Stake; and that I have won two of the three, and he only one. I deſire to know, if we agree to leave off and divide the Stake, how much falls to my Share?

IN the firſt place we muſt conſider the number of Games ſtill wanting to either Party: For it is plain, that ſuppoſing we had agreed the Stake ſhou'd be deliver'd to him that ſhou'd win the firſt twenty Games, and I had won nineteen of them, and the other only eighteen; my Chance wou'd have then been juſt ſo much better than his, as it is in the preſent Caſe, where I am ſuppoſed to have won two out of the three, and he only one; becauſe in both Caſes there remains but one Game for me to win, and two for him.

MOREOVER, to find how to ſhare the Stake, we muſt have regard to what wou'd happen, if both play'd on: For it is manifeſt, that if I win the next Game, my Number is compleated, and the Stake, which call *a*, is mine. But if the other gets the next Game, then both our Chances will be even, becauſe we want but one Game apiece, and each of them worth

worth $\frac{1}{2}a$. But it is plain, I have an equal Chance to win or loſe the next Game; and conſequently an equal Chance to gain *a*, or $\frac{1}{2}a$; which by *Prop.* 1. is worth half the Sum of them both, *i. e.* $\frac{3}{4}a$. *Q. E. I.*

My Playfellow's Share, which of courſe muſt be the remaining $\frac{1}{4}a$, might be firſt found after the ſame manner. From whence it appears, That he who would play in my room, ought to give me $\frac{3}{4}a$ for my Chance; and conſequently that whoever undertakes to win one Game, before another ſhall win two, may lay 3 to 1 Odds.

PROP. V.

Suppoſe I want one Game of being up, and my Adverſary wants three; How muſt the Stake be divided?

Let us again conſider what wou'd be the Conſequence, if I ſhou'd get the next Game; 'tis plain I ſhould then win the Stake, ſuppoſe *a*; but if the other ſhou'd get it, he wou'd ſtill want two Games, and I but one; and conſequently our Caſe wou'd then be the ſame with that mention'd in the foregoing *Propoſition*, and my Share, $\frac{3}{4}a$, as is there demonſtrated.

There-

Therefore, ſince I have an equal Chance to gain *a*, or $\frac{3}{4}a$, my Expectation muſt, by *Prop.* 1. be worth $\frac{7}{8}a$; and my Adverſary's Share, the remaining $\frac{1}{8}a$. So that my Chance is to his, as 7 to 1. *Q. E. I.*

And as the *Solution* of the foregoing Caſe is neceſſary to the ſolving this laſt; ſo is the *Solution* of this laſt neceſſary to the ſolving the following one, where I am ſuppoſed to want but one Game, and my Adverſary four; for then my Share will be found, after the ſame manner, to be $\frac{15}{16}$ of the Stake, and his, $\frac{1}{16}$.

PROP. VI.

Suppoſe I have two Games to get, and my Adverſary three.

Therefore after the next Game, I ſhall either want but one more, and he three, (in which Caſe my Share, by the foregoing *Prop.* is $\frac{7}{8}a$,) or we ſhall want two a piece, and then my Share is $\frac{1}{2}a$, both our Chances being equal. But I have an even Chance to win or loſe the next Game, and conſequently have an equal Expectation of obtaining $\frac{7}{8}a$, or $\frac{1}{2}a$, wch

by *Prop.* 1. is worth $\frac{11}{16}a$. So that eleven Parts of the Stake fall to my Share, and five to his. *Q. E. I.*

PROP. VII.

Suppoſe I want two Games, and my Adverſary four.

THEREFORE it will either fall out, that by winning the next Game I ſhall want but one more, and he four; or by loſing it I ſhall want two, and he will want three. So that, by *Schol. Prop.* 5. and *Prop.* 6. I ſhall have an equal Chance for $\frac{15}{16}a$, or $\frac{11}{16}a$, which, by *Prop.* 1. is juſt worth $\frac{13}{16}a$. *Q. E. I.*

COROLL.

FROM whence it appears, that he that is to get two Games, before another ſhall get four, has a better Chance than he that is to get one, before another gets two Games. For in this laſt Caſe, namely of 1 to 2, his Share, by *Prop.* 4. is but $\frac{3}{4}a$, which is leſs than $\frac{13}{16}a$.

PROP. VIII.

Suppoſe now there are three Gameſters, and that the firſt and ſecond want one Game a piece, and the third wants two Games.

TO

To find therefore the Share of the firſt Gameſter, we muſt again examine what he wou'd gain, if either he himſelf, or one of the other two gets the next Game: If he gets it, he wins the whole Stake *a*; if the ſecond gets it, becauſe he likewiſe wanted but one Game, he has the Stake, and the firſt gets o; and if the third gets it, then they will all three want one Game a piece, and the Share of each of them will conſequently be $\frac{1}{3}a$. The firſt Gameſter therefore has an equal Expectation of gaining *a*, or o, or $\frac{1}{3}a$, (ſince each has the ſame likelyhood of winning the next Game,) which, by *Prop.* 2. is worth $\frac{4}{9}a$. The Share of the ſecond will be likewiſe $\frac{4}{9}a$, and there will be $\frac{1}{9}a$ remaining for the third; whoſe Share might, after the ſame manner, be found ſeparately from the others, and theirs determin'd by that. *Q. E. I.*

PROP. IX.

To find the ſeveral Shares of as many Gameſters, as we pleaſe, ſome of which ſhall want more Games, others fewer; we muſt conſider what he, whoſe Share we *want to find, wou'd gain, if he, or any one of the others wins the next Game: Then adding together what he wou'd gain in all thoſe particular Caſes, and dividing the Sum by the Number of Gameſters, the Quotient gives the particular Share required.*

Suppose, for Example, there were three Gamesters, *A*, *B*, and *C*, and *A* wanted one Game of being up, and *B* and *C* wanted two apiece, and I desire to find the Share that *B* has in the Stake *q*.

First of all we must see what will happen to *B*, if either he, or *A*, or *C* wins the following Game.

If *A* wins it, he is up, and consequently *B* gets 0. If *B* himself wins it, he and *A* will still want a Game apiece, and *C* will want two; and consequently *B*, by *Prop.* 8. gets $\frac{4}{9}q$. But lastly, if *C* wins it, then *A* and *C* will still want one Game apiece, and *B* will want two; and consequently *B*, by *Prop.* 8. will in this Case get $\frac{1}{9}q$.

Now the three several Gains of *B*, in all these particular Cases, which are 0, $\frac{4}{9}q$, and $\frac{1}{9}q$, added together, make $\frac{5}{9}q$; which Sum being divided by 3, the number of Gamesters, gives $\frac{5}{27}q$, for the Share of *B*. *Q. E. I.*

The Demonstration of this is plain from *Prop.* 2. For since *B* has an equal Chance to 0, $\frac{4}{9}q$, or $\frac{1}{9}q$, his Expectation is, by that *Prop.* worth $\frac{0 + \frac{4}{9}q + \frac{1}{9}q}{3}$ *i. e.* $\frac{5}{27}q$. And 'tis evident, this Divisor 3, is the Number of Gamesters. *Q. E. D.*

But in order to find what any one will gain in every particular Case, supposing himself or any of the rest should win the next Game; the more simple and intermediate Cases must be first investigated. For as this last Case cou'd not have been resolv'd, without that calculated in *Prop.* 8. where the Games wanting were 1, 1, 2; so likewise every single Person's Share, in case the Games wanting were 1, 2, 3, cannot be found, without first determining the Case where the Games wanting are 1, 2, 2, (which is already done in *Prop.* 9.) and that likewise where the Games wanting are 1, 1, 3, which may, by *Prop.* 8. be easily calculated. And after this manner may be resolv'd all the Cases comprehended in the following Table, and others, *ad Infinitum.*

TABLE

TABLE for Three Gamesters.

Games wanting	1. 1. 2.	1. 2. 2.	1. 1. 3.	1. 2. 3.
Shares.	4. 4. 1.	17.5. 5.	13.13.1.	19.6. 2.
	9.	27.	27.	27.

Games wanting.	1. 1. 4.	1. 1. 5.	1. 2. 4.	1. 2. 5.
Shares.	40.40.1.	121.121.1.	178.58. 7.	542.179.8.
	81.	243.	243.	729.

Games wanting.	1. 3. 3.	1. 3. 4.	1. 3. 5.
Shares.	65. 8. 8.	616. 82. 31.	629. 87. 13.
	81.	729.	729.

Games wanting.	2. 2. 3.	2. 2. 4.	2. 2. 5.	2. 3. 3.	2. 3. 4.	2. 3. 5.
Shares.	34. 34. 13.	338. 338. 53.	353. 353. 23.	133. 55. 55.	451. 195. 83.	1433. 635. 119.
	81.	729.	729.	243.	729.	2187.

As to what belongs to Dice, the Queſtions propos'd concerning them are, In how many Times we may venture to throw *Six*, or any other Number with a ſingle Die, or two *Sixes* with two Dice, or three *Sixes* with three Dice; and ſuch-like.

To reſolve which, we muſt obſerve, Firſt, That there are ſix ſeveral Throws upon one Die, which have all an equal probability of coming up. That upon two Dice there are 36 ſeveral Throws, equally liable to be thrown, for any one of the ſix Throws of one Die may come up with every one of the ſix Throws of the other; and ſo 6 times 6 will make 36 Throws. So likewiſe, that three Dice have 216 ſeveral Throws; for the 36 Throws on the two Dice may happen together with any one of the 6 Throws of the third Die; and ſo 6 times 36 will make 216 Throws. After the ſame manner, 'tis plain, four Dice will have 6 times 216, *i. e.* 1296 Throws; and ſo on, may we calculate the Throws upon any Number of Dice, taking always at the addition of every Die 6 times the preceding Number of Throws.

It is farther to be obſerv'd, that upon two Dice there is only one Throw that can produce 2 or 12, and two Throws that can produce 3 or 11. For if we call the Dice *A* and *B*, 'tis plain, the throw 3 may be made up of the 1 of *A*, and 2 of *B*; or of the 1 of *B* and 2 of *A*. So likewiſe will 11 be produced by the 5 of *A*, and 6 of *B*; or by the 5 of *B*, and 6 of *A*. The Number 4 may be thrown three Ways, by 1 of *A* and 3 of *B*; or 3 of *A* and 1 of *B*; or 2 of *A* and 2 of *B*.

X may

X may likewiſe be thrown three ſeveral Ways.

V or IX has 4 ſeveral Throws.

VI or VIII has 5 Throws.

VII has 6 Throws.

Upon 3 Dice, the Numbers		may come up by		ſeveral Throws.
	3 or 18		1	
	4 or 17		3	
	5 or 16		6	
	6 or 15		10	
	7 or 14		15	
	8 or 13		21	
	9 or 12		25	
	10 or 11		27	

PROP. X.

To find in how many Throws one may undertake to throw the Number 6 with a ſingle Die.

If any one wou'd venture to throw *Six* the firſt Throw, 'tis plain there is but 1 Chance by which he might win the Stake; and 5 Chances by which he might loſe it. For there are 5 Throws againſt him, and only 1 for him. Let the Stake then be called *a*. Since therefore he has one Expectation of *a*, and 5 Expectations of 0, his Chance is, by *Prop.* 2. worth $\frac{1}{6}a$; and there remains to his Adverſary $\frac{5}{6}a$. So that he that wou'd undertake to throw *Six* the firſt Throw, muſt lay only 1 to 5.

He that wou'd venture to throw *Six* once in two Throws, may calculate his Chance after the following manner: If he throws *Six* the firſt Throw, he gains a; if the contrary happens, he has ſtill another Throw remaining, which, by the foregoing Caſe, is worth $\frac{1}{6}a$. But he has only 1 Chance to throw *Six* the firſt Throw, and 5 Chances to the contrary: Therefore before he throws, he has one Chance for a, and five Chances for $\frac{1}{6}a$, the Value of which, by *Prop.* 3. is $\frac{11}{36}a$. And conſequently there remains $\frac{25}{36}a$, to the other that lays with him. So that their ſeveral Chances, or the Values of their ſeveral Expectations, bear the Proportion of 11 to 25, *i. e.* leſs than 1 to 2.

Hence, after the ſame Method, the Chance of him who wou'd venture to throw *Six* once in three Throws, may be inveſtigated and found worth $\frac{91}{216}a$, ſo that he may lay 91 againſt 125; which is a little leſs than 3 to 4.

He who undertakes to throw it once in four Times, has a Chance worth $\frac{671}{1296}a$, and may lay 671 to 625, *i. e.* ſomething more than 1 to 1.

He who undertakes to throw it once in five Times, has a Chance worth $\frac{4651}{7776}a$, and may lay 4651 againſt 3125, *i. e.* ſomething leſs than 3 to 2.

He who undertakes the ſame in 6 Throws, has a Chance worth $\frac{31031}{46656}a$, and may lay 31031 to 15625, *i. e.* a little leſs than 2 to 1

And thus may the *Problem* be reſolv'd in what Number of Throws we pleaſe. *Q. E. I.*

But it is poſſible for us to proceed after a more compendious Method, as ſhall be ſhown in the following *Propoſition*, without which the Calculation wou'd otherwiſe be much more prolix.

PROP. XI.

To find in how many Throws one may venture to throw the Number 12 *with two Dice.*

If any one ſhou'd pretend to throw 12 the firſt throw, 'tis plain he has but one Chance of winning, *i. e.* of gaining *a*; and 35 Chances of loſing, or gaining 0, becauſe there are in all, 36 ſeveral Throws. And conſequently his Expectation, by *Prop.* 3. is worth $\frac{1}{36}a$.

He that undertakes to do it in two Throws, if it comes up the firſt Throw, will obtain *a*; and if it does not, he has yet one Throw to come, which, by what has been ſaid before, is worth $\frac{1}{36}a$. But there is only one Chance for throwing 12 the firſt Throw, and 35 Chances againſt it: Therefore ſince he has 1 Expectation of *a*, and 35 of $\frac{1}{36}a$, his Chance

Chance, by *Prop.* 3. is worth $\frac{71}{1296}a$; and his Adverſary's, the remaining $\frac{1225}{1296}a$.

From theſe Two Caſes we can determine the value of his Chance who ventures to do the ſame in Four Throws, without conſidering the Chance of him that undertakes to do it in Three.

For he that ventures to throw 12 once in four times throwing, if he does it the firſt or ſecond time, gains a; and if the contrary happens, he has ſtill two Throws more, which, by what has been ſaid before, are worth $\frac{71}{1296}a$: For which Reaſon likewiſe, he muſt have 71 Chances for throwing 12 in one of the two firſt Throws, and 1225 Chances againſt it. Therefore at his beginning to throw he has 71 Expectations of a, and 1225 Expectations of $\frac{71}{1296}a$, which, by *Prop.* 2. are worth $\frac{178991}{1679616}a$: And the value of his Chance that plays againſt him will be the remaining $\frac{1500625}{1679616}a$. Which ſhows that their Chances are to one another, as 178991 to 1500625.

From which likewiſe, without calculating any other Caſes for that Purpoſe, may be found by the ſame Way of Reaſoning, the worth of his Expectation who undertakes to throw two *Sixes* once in 8 Throws. And from thence the worth of his Expectation

who ventures to do the ſame in 16 Throws. And from the Expectation of this laſt being found, together with his alſo who ventures it in 8 Throws, may be determined the value of his Expectation who undertakes it in 24 Throws. In which Operation, becauſe the principal Queſtion is, In what Number of Throws one may lay an even Wager to throw two *Sixes*; we may cut off ſome of the hindermoſt Figures from the long Numbers that ariſe in the midſt of the Calculation, and which wou'd otherwiſe encreaſe prodigiouſly. And by this Means I find he that undertakes it in 24 Throws, wants ſomething of an even Chance to win; and that he that lays to do it in 25 Throws, has ſomething the better ſide of the Wager.

PROP. XII.

To find with how many Dice one may undertake to throw two Sixes *the firſt Throw.*

This is the ſame thing as if we wou'd know, in how many Throws one may undertake to throw two *Sixes* with a ſingle Die. He that ventures to do this in two Throws, has, by what has been before demonſtrated in *Prop.* 11. a Chance worth $\frac{1}{36}a$. He that undertakes it in three Throws, if he does not happen to throw one *Six* the firſt Throw, has yet two more to come, which as before, are worth $\frac{1}{36}a$. But if the firſt Throw chance to be a *Six*, he has two Throws more to throw one *Six*, which, by *Prop.* 10. are worth $\frac{11}{36}a$.

Now

Now 'tis plain that there is one Chance for throwing a *Six* the firſt Throw, and five Chances to the contrary: So that before he throws, he has one Chance for $\frac{11}{36}a$, 5 Chances of $\frac{1}{36}a$, which, by *Prop.* 3. are worth $\frac{16}{216}a$, or $\frac{2}{27}a$. After this manner, by continually taking one throw more, we find that we may in 10 Throws with one Die, or in one Throw with 10 Dice, undertake to throw two *Sixes*, and that with Advantage.

PROP. XIII.

Suppoſing I lay with another to take one Throw with a pair of Dice upon theſe Terms, That if the Number 7 *comes up, I ſhall win, and if* 10 *comes up, he ſhall win; and that after this Bargain made, we conſent to draw Stakes by a fair Diviſion, according to the Value of our Chances in the preſent Contract: To find what will be our ſeveral Shares.*

Because, of the 36 Throws upon two Dice, there are but 6 which conſiſt of the Number 7 a piece, and 3 which conſiſt of the Number 10 a piece; there remain 27 Throws, which, if any one of them chance to come up, will make us neither win nor loſe, and conſequently entitle each of us to $\frac{1}{2}a$. But if none of thoſe Throws ſhou'd happen, I have 6 Chances for winning a, or and Chances of loſing, or having 0, which, by *Prop.* 3. is as good as if

I had $\frac{2}{3}a$. Therefore I have from the beginning 27 Chances for $\frac{1}{2}a$, and 9 Chances for $\frac{2}{3}a$, which, by *Prop.* 3. is worth $\frac{13}{24}a$; and there remains to him that plays against me $\frac{11}{24}a$.

PROP. XIV.

If my ſelf and another play by turns with a pair of Dice upon theſe Terms, That I ſhall win if I throw the Number 7, or he if he throws 6 ſooneſt, and he to have the Advantage of the firſt Throw: To find the Proportion of our Chances.

Suppose my Chance worth x, and call the Stake a; therefore his Chance will be $=a-x$. 'Tis plain then, that whenever it comes to his Turn to throw, my Chance ought to be $= x$. But when it is my Turn to throw, my Chance muſt be worth ſomething more. Let its Value, then, be expreſs'd by y. Now becauſe of the 36 Throws upon a pair of Dice, 5 are made up of the Number 6 apiece, and may make my Adverſary win, and 31 of them are againſt him, *i. e.* promote my Turn of throwing; I have, before he begins to throw, 5 Chances of obtaining 0, and 31 Chances of obtaining y, which, by *Prop.* 3. are worth $\frac{31}{36}y$. But my Chance from the beginning was ſuppoſed worth x, and therefore $\frac{31}{36}y = x$, and conſequently

$y =$

$= \frac{36x}{31}$. It was further ſuppoſed that in my Turns of throwing, my Chance was worth y. But when I'm to throw, I have 6 Chances of gaining a, becauſe there are 6 Throws of the Number 7 apiece, which wou'd give me the Game; and 30 Chances, which will bring it to my Adverſary's Turn to throw, *i. e.* make me gain x; which, by *Prop.* 3. are worth $\frac{6a+30x}{36}$. And becauſe this is $= y$, which was before found $= \frac{36x}{31}$; therefore $\frac{6a+30x}{36} = \frac{36x}{31}$. From which Equation will be had $x = \frac{31}{61}a$, the Value of my Chance. And by conſequence my Adverſary's will be worth $\frac{30}{61}a$; ſo that the Proportion of my Chance to his is, as 31 to 30.

For a concluding Ornament to this Work, we ſhall ſubjoin the following Problems.

PROBLEM I.

A and *B* play together with a pair of Dice upon this Condition, That *A* ſhall win if he throws 6, and *B* if he throws 7; and *A* is to take one Throw firſt, and then *B* two Throws together, then *A* to take two Throws together, and ſo on both of them the ſame, till one wins. The Queſtion is, *What Proportion their Chances bear to one another?* Anſw. As 10355 to 12276.

PROB-

PROBLEM II.

THREE Gamesters, *A*, *B*, and *C*, taking 12 Counters, 4 of which are white, and 8 black, play upon these Terms; That the first of them that shall blindfold choose a white Counter shall win; and *A* shall have the first Choice, *B* the second, and *C* third; and then *A* to begin again, and so on in their Turns. *What is the Proportion of their Chances?*

PROBLEM III.

A lays with *B*, that out of 40 Cards, *i. e.* 10 of each different Sort, he will draw 4, so as to have one of every Sort. And the Proportion of his Chance to that of *B*, is found to be as 1000 to 8139.

PROBLEM IV.

HAVING chosen 12 Counters, as before, 8 black and 4 white, *A* lays with *B* that he will blindfold take 7 out of them, among which there shall be 3 black ones. *Quære, What is the Proportion of their Chances?*

PROBLEM V.

A and *B* taking 12 Pieces of Money each, play with 3 Dice on this Condition, That if the Number 11 is thrown, *A* shall give *B* one Piece; but if 14 be thrown, then *B* shall give one to *A*; and he shall win the Game that first gets all the Pieces of Money: And the Proportion of *A*'s Chance to *B*'s is found to be, as 244,140,625 to 282,429,536,481.

FINIS.

Table for Two Gamesters.

Games wanting Shares	1 2	1 3	1 4	1 5	1 6
	3 1 / 4	7 1 / 8	15 1 / 16	31 1 / 32	63 1 / 64
Games wanting Shares	2 3	2 4	2 5	2 6	3 4
	11 5 / 16	13 3 / 16	57 7 / 64	15 1 / 16	21 11 / 32
Games wanting Shares	3 5	3 6	4 5	4 6	5 6
	99 29 / 128	219 37 / 256	163 93 / 256	191 65 / 256	319 193 / 512

Four persons wanting respectively
1. 2. 3. 4. games to be out, but
stopping play, desire the stake to
fairly divided. What is each one's sh
To find the Answer ($\frac{5670}{8192}$, $\frac{1762}{8192}$, $\frac{561}{8192}$, 8
all possible cases that can happen to th
must be successively calculated beginnin
with the simplest. Thus

1 1 1 1 1 1 1 1 4	1 1 1 2 5 5 5 1 16
1 1 1 3 21 21 21 1 64	1 1 1 4 85 85 85 1 256
1 1 2 2 13 13 3 3 32	1 1 2 3 111 111 27 7 256
1 1 2 4 113 113 28 2 256	1 1 3 3 239 239 17 17 512

1	3	4	
977	60	34	
2048			

1	2	2	2
71	19	19	19
128			

2	2	3	
44	44	13	
256			

1	2	2	4
637	185	185	17
1024			

2	3	3	
415	131	131	
2048			

1	2	3	4
5670	1762	561	199
8192			

Supposing the whole stake were £8.10.8. (8192 farthings) the shares would be

£5. 18. 1½

1. 16. 8½

11. 8¼

4. 1¾


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