

Bayes, Hume, Price, and Miracles

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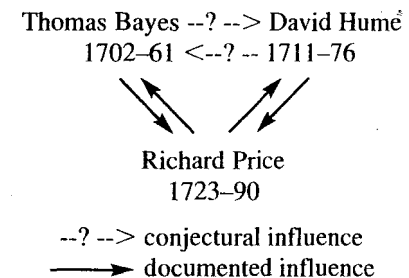
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BAYES'S THEOREM

MY TOPIC is the Bayesian analysis of miracles. To make the discussion concrete, I will set it in the context of eighteenth-century debate on miracles, and I will focus on the response to David Hume's celebrated argument against miracles that Thomas Bayes would have made and did in part make, albeit from beyond the grave, through his colleague Richard Price.²

1. My trinity: Bayes, Price, and Hume



It is irresistible to think that Bayes had read Hume and that Bayes's 'Essay Towards Solving a Problem in the Doctrine of Chances' was at least in part a reaction to Hume's sceptical attack on induction.¹ But while the Appendix of the published paper is surely making reference to Hume, that Appendix was penned not by Bayes but by Richard Price. It is also nice to think that Hume read Bayes's essay and then to speculate about what Hume's reaction would have been. But while there is a reference to Bayes's essay in Price's *Four Dissertations*, a copy of which was sent to Hume who duly acknowledged receiving and reading it (see below), there is no evidence that Hume followed up the reference. And even if he had, it is unlikely that he would have

¹ Bayes's essay was published posthumously in 1763. It was probably written in the 1740s.

understood Bayes's essay since he was largely innocent of the technical developments that were taking place in the probability calculus.

While there is no evidence of a direct connection between Bayes and Hume, the indirect connection that goes through Price is solid. Although we know little of the relationship between Bayes and Price, it must have been reasonably close since Bayes's will left £200 to be divided between Price and one John Boyl (see Barnard, 1958) and since it was Price who arranged for the posthumous publication of Bayes's essay. In the other direction, Price was a persistent critic of Hume—not just on induction but on matters of religion and ethics as well. Despite the sharp differences in opinion the two men remained on remarkably cordial terms, dining together in London and in Price's home in Newington Green (see Thomas, 1924). In the second edition of *Four Dissertations* Hume is lauded by Price as 'a writer whose genius and abilities are so distinguished as to be above any of my commendations' (1768, p. 382). And Hume in turn praised Price for the 'civility which you have treated me' (Klibansky and Mossner, 1954, 233–4). More intriguingly, Hume goes on to say that 'I own to you, that the Light in which you have put this Controversy [about miracles] is new and plausible and ingenious, and perhaps solid'. Unfortunately, Hume left the matter hanging by adding that 'I must have some more time to weigh it, before I can pronounce this Judgment with satisfaction to myself' (ibid.).

I will finish Hume's unfinished task. I will claim that Price's criticisms of Hume's argument against miracles were largely solid. More generally, I claim that when Hume's 'Of miracles' is examined through the lens of Bayesianism, it is seen to be a shambles.²

2. The context

The debate on miracles which took place in eighteenth-century Britain is rich and endlessly fascinating. One of the key philosophical problems underlying the debate was posed before the beginning of the century in Locke's *Essay Concerning Human Understanding* (1690); namely, how should one apportion belief when the two main sources of credibility, 'common observation in like cases' and 'particular testimonies', are at odds? The case of eyewitness

² A first draft of Hume's essay on miracles was written (probably) in 1737. But the essay did not appear in print until 1748 when it was published as Chapter 10 ('Of miracles') of Hume's *Philosophical Essays Concerning Human Understanding*, later called *Enquiries Concerning Human Understanding*. The internal evidence indicates that the published version of Hume's miracles essay was written in the 1740s.

testimony to miraculous events—the central focus of Hume's 'Of miracles'—presents the extreme form of this problem. The miraculous event around which the eighteenth-century debate swirled was, of course, the Resurrection of Jesus of Nazareth, though the more prudent naysayers took care to make oblique reference to this matter—Hume, for example, couches his discussion in terms of the hypothetical case of the resurrection of Queen Elizabeth.

To convey a bit of the flavour of the debate I will trace one of the many threads—the one that starts with Thomas Woolston, one of the incautious naysayers. His *Six Discourses on the Miracles of Our Savior* (1727–29) was an undisguised and broadsided attack on the New Testament miracles. As attested by Swift, it created a minor sensation:³

Here is Woolston's tract, the twelfth edition
'Tis read by every politician:
The country members when in town
To all their boroughs send them down:
You never met a thing so smart;
The courtiers have them all by heart.

What the courtiers learned from reading Woolston was that the New Testament accounts of miracles were filled with 'absurdities, improbabilities, and incredibilities'. While Woolston's charges and his sarcasm and obvious contempt for the Church authorities won him a large readership—reportedly, 30,000 copies of *Six Discourses* were printed—they also earned him a fine and a stay in prison, where he died.

Six Discourses received a number of replies, the most influential being Thomas Sherlock's *Tryal of the Witnesses* (1728), which itself went through fourteen editions. Sherlock was answered by Peter Annet (1744a,b), and Annet in turn was answered by Chandler (1744), Jackson (1744), and West (1747). And so it went.

Set in this context, Hume's essay on miracles seems both tame and derivative. It is also something of a muddle. What appears on first reading to be a powerful and seamless argument turns on closer examination to be series of considerations that don't comfortably mesh. And worse, Hume's thesis remains obscure—the possibilities range from the banal to the absurd:

(T1) One should be cautious in accepting testimony to marvelous and miraculous events, and doubly so when religious themes arise.

(T2) The testimonies of the Disciples do not establish the credibility of the resurrection of Jesus.

³ Quoted in Burns (1981). Burns's book provides a good overview of the eighteenth-century miracles debate.

(T3) In no recorded case does the testimony of eyewitnesses establish the credibility of a miracle deemed to have religious significance.

(T4) Eyewitness testimony is incapable of establishing the credibility of a miracle deemed to have religious significance.

(T5) Eyewitness testimony is incapable of establishing the credibility of any miraculous event.

Hume deserves no credit — save for pompous solemnity — for uttering various forms of the banality (T1), since even the pro-miracle proponents in the eighteenth-century debate were at pains to acknowledge the pitfalls of eyewitness testimony and the need to carefully sift the evidence, especially in the case of religious miracles. Hume believed (T2), but refused to join Annet, Sherlock, Woolston et al. in arguing the specifics. He surely also believed something in the neighbourhood of (T3), but his cursory recitation and rejection in Part II of his essay of a number of sacred and profane miracles hardly counts as much of an argument for this sweeping thesis. What Hume manages beautifully is the creation of an illusion that he was in possession of principles for evaluating evidence that allowed him to remain above the nastiness of the fray. This would have been the case if he had been in possession of principles that entailed (T4) or (T5). But there are no such principles since (T4) and (T5) are absurd. A skilful oscillation among these various theses is what helps to create the illusion of a worthy argument against miracles.

Stripped of the theology, the key issue addressed in Hume's essay is how to evaluate the evidential force of fallible witnesses. The obvious tool to apply was the probability calculus, and one of the first published applications in English was the anonymous (George Hooper?) essay 'A Calculation of the Credibility of Human Testimony', which appeared in the 1699 *Transactions of the Royal Society*. Some advances were made in the eighteenth century — for example, by Price — but it was not until the work of Laplace (1812, 1814) and Charles Babbage (1838) that definitive results were obtained. Hume's pronouncements on the key issue have that combination of vagueness, obscurity, and aphorism that philosophers find irresistible, and, consequently, have led to an endless and largely unfruitful debate in the philosophical literature, the contributors to which are willing to go to extraordinary lengths in an attempt to show that Hume had something interesting to say on the matter.

To state what should be obvious, but still manages to elude some commentators, the core issues in Hume's essay are independent of the theological subtleties of that vexed term 'miracle'. For what an eyewitness testifies to in the first instance is the occurrence of an event which can be characterized in

purely naturalistic terms, e.g., the return to life of a dead man. How such an event, if its credibility is established by eyewitness testimony, can serve theological purposes is a matter I will take up in due course. But first I have to substantiate my negative evaluation of Hume's treatment of eyewitness testimony. At the same time I hope to show how the issues can be advanced with the help of Bayes's apparatus.

3. Hume's 'proof' against miracles

Here is Hume's 'proof', such as it is:

A miracle is a violation of the laws of nature; and as a firm and unalterable experience has established these laws, the proof against a miracle, from the very nature of the fact, is as entire as any argument from experience can possibly be imagined. (1748, p. 114)

For Hume 'proofs' are 'such arguments from experience as leave no room for doubt or opposition' (1748, p. 56, fn. 1). He thought his proof against miracles fitted this bill since it is 'full and certain when taken alone, because it implies no doubt, as in the case of all probabilities' (Greig, 1932, Vol. I, p. 350). In probability language, Hume seems to be saying that if L is a law-like generalization, such as 'No dead man returns to life', and E records uniform past experience in favour of L , then $P(L|E) = 1$; hence, if M asserts a violation of L , then it follows from the rules of probability that $P(M|E) = 0$.

Price opposed Hume's wildly incautious and inaccurate account of inductive practice: 'It must be remembered, that the greatest uniformity and frequency of experience will not offer a proper proof, that an event will happen in a future trial.' But he continued with a streak of dark pessimism: 'or even render it so much as probable that it will always happen in all future trials' (1768, pp. 392–3). Here Price is reporting a result that follows from Bayes's interesting but ultimately flawed attempt to justify a particular prior probability assignment that had the consequences that, in the case of a domain with a countably infinite number of individuals, (i) $P(L|E) = 0$ no matter how extensive the past record E of uniform experience in favour of L is, and (ii) nevertheless, the probability that the next instance conforms to L approaches 1 as the number of past positive instances increases without bound. Leaving aside the issue of the justification of prior probabilities, the relevant thing to note for present purposes is that Hume's account of inductive practice is descriptively inaccurate: scientists don't think that uniform

experience in favour of a lawlike generalization leaves no room for doubt—if they did it would be hard to explain their continued efforts to search for exceptions.

But the crucial point is that Hume recognized that his ‘proof’ only applies when the evidence consists of uniform experience (recall the qualifier ‘when taken alone’). The real issue is joined when that proof is opposed by a counterproof from eyewitness testimony. Many commentators from Hume’s day to the present have read Hume as saying that this contest has a preordained outcome. The idea is that the probative force of eyewitness testimony derives from the observation of the conformity between the testimony and reality; but whereas our experience in favour of the relevant lawlike generalization and, thus, against an exception that constitutes a miracle, is uniform, our experience as to the trustworthiness of witnesses is anything but uniform. Thus, Price took Hume to be arguing that to believe a miracle on the basis of testimony is to ‘prefer a weaker proof to a stronger’ (1768, p. 385).⁴ Such a reading makes Hume’s essay into a puzzle: if this was Hume’s argument, why did the essay have to be more than one page long? And apart from the puzzle, the position being attributed to Hume implies the absurd thesis (T5).

The anti-miracle forces in the eighteenth-century debate sometimes asserted that uniform experience always trumps testimony. Such assertions were met with variations of Locke’s example of the king of Siam who had never seen water and refused to believe the Dutch ambassador’s report that, during the winter in Holland, water became so hard as to support the weight of an elephant. Thus, in the *Trial of the Witnesses* (1728), published almost a decade before Hume got the idea for his miracle essay,⁵ Sherlock puts the rhetorical question: ‘[W]hen the Thing testify’d is contrary to the Order of Nature, and, at first sight at least, impossible, what Evidence can be sufficient to overturn the constant Evidence of Nature, which she gives us in the constant and regular Method of her Operation?’ (1728, p. 58). Sherlock answers his rhetorical question with a version of Locke’s example: what do the naysayers against the Resurrection have to say ‘more than any man who never saw Ice might say against an hundred honest witnesses, who assert that Water turns into Ice in cold Climates?’ (1728, p. 60). That Hume’s contemporaries were not impressed by his miracles essay is partly explained by the fact that he seemed to be offering a warmed-up version of the past debate, and

⁴ And C. D. Broad took Hume to be saying that ‘we have never the right to believe any alleged miracle however strong the testimony for it may be’ (1916–17, p. 80).

⁵ Hume tell us that he got the idea during his stay in La Flèche (1736–37); see Greig (1932), Vol. I, p. 361.

a defective version at that, since the 1748 edition of the *Enquiries* is silent on the issues raised by Locke’s example.⁶

Let’s be charitable to Hume by not subscribing to the reading that takes him to be saying that uniform experience always trumps eyewitness testimony. The issue then becomes how to tell when the balance tips in favour of one or the other. Hume’s famous ‘maxim’ might be thought to provide just such a prescription.

4. Hume’s maxim

Hume announces the ‘general maxim’

That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavours to establish. (1748, pp. 115–16)

Most commentators have seen profound wisdom here. I see only triviality.

Suppose that we are in a situation where witnesses have offered testimony $t(M)$ to the occurrence of the miraculous event M . Let E be the record of our past experience in favour of the lawlike generalization to which M is an exception. Then, as good Bayesians, our current degree of belief function should be the conditionalization on $t(M)\&E$ of the function $P(\cdot)$ we had before obtaining this evidence. Thus, the relevant probability of the event which the testimony endeavours to establish is $P(M|t(M)\&E)$, while the relevant probability of the falsehood of the testimony is $P(\sim M|t(M)\&E)$. To say that the falsehood of the testimony is more miraculous than the event which it endeavours to establish is just to say that the latter probability is smaller than the former, i.e.

$$P(M|t(M)\&E) > P(\sim M|t(M)\&E) \quad (1)$$

which is equivalent to

$$P(M|t(M)\&E) > 0.5. \quad (2)$$

On this reading, Hume’s maxim is the correct but unhelpful principle that no testimony is sufficient to establish the credibility of a miracle unless the testimony makes the miracle more likely than not.

⁶ Hume’s ‘Indian prince’ example makes its appearance in a hastily added endnote to the 1750 edition. In later editions a paragraph and a long footnote on the Indian prince were added to the text.

A number of other renderings of Hume's maxim have been offered, but either they fail to do justice to the text of Hume's essay or else they turn Hume's maxim into a false principle (see Earman, 2000 for details). For example, Gillies (1991) and Sobel (1991) translate Hume's maxim as

$$P(ME) > P(\sim M \& t(M)|E) \quad (GS)$$

(GS) does provide a necessary condition for the credibility of a miracle in the sense of (1)–(2). But the context in which the maxim is enunciated makes it clear that Hume intended to provide a sufficient as well as a necessary condition for testimony to establish the credibility of a miracle.⁷ In modern conditional probability notation Price's (1767) reading could be rendered in two ways by moving either the $\sim M$ or the $t(M)$ in (GS) to the right of the modulus:

$$P(ME) > P(\sim M|t(M)\&E) \quad (P)$$

or

$$P(ME) > P(t(M)|\sim M\&E) \quad (P')$$

Hume's text is ambiguous enough that either (P) or (P') could, without forcing, serve as a translation. But each of these conditions can fail even when (1)–(2) hold.

In closing this section I want to acknowledge Colin Howson's (2000) insight that Hume's maxim takes on a different complexion if 'establish a miracle' means render certain rather than render credible. This reading, however, makes Hume's argument ineffective against his more sophisticated eighteenth-century opponents who took the aim of natural religion as establishing the likelihood rather than the certainty of religious doctrines. John Wilkins, a founder of the Royal Society and a spokesman for liberal Anglicans, held that

'Tis sufficient that matters of Faith and Religion be propounded in such a way, as to render them highly credible, so as an honest and teachable man may willingly and safely assent to them, and according to the rules of Prudence be justified in so doing. (1699, p. 30)

In a similar vein, Samuel Clarke wrote that

[S]uch moral Evidence, or mixt Proofs from Circumstances and Testimony, as most Matters of Fact are only capable of, and wise and honest Men are always satisfied

⁷ Hume writes: 'If the falsehood of his testimony would be more miraculous, than the event which he relates: *then, and not till then*, can he pretend to commend my belief or opinion' (p. 116). I take the italicized phrase to have the force of 'if and only if'.

with, ought to be accounted sufficient for the present Case [the truth of the Christian revelation]. (1705, p. 600)

And John Tillotson, on whom Hume bestows (ironic?) praise at the beginning of his miracles essay, wrote:

And for any man to urge that tho' men in temporal affairs proceed upon moral assurance, yet there is a greater assurance required to make men seek Heaven and avoid Hell, seems to me highly unreasonable. (1728)

5. Hume's diminution principle

This principle is enunciated by Hume in the following passage:

[T]he evidence, resulting from the testimony, admits of a diminution, greater or less, in proportion as the fact is more or less unusual. (1748, p. 113)

In citing the proverbial Roman saying, 'I should not believe such a story were it told me by Cato', Hume intimates that in the case of a miracle the diminution of the evidential force of testimony is total.

Price responded that 'improbabilities as such do not lessen the capacity of testimony to report the truth' (1768, p. 413). This is surely right, as was Price's further claim that the diminution effect operates through the factors of the intent to deceive and the danger of being deceived, either by others or by oneself. Unfortunately Price overstepped himself in claiming that when the first factor is absent, testimony 'communicates its own probability' to the event (1768, p. 414). It was left to Laplace (1814) to give a correct and thorough Bayesian analysis of when and how the intent to deceive and the danger of being deceived give rise to a diminution effect. Two cases suffice to illustrate the results that can emerge from the analysis.

The first case — one considered by Price — concerns a witness who testifies $t(W_{79})$ that the winning ticket in a fair lottery with N tickets is #79. Assuming that when the witness misreports, she has no tendency to report one wrong number over another, then

$$P(W_{79}|t(W_{79})\&E) = P(t(W_{79})|(W_{79})\&E) \quad (3)$$

where E records the background knowledge of the lottery.⁸ And this is so regardless of how large N is and, thus, regardless of how small the prior probability $P(W_{79}|E) = 1/N$. In such a case the testimony does, in Price's words,

communicate its own probability to the event. But, as Price was well aware, (3) does not hold if the witness has some ulterior interest in reporting #79 as the winner.

The second case concerns the testimony $t(W)$ that a ball drawn at random from an urn containing one white ball and $N - 1$ black balls is white. As long as there is a non-zero probability that the witness misperceives the colour of the ball drawn, or else there is a non-zero probability that the witness misreports the correctly perceived colour, the posterior probability $P(W|t(W)\&E)$ does diminish as N is increased and, thus, as the prior probability $P(W|E) = 1/(N - 1)$ is reduced.

The difference between the two cases lies in the fact that in the urn case, as the prior probability is reduced the likelihood factor $P(t(W)|\sim W\&E)$ $P(t(W)|W\&E)$ remains the same because the visual stimulus presented to the witness when either a black or a white ball is drawn is independent of the numbers of black and white balls in the urn. By contrast, in the lottery case the corresponding likelihood factor changes in such a way as to cancel the change in the prior factor, quenching the diminution effect.

Two comments are in order. First, I see no a priori reason to think that cases of reported miracles are always or even mostly like the urn case in that they are subject to the diminution effect. Even when the reported miracle is subject to the diminution effect, Hume gets his intended moral only for one order of quantifiers; namely, for any given fallible witness (who may practise deceit or who is subject to misperception, deceit, or self-deception), there is a story so a priori implausible that we should not believe the story if told by that witness. It does not follow that there is a story so a priori implausible (unless, of course, the prior probability is flatly zero) that for any fallible witness, we should not believe the story if told by that witness. To get this latter implication one needs the extra postulate that there are in-principle bounds below which the probability of errors of reporting cannot be reduced. Hume's cynicism suggests such a postulate, but cynicism is not an argument.

6. Multiple witnesses

Hume makes some nods to the importance of multiple witnessing, but he seems not to have been aware of how powerful a consideration it can be. In fairness, the power was clearly and fully revealed only in the work of Laplace (1812, 1814) and more especially the work of Babbage (1838), whose *Ninth Bridgewater Treatise* devotes a chapter to a refutation of Hume's argument against miracles.

Suppose that there are N fallible witnesses, all of whom testify to the occurrence of an event M . For simplicity suppose that the witnesses are all equally fallible in that for all $i = 1, 2, \dots, N$

$$P(t_i(M)|M\&E) = p, \quad P(t_i(M)|\sim M\&E) = q \quad (4)$$

And suppose that conditional on the occurrence (or non-occurrence) of M , the testimonies of the witnesses are independent in the sense that

$$\begin{aligned} &P(t_1(M)\&t_2(M)\&\dots\&t_N(M)|\pm M\&E) \\ &= P(t_1(M)|\pm M\&E) \times P(t_2(M)|\pm M\&E) \times \dots \times P(t_N(M)|\pm M\&E) \end{aligned} \quad (5)$$

where $\pm M$ stands for M or $\sim M$, and where the choice is made uniformly on both sides of the equality. Then the concurrent testimony of all N witnesses gives a posterior probability

$$P(M|t_1(M)\&t_2(M)\&\dots\&t_N(M)\&E) = \frac{1}{1 + \left[\frac{P(\sim M|E)}{P(M|E)} \right] \left(\frac{q}{p} \right)^N} \quad (6)$$

The witnesses may be very fallible in the sense that q can be as close to 1 as you like. But as long as they are minimally reliable in the sense that $p > q$, it follows from (6) that the posterior probability of M can be pushed as close to 1 as you like by a sufficiently large cloud of such fallible but minimally reliable witnesses—provided, of course, that $P(M|E) > 0$. Using this result Babbage was able to give some nice examples where just twelve minimally reliable witnesses can push the posterior probability of an initially very improbable event to a respectably high level.

In sum, if we set aside Hume's wildly optimistic account of induction on which $P(M|E) = 0$, then he must agree that fallible multiple witnesses can establish the credibility of a miracle, provided that their testimonies are independent in the sense of (5) and that they are minimally reliable in the sense that $p > q$. Which of these provisos would Hume have rejected? The answer is 'Neither', at least in some cases of secular miracles, such as in his hypothetical example of eight days of total darkness around the world.

[S]uppose, all authors, in all languages, agree, that, from the first of January 1600, there was total darkness over the whole earth for eight days: Suppose that the tradition of this extraordinary event is still strong and lively among the people: that all travellers, who return from foreign countries, bring us accounts of the same tradition without the least variation or contradiction: It is evident, that our present philosophers, instead of doubting the fact, ought to receive it as certain ... (1748, pp. 127–8)

But in cases to which religious significance is attached, Hume professed to be unswayed by even the largest cloud of witnesses. In the hypothetical case of the resurrection of Queen Elizabeth, Hume declared that he would 'not have the least inclination to believe so miraculous an event' (1748, p. 128), even if all the members of the court and Parliament proclaimed it. Here it is less plausible than in the previous case that the independence assumption is satisfied since the witnesses can be influenced by each other and the general hubbub surrounding the events. But there seems to be no in-principle difficulty in arranging the circumstances so as to secure the independence condition. The minimal reliability condition becomes suspect if the alleged resurrection is invested with religious significance because witnesses in the grip of religious fervour tend to be more credulous and because they may give in to the temptation to practise deceit in order to win over the unconverted. But it is an insult to the quality of the eighteenth-century debate to think that the participants needed a sermonette on this topic from Hume. The pro-miracle proponents were acutely aware of the need to scrutinize the contextual factors that might give clues as to the reliability of the witnesses. The conclusions they drew from their scrutiny may have been mistaken, but if so, the mistakes did not flow from a failure to heed the empty solemnities of Hume's essay. Hume's treatment of the hypothetical Queen Elizabeth case drips with cynicism:

[S]hould this miracle be ascribed to any new system of religion; men, in all ages, have been so much imposed on by ridiculous stories of that kind, that this very circumstance would be a full proof of cheat, and sufficient, with all men of sense, not only to make them reject the fact, but even reject it without farther examination. (1748, pp. 128–9)

The cynicism remains just cynicism unless it is backed by an argument showing that, in principle, the witnesses cannot be minimally reliable and independent when the alleged miracle is ascribed to system of religion. Such an argument is not to be found in Hume's 'Of miracles'.

7. Miracles as a just foundation for religion

Grant that nothing in principle blocks the use of eyewitness testimony to establish the credibility of a miracle—say, a resurrection—of supposed religious significance. It might still seem that Hume has safe ground to which to retreat. On his behalf one can argue that to serve as a 'just foundation for religion', the miracle must not only satisfy Hume's first definition of 'miracle' as a violation of a (putative) law of nature but must also satisfy the

second definition, according to which a miracle is 'a transgression of a law of nature by a particular volition of the Deity, or by the interposition of some invisible agent' (1748, p. 115n). But (still continuing on Hume's behalf, now in the voice of John Stuart Mill), nothing can ever prove that the resurrection is miraculous in the sense of Hume's second definition because 'there is still another possible hypothesis, that of its being the result of some unknown natural cause; and this possibility cannot be so completely shut out as to leave no alternative but that of admitting the existence and intervention of a being superior to nature' (1843, p. 440).

This line is used over and over again across the decades by commentators sympathetic to Hume. It is apt to strike the innocent reader as a powerful consideration, but when the context is filled in it is seen as unavailing against the more sophisticated eighteenth-century pro-miracles proponents. For, to repeat, the role these proponents saw for miracles was not that of a direct and full proof of the presence of God by marks of a supernatural intervention in human affairs. Rather, the miraculous figured in an argument whose goal was to render religious doctrines highly credible and, ideally, to give them the kind of moral assurance needed to render a jury verdict of beyond reasonable doubt. And to fulfil this role, miracles need not be conceived as supernatural interventions; they need only serve as probabilistic indicators of the truth of the religious doctrines.

To illustrate how miracles can serve to confirm religious doctrines, suppose that testimonial evidence $t(M)$ has incrementally confirmed M :

$$P(M|t(M)\&E) > P(M|E) \quad (7)$$

Suppose also that the testimony bears on the religious doctrine D only through M in that

$$P(D|\pm M\&\pm t(M)\&E) = P(D|\pm M\&E) \quad (8)$$

And suppose finally that

$$P(M|D\&E) > P(M|\sim D\&E) \quad (9)$$

Then it follows that

$$P(D|t(M)\&E) > P(D|E) \quad (10)$$

that is, $t(M)$ incrementally confirms D .

Condition (8) is surely unobjectionable. Condition (9) is the sense in which M serves as a probabilistic indicator of the truth of D , and to fulfil this

role M need not be the result of a supernatural intervention that disrupts the order of nature. And condition (9) is seemingly easy to satisfy. For example, isn't it obvious that the miracle of the loaves is more likely on the assumption that Christian doctrine is true than on the assumption that it is false? In fact, it is not obvious. In general, whether or not (9) is satisfied depends on what alternatives are included in $\sim D$ and what their prior probabilities are.⁸ Consider, for example, the case where a high prior is given to the possibility that there is a non-Christian deceiver God who actualizes a world containing events designed to mislead people into falsely believing Christian doctrine. And even when D is incrementally confirmed by $t(M)$, there is no assurance that evidence of other miracles will push the probability of D anywhere near that required for moral certainty—again, it depends on the available alternatives and their prior probabilities.

So the discouraging word is that Bayesianism does not pave an easy road for religion. But by the same token there is no obvious difference here with theoretical physics: whether and how much a physical theory T that postulates unobservable properties of unobservable elementary particles is confirmed by direct or testimonial evidence about streaks in cloud chambers depends on what alternatives to T are entertained and what priors these alternatives are assigned.

8. Parting company

Let us agree for present purposes that the attempts by Bayes and others to objectify priors do not succeed.⁹ And, again for present purposes, let us agree that the subjectivist form of Bayesianism captures the logic of inductive reasoning. This form of Bayesianism imposes two and only two constraints on degrees of belief: a synchronic constraint which requires that at any given time the degrees of belief conform to the probability axioms, and a diachronic constraint which requires that the degree of belief function changes by conditionalization on the evidence that is acquired.¹⁰ In so far as degrees of belief of an agent conform to these strictures, they are deemed to be rational.

By these lights it is rational to give high degrees of belief to miracles; indeed, we have seen that given the priors that many of us—including, presumably, Hume himself—start with, and given various kinds of eyewitness

⁸ An exception is the hypothetico-deductive case where D entails M .

⁹ For a critique of Bayes's attempted justification of his favoured prior probability assignment, see my book (1992).

¹⁰ Some good Bayesians drop the diachronic constraint (e.g. Howson, 2000). But for present purposes it does no harm to impose it.

testimony that it is in principle possible to obtain, it would be irrational not to give a high credibility to miracles. Further, miracles can be used to support rational credence in theological doctrines.

Thus far I have marched shoulder to shoulder with the pro-miracle forces. But now I drop out of the parade, and this for two reasons. First, my degree of belief function—which I immodestly assume to satisfy the Bayesian strictures—disagrees with, for example, Professor Swinburne's (1970, 1979) function—which I have no doubt satisfies the strictures.¹¹ Second, and more important, I think that these differences are matters of taste in that there is no objective basis to prefer one over the other. One way to find objectivity in the framework of subjective Bayesianism is through an evidence-driven merger of opinion (aka 'washing out of priors'¹²). Such a consensus, however, is hollow unless it is in principle possible that the accumulating evidence produces the merger of opinion by driving the posterior probability to 1 on the true hypotheses and 0 on the false hypotheses. But for theological hypotheses, whose truth values do not supervene on the totality of empirical evidence—no matter how liberally that evidence is construed—the desired convergence to certainty is impossible.

To add some precision to this claim I will introduce a bit of apparatus. Let H and H' be the rival (incompatible) hypotheses at issue. The set of 'possible worlds' allowed by these competing hypotheses are the models $\mathcal{M} := \mathcal{M}^H \cup \mathcal{M}^{H'}$, where $\mathcal{M}^H := \text{mod}(H)$, $\mathcal{M}^{H'} := \text{mod}(H')$, and $\text{mod}(X)$ stands for the set of models that satisfy X . Let \mathcal{M}_O^H and $\mathcal{M}_O^{H'}$ stand for the empirical submodels of \mathcal{M}^H and $\mathcal{M}^{H'}$ respectively. Exactly what is included in these submodels is left vague, but my intent is to be as liberal as possible. For example, they may include not only states of affairs that are directly observable by the unaided senses, but also states of affairs that can only be inferred using elaborate measuring devices and the background theories of these devices. I will say that the truth of the matter as regards H vs H' is *strongly underdetermined* by the empirical iff H and H' have the same empirical content in that there is a one-one map $f: \mathcal{M}^H \rightarrow \mathcal{M}^{H'}$ such that for all $m \in \mathcal{M}^H$ the corresponding empirical submodels $m_O \in \mathcal{M}_O^H$ and $f(m)_O \in \mathcal{M}_O^{H'}$ are isomorphic.¹³ I call this strong empirical underdetermination because for any empirical situation (submodel) allowed by either hypothesis, the 'same' situation

¹¹ It is only here that I part company with Professor Swinburne, from whom I learned to apply Bayesianism to religious matters.

¹² This phrase is misleading since the likelihoods, which in many cases are just as subjective as the priors, have to wash out too.

¹³ A weaker form of empirical underdetermination would obtain if H and H' had some common empirical submodels, i.e. there are $m \in \mathcal{M}^H$ and $m' \in \mathcal{M}^{H'}$ such that m_O and m'_O are isomorphic.

(as expressed in isomorphic submodels) extends to a full model in which H is true and H' is false, and it also extends to a different full model in which H' is true and H is false. For such a pair of rival hypotheses there can be no empirical procedure — Bayesian or non-Bayesian — which is reliable over all the possible worlds \mathcal{M} in detecting the truth of the matter as regards H and H' . For there is no procedure which for each $m \in \mathcal{M}$ operates on the empirical submodel m_O of m and produces the truth values of H and H' in m .¹⁴ If one prefers, the same point can be made in terms of evidence statements. A condition for an evidence statement E to be counted as empirical is that it be treated symmetrically with respect to any pair of isomorphic empirical submodels m_O and m'_O : either $m_O \models E$ and $m'_O \models E$ or else $m_O \models \sim E$ and $m'_O \models \sim E$. The set \mathcal{E} of all such evidence statements constitutes what can be called the *empirical evidence matrix*. A possible world m fills in the truth values for the statements in the matrix, the array of which is denoted by \mathcal{E}^m . The point now becomes that in cases of strong underdetermination there is no procedure — Bayesian or otherwise — which for each $m \in \mathcal{M}$ looks at some or all of the array \mathcal{E}^m and produces the truth values for H and H' in m .

One might hope that adding background knowledge could make a reliable empirical learning procedure possible since such knowledge will cut down on the set of possible worlds over which the procedure has to be reliable. But this hope is vain when the underdetermination is of the strong variety and when the background knowledge is empirical. Background knowledge may be either propositional or probabilistic. In the former case, the knowledge K reduces the set of possible worlds \mathcal{M} to $\mathcal{M}^* := \mathcal{M}^{*H} \cup \mathcal{M}^{*H'}$ where $\mathcal{M}^{*H} := \text{mod}(H \& K)$ and $\mathcal{M}^{*H'} := \text{mod}(H' \& K)$. In the case of probabilistic knowledge, \mathcal{M}^{*H} and $\mathcal{M}^{*H'}$ are singled out as subsets of measure one relative to some measure imposed on \mathcal{M} . I propose that whatever counts as empirical background knowledge must satisfy the constraint that $I(\mathcal{M}^{*H}) = \mathcal{M}^{*H'}$, where I is the same mapping that exhibits the original strong underdetermination. The idea is that empirical knowledge should be characterizable directly in terms of how it operates on the empirical submodels and, thus, must treat isomorphic empirical submodels symmetrically. Two cases have to be considered. (a) The background knowledge is so strong that either $\mathcal{M}^{*H} = \emptyset$ or $\mathcal{M}^{*H'} = \emptyset$. But in this case the background knowledge amounts to knowledge of the non-empirical since it fixes the truth value of one (or more) of the competing hypotheses to False. (b) $\mathcal{M}^{*H} \neq \emptyset$ and $\mathcal{M}^{*H'} \neq \emptyset$. In this case the constraint

¹⁴ More precisely, there are no functions $f_H: \mathcal{M}_O^H \rightarrow \{T, F\}$ and $f_{H'}: \mathcal{M}_O^{H'} \rightarrow \{T, F\}$ such that for any $m_O \in \mathcal{M}_O^H$ if m_O is the empirical submodel of $m \in \mathcal{M}^H$ then $f(m_O) = T$ (respectively, F) if $m \models H$ (respectively, if $m \models \sim H$), and similarly for $f_{H'}$.

on what counts as empirical background knowledge implies that the strong empirical underdetermination remains intact even after the empirical background knowledge is applied. It follows that only a priori background knowledge of the non-empirical can lead to a reliable empirical procedure for ascertaining the truth of hypotheses subject to strong empirical underdetermination. Whatever its metaphysical legitimacy, such background knowledge is no part of scientific methodology.

To complete my case I need two additional premises: first, that a characteristic feature of religious doctrines is that, even on the most liberal construal of 'the empirical', they are subject to strong empirical underdetermination; and second, that while beliefs or degrees of belief that are formed in a manner that does not reflect a reliable connection between truth and empirical evidence — either because no such reliable connection exists, or else because a reliable connection exists but the belief formation method does not reflect it — may possibly be counted as 'rational', they cannot qualify as objective or scientific. It follows that the project of natural religions is impossible, at least in so far as that project presupposes that theology can proceed, as does science, in gathering empirical evidence and forming objective opinions on the basis of that evidence.

This sour conclusion can be avoided by attacking my additional premises; in particular, it could be claimed either that theological hypotheses are not underdetermined by empirical evidence, or else that I have imposed an impossibly high standard of objectivity on science since high-level hypotheses in theoretical physics are as underdetermined as theological hypotheses. I will not try to respond to the first attack since doing so would involve a substantial discussion of theology, which is something I cannot provide here. As for the second attack, I would turn it around and maintain that, in so far as the hypotheses of theoretical physics are subject to strong empirical underdetermination, theoretical physics is a kind of theology whose high priests should be accorded no more respect, *qua* scientists, than the high priests of Christianity or Buddhism. But I would add that, despite the fact that it is asserted in the literature on scientific realism that underdetermination is a commonplace, I have seen very few interesting examples of it in the history of science. Perhaps the sparsity of examples is due either to a lack of imagination or, more interestingly, to the tacit imposition of a set of criteria for selecting hypotheses to be seriously entertained. In the latter case the proponents of natural religion may be able to argue that what is sauce for the goose is sauce for the gander and that by using analogous selection criteria theologians can match theoretical physicists in overcoming or greatly reducing the scope of underdetermination. These issues merit a vigorous and thorough examination.

9. Conclusion

I trust that I have displeased all parties. I hope to have upset the devotees of Hume's miracles essays by showing that a Bayesian examination reveals Hume's seemingly powerful argument to be a shambles from which little emerges intact, save for posturing and pompous solemnities. At the same time I hope I have given no comfort to the pro-miracle forces. I personally do not give much credibility to religious miracles and religious doctrines. And while I acknowledge that those who do can be just as rational as I am, I suspect that degrees of belief in religious doctrines cannot have an objective status if a necessary condition for such a status is the existence of a reliable procedure for learning, from all possible empirical evidence, the truth values of these doctrines. The proof, or disproof, of this suspicion would, I think, constitute an important contribution to the philosophy of religion.

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