1 Determinism: What We Have Learned and What We Still Don't Know

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1 Introduction

The purpose of this essay is to give a brief survey of the implications of the theories of modern physics for the doctrine of determinism. The survey will reveal a curious feature of determinism: in some respects it is fragile, requiring a number of enabling assumptions to give it a fighting chance; but in other respects it is quite robust and very difficult to kill. The survey will also aim to show that, apart from its own intrinsic interest, determinism is an excellent device for probing the foundations of classical, relativistic, and quantum physics.

The survey is conducted under three major presuppositions. First, I take a realistic attitude toward scientific theories in that I assume that to give an interpretation of a theory is, at a minimum, to specify what the world would have to be like in order for the theory to be true. But we will see that the demand for a deterministic interpretation of a theory can force us to abandon a naively realistic reading of the theory. Second, I reject the "no laws" view of science and assume that the field equations or laws of motion of the most fundamental theories of current physics represent science's best guesses as to the form of the basic laws of nature. Third, I take determinism to be an ontological doctrine, a doctrine about the temporal evolution of the world. This ontological doctrine must not be confused with predictability, which is an epistemological doctrine, the failure of which need not entail a failure of determinism. From time to time I will comment on ways in which predictability can fail in a deterministic setting. Finally, my survey will concentrate on the Laplacian variety of determinism according to which the instantaneous state of the world at any time uniquely determines the state at any other time.

The plan of the survey is as follows. Section 2 illustrates the fragility of determinism by means of a Zeno-type example. Then sections 3 and 4 survey successively the fortunes of determinism in the Newtonian and the special relativistic settings. The implications of ordinary nonrelativistic quantum mechanics and relativistic quantum field theory for determinism are taken up on section 5. Determinism in classical general relativistic physics is discussed in section 6. Section 7 contains some, necessarily speculative, comments on how determinism may fare in a quantum theory of gravity. Conclusions are presented in section 8.

2 Zeno's Revenge: An Illustration of the Fragility of Determinism

Suppose that the world is populated with billiard balls. And suppose that the laws of motion for this world consist precisely of the specifications that when two balls collide they obey the standard laws of elastic impact and that between collisions they move uniformly and rectilinearly. And finally suppose that atomism is false so that billiard balls of arbitrarily small size can exist. Then à la Zeno we can string a countably infinite number of unit mass billiard balls in the unit interval. Assume that at $t = t^*$ all the balls in this infinite string are at rest and that coming from the right is a cue ball of unit mass moving with unit speed (see figure 1.1a). In a unit amount of time an infinite number of binary collisions take place, at the end of which each ball is at rest in the original position of its left successor in the series (see figure 1.1b).[1] The time reverse of this process has all the balls initially at rest. Then suddenly a ripple goes through the string, and the cue ball is ejected to the right. Futuristic Laplacian determinism is violated since it is consistent with the laws of elastic impact that the string does not self-excite but remains quiescent for all time (Laraudogoitia 1996).[2]

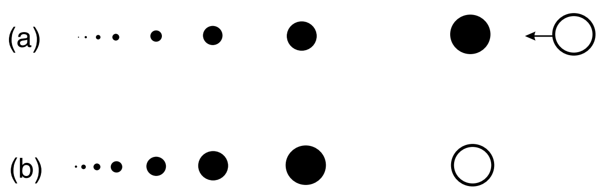

Figure 1.1
Zeno's revenge.

I see no non–ad hoc way to save determinism in this setting. If we cherish determinism we can only thank the Creator that he did not place us in a world where atomism is false and where Zeno can have his revenge.

3 Determinism in Newtonian Physics

One theme that will be sounded again and again in this section is that classical spacetimes provide unfriendly and even hostile environments for determinism. A related theme is that determinism in classical physics is inextricably linked to basic philosophical issues about the nature of space, time, and motion. To illustrate the latter theme I assert that Laplacian determinism implies that it cannot be the case that both (i) space is 'absolute' in the sense that it is a 'container' for bodies (e.g., shifting all the bodies in the universe one mile to the east results in a new state distinct from the original state), and (ii) all motion is the relative motion of bodies. The argument is simple. Assumption (ii) implies that only relative particle quantities, such as relative particle positions, relative particle velocities, relative particle accelerations, and so forth, and not absolute position, velocity, acceleration, and so forth, are well-defined quantities. The appropriate classical spacetime setting that supports this conception of motion has three elements: planes of absolute simultaneity, which reflect the observer-independent nature of coexistence; a metric (assumed to be $\mathbb{E}^3$) on the instantaneous three-spaces, which measures the spatial distance between simultaneous events; and a time metric, which measures the lapse of time between nonsimultaneous events.[3] But if (i) is maintained in this spacetime setting, not even a weakened form of Laplacian determinism can hold for particle motions. In coordinates adapted in the natural way to the spacetime structure, the symmetries of the spacetime have the form

$$\mathbf{x} \to \mathbf{x}' = \mathbf{R}(t)\mathbf{x} + \mathbf{a}(t) \tag{1a}$$

$$t \to t' = t + const \tag{1b}$$

where $\mathbf{R}(t)$ is a time dependent orthogonal matrix and $\mathbf{a}(t)$ is an arbitrary smooth function of time. We can choose $\mathbf{a}(t)$ and $\mathbf{R}(t)$ such that $\mathbf{a}(t) = 0 = \mathbf{R}(t)$ for $t \leq t^*$ but $\mathbf{a}(t) \neq 0$ or $\mathbf{R}(t) \neq 0$ for $t > t^*$. Since a symmetry of the spacetime should be a symmetry of the laws of motion, the image under (1) of a solution of the equations of motion should also be a solution. But for

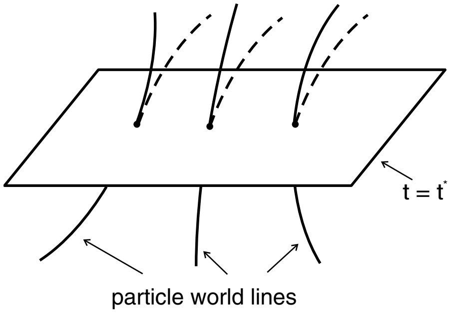

Figure 1.2
An apparent violation of Laplacian determinism.

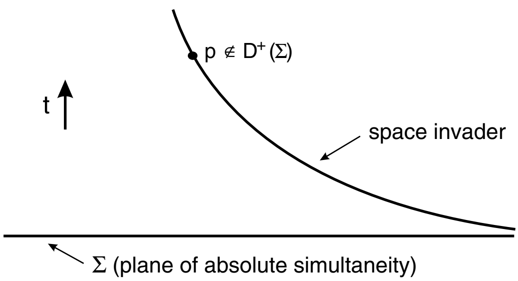

Figure 1.3
Domain of dependence in classical spacetime.

the chosen forms of $\mathbf{a}(t)$ and $\mathbf{R}(t)$ the two solutions will agree for all $t \leq t^*$ but will disagree for $t > t^*$ since the two solutions (as indicated by the solid and the dashed world lines of figure 1.2) entail different future positions for the particles in the container space.

To save determinism one can reject (i) and claim that the alleged violation of determinism is spurious on the grounds that once the container view of space is rejected there is no temptation to see the dashed and solid lines of figure 1.3 as different future histories rather than as different representations of the same history. To make good on this point of view it would have to be shown how to concoct deterministic and empirically adequate equations of motion that are formulated entirely in terms of relative particle quantities. As the history of mechanics shows, this is not an easy row to hoe. But that is a story I don't have time to recount here.

The alternative way to save determinism is to reject (ii) and beef up the structure of the spacetime by adding, say, inertial structure to make well-defined quantities like absolute acceleration. This additional structure linearizes the transformations (1) to

$$\mathbf{x} \to \mathbf{x}' = \mathbf{R}\mathbf{x} + \mathbf{v}t + \mathbf{c} \tag{2a}$$

$$t \to t' = t + const \tag{2b}$$

where $\mathbf{R}$ is now a constant orthogonal matrix and $\mathbf{v}$ and $\mathbf{c}$ are constants. The transformations (2) are, of course, the familiar Galilean transformations. In this spacetime setting (also known as neo-Newtonian spacetime) the above construction that undermines Laplacian determinism doesn't work since if a transformation from (2) is the identity map for all $t \leq t^*$ it is the identity map for all time.

However, in neo-Newtonian spacetime other threats to determinism arise. Consider Newtonian gravitational theory written as a field theory. The gravitational potential φ is governed by the Poisson equation

$$\nabla^2\varphi = 4\pi\rho \tag{3}$$

where ρ is the mass density. The gravitational force acting on a massive test body moving in the Newtonian gravitational field is proportional to $-\nabla\varphi$. Even the weakest form of Laplacian determinism fails because if φ is a solution to (3), then so is $\varphi' = \varphi + f$, where $f(\mathbf{x}, t)$ is any function linear in $\mathbf{x}$. By choosing $f(\mathbf{x}, t)$ to be 0 for all $t \leq t^*$ but different from 0 for $t > t^*$, we produce solutions for which the test body feels exactly the same gravitational force and has exactly the same motion in the past but feels different forces and, hence, executes different motions in the future. The nondeterministic solutions can be killed by excluding the homogeneous solutions to (3). This exclusion amounts to a declaration that the Newtonian gravitational field has no degrees of freedom of its own and is only an auxiliary device for describing direct particle interactions. Let us then turn to the pure particle description of Newtonian gravitation.

Consider a finite number of point mass particles interacting via Newton's $1/r^2$ force law. Let's simply ignore problems about collision singularities by focusing on solutions that are collision-free. Nevertheless, after many decades of work, it has been established that noncollision singularities can occur; that is, drawing on the infinitely deep $1/r$ potential well, the particles can accelerate themselves off to spatial infinity in a finite amount of time (Xia 1992). The time reverse of such a process is an example of 'space invaders', particles appearing from spatial infinity without any prior warning. To put it crudely, you can't hope to have Laplacian determinism for open systems, and for the type of interaction under discussion, the entire universe is an open system.

To save determinism from this threat, three approaches can be taken. The first, and least interesting, is to impose boundary conditions at spatial infinity to rule out space invaders. This smacks of making determinism true by creating a postulate of wishful thinking. The second is to maintain the idealization of point mass particles while adding to Newton's $1/r^2$ attractive force a short-range repulsive force which doesn't affect predictions for particles with large spatial separations, but which is such that the total potential well is no longer infinitely deep. One would then have to show that the combined force law gives rise to a well-posed Laplacian initial value problem. I am not aware of any results to this effect, but I see no in-principle obstacles to achieving them. The third alternative is to move from point mass particles to a particle with a finite radius, and to postulate that when two particles collide they obey, say, the laws of elastic impact. Even if it is assumed that atomism is true in a form that excludes the Zeno examples of section 1, this tack can run aground on at least two shoals. (a) If triple collisions occur the result is generally underdetermined since there will be more unknowns than there are governing equations. (b) Even if attention is restricted to binary collisions the uniqueness of solutions can fail if an infinite number of particles are present in the universe. Turn off the gravitational interactions of the particles and suppose that they interact only upon contact. Lanford (1974) constructed a solution[4] in which all the particles are at rest for all $t \leq t^*$ but for any $t > t^*$ all but a finite number of particles are in motion. Thus, the equations of motion don't determine whether a quiescent past is to be extended into the future by a 'normal' solution in which the particles continue to be quiescent or by an 'abnormal' solution in which the particles appear to self-excite. Determinism can be saved either by banning infinities of particles or by imposing boundary conditions at infinity which prevent a too rapid increase in the velocities of particles as one goes out to infinity. Either move smacks of making the world safe for determinism by fiat. A more interesting saving move would be to show that analogues to Lanford's solution cannot be constructed if elastic collisions are replaced by a smooth short-range repulsive force. I am not aware of any results to this effect.

Let us now leave particles to consider pure field theories. A familiar field equation in Newtonian physics is the Fourier heat equation

$$\nabla^2\Phi = \kappa\frac{\partial\Phi}{\partial t} \tag{4}$$

where κ is the coefficient of heat conduction. That (4) fails to be invariant under the Galilean transformations (2) is no cause for concern since the Φ-field is supposed to be a property of a medium, and this medium picks out a distinguished rest frame. What is of concern is that disturbances in the Φ-field are propagated infinitely fast. As a result, (4) admits smooth solutions Φ^* with the by now familiar determinism-wrecking property that $\Phi^* = 0$ for all $t \leq t^*$ but is non-zero for $t > t^*$—the field theoretic version of space-invading particles. Because of the linearity of (4), if Φ is a solution, so is $\Phi + \Phi^*$. Once again determinism can be saved by paring down the set of solutions by imposing boundary conditions at infinity. Perhaps a more interesting move is to declare that heat is nothing but the kinetic energy of molecules and, thus, that the indeterminism in (4) is not disturbing since (4) is not a fundamental law. The fate of determinism then reverts to properties of the fundamental laws, which are taken to be the laws governing particle motion. Even if one is inclined to follow this line, it should not be allowed to disguise the disturbing point that the Newtonian setting is inimical to deterministic field laws. The natural language for formulating laws governing field quantities is that of partial differential equations. But the type of pde that admits existence and uniqueness for a Laplacian initial value problem are of the hyperbolic type, and hyperbolic pdes require a null cone structure for spacetime that exists naturally in the relativistic setting but can only be artificially introduced in the Newtonian setting.

Although it is outside of my main focus, I will close this section with a few comments on epistemological matters. Ontological determinism is compatible with sensitive dependence on initial conditions. When measurement procedures for ascertaining the values of state variables are not error-free—as would seem to be the case for any actual measurement—sensitive dependence on initial conditions means that the link between determinism and prediction is weakened and even broken. Furthermore, a strong form of sensitive dependence on initial conditions (positive Liapunov components) plus the compactness of phase space implies 'chaos' in the form of higher-order ergodic properties, such as the Bernoulli property (see Belot and Earman 1997), which means that on a macroscopic scale a deterministic system can behave in a seemingly random and stochastic fashion. This raises the issue of whether and how critters such as us can be justified in believing that the stochastic behavior we are observing is due to indeterminism in the form of an irreducibly stochastic element or to deterministic chaos (see Suppes 1993).

4 Determinism in Special Relativistic Classical Physics

The prospects for determinism brighten considerably when we leave classical spacetimes for Minkowski spacetime, the spacetime setting for special relativistic theories. The combination of the null cone structure for Minkowski spacetime plus the prohibition of superluminal propagation kills space invaders and solves in a non–ad hoc way the open systems problem. Fudge-free examples of Laplacian determinism—no boundary conditions at infinity needed—are now possible. Indeed, physicists are so convinced that Laplacian determinism is the norm in this setting that they sometimes use it to draw a distinction between fundamental and nonfundamental fields. A 'fundamental field' (such as the source-free electromagnetic field and the scalar Klein–Gordon field) is one whose field equations (respectively, the source-free Maxwell equations and the massive Klein–Gordon equations) have a Laplacian initial value problem that admits global existence and uniqueness proofs: given the appropriate initial data on a Cauchy surface (a spacelike hypersurface which intersects each timelike curve without an end point) there exists one and only one global solution (i.e., a solution valid for all of Minkowski spacetime) whose restriction to the chosen Cauchy surface agrees with the given initial data. The failure of field equations to admit even local existence and uniqueness proofs is taken as an indication of an incompleteness of description. The failure of global existence and uniqueness—say, because the solutions develop singularities after a finite amount of time—is taken as an indication of an illicit idealization of description.

A feel for why the relativistic setting is friendlier to Laplacian determinism than the Newtonian setting can be gained by introducing the notion of the *domain of dependence* $D(\Sigma)$ of some subset $\Sigma \subset M$ of points Σ of the spacetime manifold M. The future (respectively, past) domain of dependence $D^{+}(\Sigma)$ (respectively, $D^{-}(\Sigma)$) is defined as the set of all $p \in M$ such that every past (respectively, future) inextendible causal curve through p intersects Σ. The total domain of dependence $D(\Sigma)$ of Σ is then defined as

$D^+(\Sigma) \cup D^-(\Sigma)$. The idea is that, if the laws of physics are cooperative, the state on Σ suffices to fix the state throughout $D(\Sigma)$; but for a point $q \notin D(\Sigma)$ it is hopeless to try to determine the state at q from the state on Σ since events at q can be influenced by a causal process that does not register on Σ. Using $D(\Sigma)$ we can give a general definition of a Cauchy surface that applies to general relativistic spacetimes (to be discussed in section 6) as well as to Minkowski spacetime and also—with suitable allowances—to classical spacetimes: it is a global time slice Σ (i.e., a spacelike hypersurface without edges) such that Σ is achronal (i.e., is not intersected more than once by any future directed timelike curve) and such that $D(\Sigma)$ is the entire spacetime M. The $t = const$ level surfaces of any inertial time t for Minkowski spacetime are, of course, Cauchy surfaces. But the level surfaces of absolute simultaneity of Newtonian spacetime are *not* Cauchy (see figure 1.3); indeed, in this case $D(\Sigma) = \Sigma$ for Σ as $t = const$.

If determinism fares better in Minkowski spacetime, prediction certainly does not. The basic problem is that the very null cone structure that helps to make the special relativistic setting friendly to determinism makes it impossible to acquire the information needed for a prediction prior to the occurrence of the events to be predicted. To make this precise a few additional definitions are needed. Define the *causal* (respectively, *chronological*) *past* of a point $p \in M$, $J^-(p)$ (respectively, $I^-(p)$) to be the set of all $q \in M$ such that there is a future-directed causal (respectively, timelike) curve from q to p. Then for a point $p \in M$ take the *domain of prediction* $P(p)$ of p to be the set of all $q \in M$ such that (i) every past inextendible causal curve through q enters $J^-(p)$, and (ii) $I^-(q) \nsubseteq I^-(p)$.[5] Condition (i) is needed to assure that an observer at p can, in principle, have causal access to all the processes that can influence the events at q, and condition (ii) is needed to assure that from the perspective of an observer at p the events to be predicted at q have not already occurred. The reader can now verify that for any point p of Minkowski spacetime, $P(p) = \varnothing$.

5 Determinism in Nonrelativistic Quantum Mechanics and Relativistic Quantum Field Theory

To illustrate how using determinism to probe the foundations of physics can lead to interesting results, consider the following little puzzle. No field equation for a scalar field ψ that is first order in time and Galilean invariant can be Laplacian deterministic. For we can choose a Galilean transformation (2) with the property that it is the identity on the slice $t = 0$ but non-identity for $t > 0$. By Galilean invariance this transformation will carry a solution of the field equation to another solution, but the new solution has the same initial data $\psi(\mathbf{x}, 0)$ as the original solution but different values for $t > 0$. The puzzle concerns the Schrödinger equation

$$\hat{H}\psi = i\hbar\frac{\partial\psi}{\partial t} \tag{5}$$

where $\hat{H}$ is the Hamiltonian operator. For a particle with mass m moving in an external potential $V(\mathbf{x})$, $\hat{H} = \frac{-\hbar^2}{2m}\nabla^2 + \mathrm{V}$. This equation seems to contradict what was said above since it is first order in time, is (presumably) Galilean invariant, and is Laplacian deterministic in that $\psi(\mathbf{x}, 0)$ uniquely determines $\psi(\mathbf{x}, t)$ for $t > 0$. The resolution is that ψ does not behave like a scalar under a Galilean transformation. In fact, the transformation properties of ψ needed to guarantee Galilean invariance of (5) implies a 'superselection rule' for mass which means that states corresponding to different mass cannot meaningfully be superposed.

In some respects quantum systems behave *more* deterministically and *more* predictably than their classical counterparts. As an example of the former, consider again the problem of a finite number of point mass particles interacting via Newton's $1/r^2$ law. The quantum Hamiltonian operator $\hat{H}$ for this system is (essentially) self-adjoint, which implies that the evolution operator $\hat{U}(t) := \exp(-i\hbar\hat{H}t)$ is unitary and is defined for all t. QM has magically smoothed away the both the collision and noncollision singularities of classical mechanics. This magic does not work for all of the singularities of classical mechanics. For example, one could try to overcome the Lanford type singularity discussed above in section 3 for infinite billiards by modeling the collisions of the balls with a short range repulsive force and then quantizing. Unfortunately, there is no well-defined quantum dynamics for such a system if the repulsive force is sufficiently strong (Radin 1977).

QM also overcomes one form of unpredictability that haunts classical mechanics since, in one sense, QM does not recognize any sensitive dependence on initial conditions. From the linearity and unitarity of the time evolution operator, it follows that $\|\psi_1(0) - \psi_2(0)\| = \|U(t)(\psi_1(0) - \psi_2(0))\| = \|U(\mathrm{t})\psi_1(0) - U(t)\psi_2(0)\| = \|(\psi_1(t) - \psi_2(t))\|$: in words, if at $t = 0$ two states are nearby, they remain nearby for all $t > 0$ in the Hilbert space norm $\| \bullet \|$. This simple fact has caused some consternation since it isn't evident how, consistent with the correspondence principle, chaos can emerge from QM in some appropriate classical limit (Belot and Earman 1997).

None of the remarks so far touches the core problem of determinism in QM. But that problem is difficult to discuss, or even to formulate, because it is bound up with a contentious issue about the nature of quantum observables; namely, under what conditions do the quantum observables, as represented by self-adjoint operators on a Hilbert space, take on definite values? One answer endorsed by standard textbooks on QM is that an observable A takes the definite value a at t iff the state $\psi(t)$ is an eigenstate of the operator $\hat{A}$ with eigenvalue a.[6] This eigenvector-eigenvalue rule—or any value assignment rule which says that an observable has a specified definite value just in case the state vector has some specified characteristic—together with deterministic Schrödinger evolution for ψ would mean that Laplacian determinism holds for quantum systems.[7]

The well-known difficulty with the eigenvector-eigenvalue value assignment rule is that it implies the unacceptable result that, after an interaction with an object system, the pointer on the dial of a measuring instrument that couples to the observable whose value for the object system we wish to ascertain has no definite value if the initial state of the object system was a superposition of eigenstates of the observable being measured. One attempt to overcome this embarrassment involves 'state vector reduction': at some juncture during the measurement process, Schrödinger evolution ceases and the state vector jumps into a simultaneous eigenstate of the object observable and the macro pointer position observable that serves as an indicator of the value of the object observable. The violation of the Schrödinger equation is, of course, a violation of Laplacian determinism; but statistical determinism is maintained by the assumption that the propensity of the state vector to collapse into a given eigenstate is governed by the Born rule probability calculated from the state vector just before collapse.

The collapse solution to the measurement problem comes at a high cost. The original theory with an embarrassing consequence has been replaced by a nontheory: the objection is not simply that the state vector reduction is a miracle—a violation of the Schrödinger equation—but that since 'measurement' is a term of art whose application is not specified by the theory, exactly when and under what circumstances the reduction takes place is left dangling. Some physicists have proposed to bridge this gap by describing a mechanism for state vector reduction by means of a nonlinear wave equation or a stochastic differential equation (see Ghirardi et al. 1986 and Pearle 1989). The extant state vector reduction schemes are nondeterministic; but it is an open question whether they must have this property.

The major alternative route to a solution of the measurement problem involves a modification of the eigenvector-eigenvalue value assignment rule rather than of the state vector dynamics. The different paths that branch off this no-collapse route can lead to quite different conclusions about determinism, as is illustrated by the Bohm interpretation, the modal interpretations, and the many worlds/many minds interpretations. According to the first, particles always have definite positions, and these positions evolve deterministically via an equation of motion that is parasitic on the Schrödinger evolution of the state vector. Assuming that all laboratory measurements can be reduced to recording positions, the Bohm interpretation offers a deterministic explanation of how and when other quantum mechanical observables, such as spin, take on definite values. It is shown that the orthodox quantum mechanical probabilities are recovered for all times on the assumption that the initial probability distribution at $t = 0$ over particle positions $\mathbf{q}$ is equal to the quantum mechanical prescription $|\psi(\mathbf{q}, 0)|^2$.

By contrast, modal interpretations don't seek to preserve determinism but settle for the more modest aim of breaking the eigenvector-eigenvalue link open wide enough to make sure that measurements have definite outcomes. This aim is accomplished by a value assignment rule which makes use of a special decomposition of the vector for composite systems (the 'biorthogonal decomposition'). This decomposition identifies a privileged class of observables which are said to possess definite values without saying which particular values are possessed. Some modal theorists want to add to their value assignment rule a dynamics for possessed values. There is a plethora of such dynamical schemes (see Dickson 1998). Typically these schemes are not deterministic. However, a recent proposal by Ax and Kochen (1999) restores determinism by construing the phase of the state vector as an additional hidden parameter—the different unit vectors in a ray of Hilbert space are taken to correspond to different members of an ensemble, and the apparent indeterminism of QM is due to a random distribution of initial phases.

Many worlds interpretations come in two main versions: the literal version according to which a measurement event corresponds to a physical splitting of the world + observer (qua physical object), and the figurative version according to which it is not the physical world but the mind or mental state of the observer that splits. On the literal version determinism fails if there is a fact of the matter about which postmeasurement observer (qua physical object) is identical with the premeasurement observer. By the same token, measurement is not an indeterministic process if genidentity of observers is denied. A parallel conclusion holds for the many minds version, with minds in place of bodies. It is hard to take any of this seriously, not so much because of the metaphysical extravagances, but because it is not at all evident that the many worlds/many minds interpretation does resolve the measurement problem: it doesn't explain exactly when and under what circumstances the splitting takes place, and it doesn't explain why the splitting takes place in some bases of the Hilbert space but not others.

There is no easy way to summarize the status of determinism in QM, but two points need to be underscored. The proponents of the Bohm interpretation claim that it recovers all quantum mechanical predictions, statistical and nonstatistical, about experiments. If this is correct, it means that in a world of ordinary nonrelativistic QM the fate of Laplacian determinism would have to be decided by nonempirical factors. The second remark concerns the claim, sometimes heard, that determinism is defeated by various no-go results for hidden variables—for example, those of the Kochen–Specker type and those that flow from the Bell inequalities. What these no-go results show is that, consistent with certain plausible mathematical restrictions on value assignments and/or with the statistical predictions of QM, some set of quantum observables cannot be assigned simultaneously definite values lying in the spectra of these observables. By itself this hardly defeats Laplacian determinism; nor is it even a terribly surprising conclusion—after all, many classical quantities are best construed as having a dispositional character in that they only take on definite values in limited contexts. That it is impossible to view the values of quantum observables as supervening on more fundamental quantities whose temporal evolution is deterministic would require a different kind of proof, a proof whose existence is highly dubious in view of the example of the Bohm interpretation.

We saw that moving classical mechanics from the Newtonian setting to the special relativistic setting improves the fortunes of determinism. The same cannot be said for quantum mechanics; for the fortunes of determinism in QM are bound by the knots of the measurement problem and the value assignment problem, and in the relativistic setting these knots are drawn even tighter. State vector reduction is supposed to take place instantaneously. Transposing this notion to the relativistic setting seems to require either breaking Lorentz invariance or relativizing states to spacelike hyperplanes (Fleming 1996), neither of which is an attractive option. The relativistic setting is also unkind to the modal interpretation because it is difficult to reconcile the value assignment rule of this interpretation with Lorentz invariance (Dickson and Clifton 1998). Nor does the Bohm interpretation fit comfortably with relativistic quantum field theory (QFT) since the ontology of this theory is best construed not in terms of particles but in terms of local field observables. The general idea of Bohmian dynamics can be applied to observables other than particle position. But in the absence of a demonstration that in measurement contexts the field observables chosen for Bohmian treatment will exhibit the localization needed to explain measurement outcomes, one of the major reasons for finding the Bohm interpretation attractive has been lost (see Saunders 1999). Furthermore, when QFT is done on curved spacetime the dynamics for the quantum field may not be unitarily implementable (see Arageorgis et al. 2002), making it harder to construct a field theoretic analogue of the Bohm dynamics for particle position.

6 Determinism in Classical General Relativistic Physics

In Einstein's general theory of relativity (GTR) we have to deal not with one spacetime setting but many since different solutions to the Einstein gravitational field equations (EFE)

$$R_{ab} - \frac{1}{2}Rg_{ab} = \kappa T_{ab} \tag{6}$$

give different spacetime structures. (Here g_{ab} is a Lorentz signature metric, R_{ab} is the Ricci tensor [which is constructed from g_{ab} and its derivatives], $R := Tr(R_{ab})$ is the Ricci curvature scalar, and T_{ab} is the stress-energy tensor [which describes the distribution of the matter-energy].) A model of GTR is a triple M, g_{ab}, T_{ab}, where M is a four-dimensional manifold (without boundary) and the metric g_{ab} is defined on all of M. A dynamically possible model is one that satisfies (6) for all points of M. (Additionally, one may want to impose on T_{ab} various so-called energy conditions that guarantee, for example, nonnegative energy densities.)

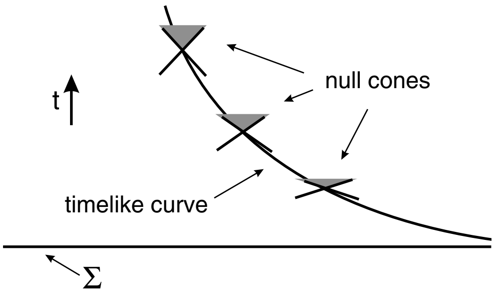

Figure 1.4
Behavior of null cones in anti–de Sitter spacetime.

To first approximation we can take a spacetime M, g_{ab} of a dynamically possible model as a fixed background on which a test field propagates. And we can ask whether this propagation exhibits Laplacian determinism. For some spacetimes coming from dynamically possible models of GTR the issue of global Laplacian determinism for a test field cannot even be stated since there may not exist any global time slices, as is the case with Gödel spacetime. In other cases the issue can be posed but has a negative answer because the spacetime admits global time slices but none of them is a Cauchy surface, as illustrated by the (covering space) of anti–de Sitter spacetime (see figure 1.4) where, in effect, the space invaders of Newtonian physics have returned in a general relativistic guise. Here again we have an illustration of the moral that determinism succeeds only with a little—or a lot—of help from its friends. Here the friends must find a non-question-begging way to exclude such pathologies from the causal structure of general relativistic spacetimes.

Treating spacetime as a fixed backdrop against which nongravitational physics takes place is inimical to the spirit of GTR which implies that spacetime structure is dynamical. Strictly speaking, the evolution of the test field has to be treated as part of the general problem of the coevolution of the metric and matter fields as governed by the coupled Einstein matter field equations. For the sake of simplicity I will initially concentrate on the initial value problem for the source-free ($T_{ab} \equiv 0$) EFE

$$R_{ab} - \frac{1}{2}Rg_{ab} = 0 \tag{7}$$

Specifying the metric field and its normal derivative on some spacelike slice Σ does not suffice to determine, via (7), the values of the field at points of M to the future or the past of Σ. Indeed, specifying g_{ab} on Σ and the entire causal past of Σ does not suffice to determine g_{ab} at points to the future. For if M, g_{ab} is a solution to the (7) and $d : M \to M$ is any diffeomorphism of M, then M, d^*g_{ab} is also a solution.[8] By now you know the trick: choose d such that $d = id$ for all points of M on or to the past of Σ but $d \neq id$ for points of M to the future of Σ. Thus we get two solutions of (7) such that the metric fields are the same in the past but different in the future.[9]

Determinism can be saved by requiring of it only that the values of genuine physical magnitudes or 'observables' be determined, via the field equations, from initial data. What then are the observables of GTR? If we have faith in determinism we can close the circle and require that whatever counts as an observable be such that its future and past values be determined by appropriate initial data (Bergmann 1961). From the above construction we see that the observables of GTR must be diffeomorphically invariant quantities.[10] This means that none of the familiar local field quantities—such as the metric field or scalar fields formed from the metric field—are observables. And in some situations, such as solutions with compact time slices, not even quasi-local quantities such as the integral of the Ricci curvature over a time slice are observables.

Here I want to emphasize the contrast between the Newtonian and general relativistic cases. In the Newtonian case it was possible to save determinism from the kind of threat under discussion either by abandoning the notion that the spacetime manifold is a container for events or alternatively by retaining this notion and adding sufficient absolute background structure to the spacetime. But in the general relativistic setting that eschews absolute objects, there is no choice; making GTR a safe haven for Laplacian determinism necessitates a radical revision of the surface level ontology and ideology of GTR as a theory of tensor fields on manifolds. One of the disconcerting features of the revision is that it seems to entail a completely frozen block universe—not only is there no shifting nowness, but there is a total absence of ordinary change since the values of observables of GTR do not change with time (see my 2002). So as not to sidetrack the discussion into these interesting but controversial matters, I will continue to discuss determinism in GTR in terms of the vocabulary of the surface level structure, which means that the statement of uniqueness results for the initial value problem will have to contain an escape clause of 'up to a diffeomorphism'.

Suppose then we are given a three manifold Σ and initial data on Σ for the source-free gravitational field. Does this data fix an appropriately unique solution to the source-free field equations (7)? The answer is yes, at least locally: there exists a unique (up to diffeomorphism) maximal development M, g_{ab} of the initial data for which Σ is a Cauchy surface; that is, M, g_{ab} cannot be extended as a solution to the source-free equations in any way that keeps Σ a Cauchy surface.[11]

Extending this local result, which gives no information about the size of the maximal solution for which Σ is a Cauchy surface, to a global result—especially to the very strong global result that requires the unique maximal development for which the initial value hypersurface is a Cauchy surface to be maximal simpliciter—can run into various kinds of problems. First, we might have made a poor choice of the initial value hypersurface; for instance, we might have chosen Σ to be the spacelike hyperboloid of Minkowski spacetime pictured in figure 1.5. Obviously the maximal development of the initial data, induced on Σ by the Minkowski metric, for which Σ is a Cauchy surface—namely $D(\Sigma)$—can be properly extended as a solution of the source-free field equations. Second, and more interestingly, there might be no good choice for the initial value surface, as in the case of an inextendible spacetime, which is a solution of (7) and does not possesses any global Cauchy surfaces. This disturbing possibility can arise for two reasons. (a) The causal structure might be like that illustrated in figure 1.4. (b) A singularity might develop in a finite time from the regular initial data, as is indicated schematically in figure 1.6. Singularities can arise, of course, in the Newtonian and special relativistic settings, but because of the existence

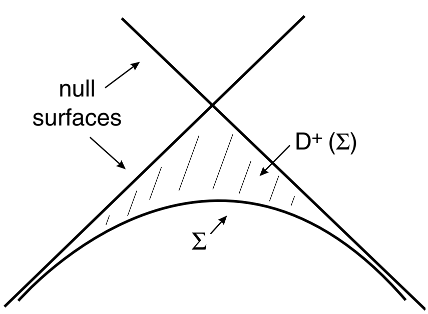

Figure 1.5
A bad choice of initial value hypersurface.

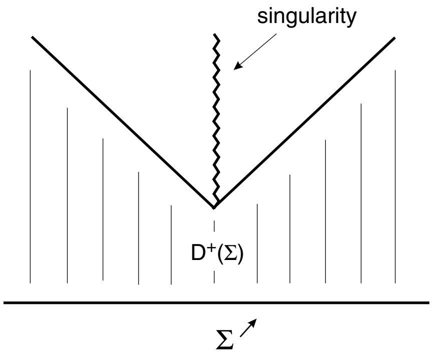

Figure 1.6
A naked singularity.

of a fixed background metric these singularities are easily characterizable—for example, they occur at regions of spacetime where some physical field 'blows up' or becomes discontinuous. But in the general relativistic setting there is no fixed background metric, and the singularities at issue are singularities in the metric structure of spacetime itself. Since by definition a spacetime is a pair M, g_{ab} where g_{ab} is defined at all points of M, we cannot think of singularities as objects that have spacetime locations. Attempts have been made to characterize spacetime singularities in GTR as boundary points that are attached to the manifold M; but these constructions all seem to involve counterintuitive features—for example, the boundary points may not be Hausdorff separated from interior points. And to make things even more complicated, the singularities that lead to a breakdown of Laplacian determinism need not be of the intuitive sort involving, say, a 'blow up' or wild oscillation of curvature scalars. In fact, one could simply dub any spacetime pathology which prevents moving from the local to global versions of Laplacian determinism in GTR a 'naked singularity'. Then the task would be to classify and characterize these pathologies.[12]

Roger Penrose's cosmic censorship conjecture surmises that the pathologies of naked singularities are not as bad in practice as they might seem in principle. Cosmic censorship comes in two versions, weak and strong. The weak version asserts that, generically, if naked singularities do develop from the gravitational collapse of suitable matter fields, then they will be contained inside black holes so that those fortunate observers who remain outside the black hole event horizon cannot 'see' the singularity and are,

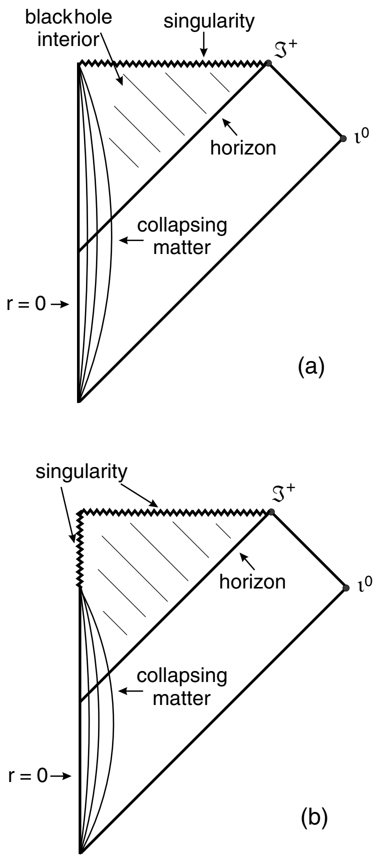

Figure 1.7
Weak and strong cosmic censorship in black hole formation.

thus shielded by the one-way causal membrane of the horizon from whatever nondeterministic effects emanate from the singularity.[13] The strong version of cosmic censorship conjectures that, generically, naked singularities do not develop from suitable matter fields, so that even the unfortunate observers who fall into a black hole do not 'see' the singularity. The strong and weak versions are illustrated respectively in figures 1.7a and 1.7b, which represent black hole formation in spherical gravitational collapse.[14] The weasel phrases 'generically' and 'suitable matter fields' are essential to warding off potential counterexamples. For instance, the weak version of cosmic censorship can be violated using collapsing dust matter.

But it seems wrong to lay the blame for the resulting naked singularity on the process of gravitational collapse itself since, even in Minkowski spacetime singularities in dust matter fields can develop from regular initial data. Thus, 'suitable' matter fields should be restricted to the 'fundamental' matter fields for which global existence and uniqueness properties hold in Minkowski spacetime (see section 4). But this restriction is not by itself enough to save weak cosmic censorship since special initial configurations of the Klein–Gordon field (certainly a fundamental field) can lead to naked singularities in gravitational collapse. Making precise the notion of non-special or generic initial conditions would require either a suitable measure on the space of solutions to Einstein's equations (in which case, generic = except for a set of measure zero) or a suitable topology (in which case, generic = complement of a set whose closure has an empty interior). Since it is not clear what the relevant measure or topology is, the cosmic censorship conjecture is also unclear. Despite these difficulties however, a growing body of inductive evidence—for example, from the stability of black holes—suggests that some interesting form of weak cosmic censorship is probably true (Wald 1998). Opinion on the strong cosmic censorship conjecture is more mixed, in part because it is harder to state in a form that is both general and free of obvious counterexamples. Proving a general form of either the weak or strong version seems to be beyond the capabilities of present techniques for investigating the global existence and uniqueness of the solutions to nonlinear pdes.

Supposing that cosmic censorship fails, it remains unclear how the fortunes of determinism are affected. Classical GTR places no restrictions on what nondeterministic influences can emerge from the naked singularities. But this does not mean that anything goes. If we are optimistic, we can suppose that even at the prequantum level there are additional lawlike regularities, beyond those codified in Einstein GTR, that govern these influences. And a quantum theory of gravity may add further restrictions or, even better, get rid of the singularities altogether.

In closing this section I will note that while the fortunes of determinism becomes more perilous in passing from special to general relativistic physics, the fortunes of prediction perk up. For example, in various general relativistic spacetimes it is possible to have nonempty domains of prediction; indeed, it is possible for there to be points $p \in M$ such that $P(p) = M$ (see Hogarth 1993).

7 Implications of Quantum Gravity

Among the challenges facing physics in the twenty-first century, that of producing a quantum theory of gravity is one of the most important and the most difficult. At the present moment there are two leading approaches: the loop formulation, which is a version of the canonical quantization program that aims to produce a quantum theory of gravity by quantizing GTR, and string theory, which does not start with classical GTR but seeks to explain it as an outcome of some low-energy limit of vibrating string or branes. Both programs hold out the hope that the singularities of classical GTR will be smoothed away, which would be a boon for determinism. This hope is fueled by the ability of the quantum to smooth away classical singularities. In section 5 we saw a demonstration of this ability at work in ordinary QM which smooths away some of the collision and noncollision singularities of Newtonian point mass mechanics. Another demonstration comes from the fact that quantum particles shot at the timelike singularities (which violate cosmic censorship) of some general relativistic spacetimes have a well-defined temporal evolution (Horowitz and Marolf 1995). However, there is an argument to the effect that we shouldn't hope that quantum gravity will smooth away all of the singularities of classical GTR; for if, for instance, a quantum theory gravity smoothed away the singularity of the negative mass Schwarzschild solution, the theory would admit a nonsingular solution with an unstable ground state, which is a physical disaster (Horowitz and Myers 1995). String theorists can remain optimistic on the basis of the fact that their theory must entail a stable ground state; thus, insofar as string theory smooths away classical singularities, it must also contain a mechanism that excludes solutions such as a smoothed version of the negative mass Schwarzschild solution (Johnson et al. 2000).

In the absence of a completed theory of quantum gravity, some insight into what may result from combining GTR and QM can be obtained from attempting to do QFT on a curved spacetime background. In even the simplest case of a linear scalar field, determinism is crucial to the attempt: the standard construction of the algebra of field observables relies on the deterministic character of (real) solutions to the Klein–Gordon equation, which holds when the spacetime background is globally hyperbolic (= admits a Cauchy surface). When global hyperbolicity fails, there is no accepted procedure for constructing the field algebra, and when the reason for failure of global

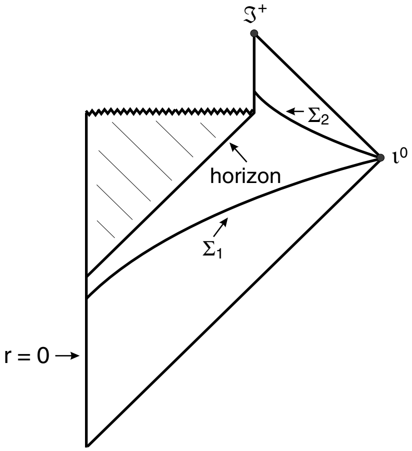

Figure 1.8
Black hole evaporation.

hyperbolicity is the presence of nasty causal structure—such as closed timelike curves—there may exist no global field algebra consistent with the natural demand that the algebra, when restricted to sufficiently small globally hyperbolic neighborhoods, should agree with the algebra obtained by applying standard construction to those neighborhoods (see Fewster 1999).

The next step up the ladder to quantum gravity involves the so-called semiclassical approximation in which the expectation value of the (renormalized) stress-energy tensor for quantum fields on a curved spacetime is inserted on the right hand side of the EFE (6), and the backreaction effects on the spacetime metric are calculated. Hawking found that the presence of quantum fields means that the black hole is not black but instead radiates with a thermal spectrum and that the backreaction effect of this Hawking radiation is to cause the black hole to evaporate (Wald 1994). If the evaporation is complete, the likely outcome—insofar as it can be described in terms of classical general relativistic spacetime structure—is a naked singularity (see figure 1.8). As a consequence, a pure quantum state at the time Σ_1, before black hole formation, evolves into a mixed state at the postevaporation time Σ_2. This pure-to-mixed transition is necessarily nonunitary, and the 'information loss' it involves has been the subject of a heated discussion in the physics literature (see Belot, Earman, and Ruetsche 1999 for a review). If the pure-to-mixed transition survives in the final quantum theory of gravity, then so does a semblance of singularities as a sink for the missing information (Wald 1999).

8 Conclusion

One might have hoped that this survey would provide an answer to the question: If we believe modern physics, is the world deterministic or not? But there is no simple and clean answer. The theories of modern physics paint many different and seemingly incommensurable pictures of the world; not only is there no unified theory of physics, there is not even agreement on the best route to getting one. And even within a particular theory—say, QM or GTR—there is no clear verdict. This is a reflection of the fact that determinism is bound up with some of the most important unresolved foundations problems for these theories. While this linkage makes for frustration if one is in search of a quick and neat answer to the above question, it also makes determinism an exciting topic for the philosophy of science.

Acknowledgment

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Notes

1. There is, of course, a violation of conservation of energy and momentum in the overall process even though both energy and momentum are conserved in each collision.

2. This example also illustrates a violation of even the weakest version of Laplacian determinism according to which the future state of the world is determined by its entire past history.

3. For more details on various classical spacetime structures, see my (1992).

4. Solution in the sense that only binary collisions occur and each such collision obeys the laws of elastic impact.

5. This is a slight modification of the definition given in Geroch (1977). Other definitions of prediction for general relativistic spacetimes are studied in Hogarth (1993).

6. This rule implies that an observable with a pure continuous spectrum never takes on a definite value. This awkwardness can be overcome by talking about approximate eigenstates.

7. At least on the reasonable understanding that the failure of the theory to determine the nonexistent value of an observable does not represent a failure of determinism.

8. A diffeomorphism of M is a one-one map of M onto itself that preserves the differentiable structure (e.g., if M is a C^∞ manifold, the mapping must be C^∞). d^*g_{ab} denotes the dragging along of g_{ab} by d.

9. This is a version of the notorious 'hole argument' which led Einstein to abandon his search for generally covariant field equations from 1913 until late 1915; see Norton (1987).

10. In 1916 Einstein took the 'observables' of GTR (although he did not use this term) to be spatiotemporal coincidences, such as the intersection of two light rays (see Einstein 1916 and Howard 1999). In order to be adequate to the content of GTR, Einstein's notion of coincidence observables has to be extended to fields.

11. M', g'_{ab} is said to be a (proper) extension of M, g_{ab} iff the latter can be isometrically embedded as a (proper) subset of the former. For the details of the initial value problem in GTR, see Wald (1984).

12. For a more detailed discussion of spacetime singularities, as well as the cosmic censorship hypothesis (introduced below), see my (1995).

13. Being visible is not sufficient to make a singularity 'naked' in the relevant sense; for example, the initial big bang singularity in Friedmann-Robertson-Walker spacetimes is not 'naked' because all of these spacetimes possess Cauchy surfaces.

14. Figures 1.7 and 1.8 use the conventions of conformal diagrams, which preserve causal relations but distort metrical relations in order to bring infinities in to a finite distance; namely, null directions lie at 45° with respect to the vertical; $\mathfrak{I}^+$ stands for future null infinity, the terminus of outgoing light rays; and ι^o stands for spacelike infinity. The center of the spherical symmetry is labeled by $r = 0$.

References

Arageorgis, A., J., Earman, and L. Ruetsche. 2002. "Weyling the Time Away: The Aon-unitary Implementability of Quantum Field Dynamics on Curved Spacetime." *Studies in History and Philosophy of Modern Physics* 33: 151–184.

Ax, J., and S. Kochen. 1999. "Extension of Quantum Mechanics to Individual Systems." Quant-ph/9905077.

Belot, G. and J. Earman. 1997. "Chaos Out of Order: Quantum Mechanics, the Correspondence Principle and Chaos." *Studies in History and Philosophy of Modern Physics* 28: 147–182.

Belot, G., J. Earman, and L. Ruetsche. 1999. "The Hawking Information Loss Paradox: The Anatomy of a Controversy." *British Journal for the Philosophy of Science* 50: 189–229.

Bergmann, P. G. 1961. "Observables in General Relativity." *Reviews of Modern Physics* 33: 510–514.

Dickson, M. 1998. "On the Plurality of Dynamics." In J. Hellman and R. Healey, eds. *Quantum Measurement: Beyond Paradox*. Minneapolis, Minn.: University of Minnesota Press.

Dickson, M. and R. Clifton. 1998. "Lorentz Invariance in Modal Interpretations." In D. Dieks and P. Vermaas, eds. *The Modal Interpretation of Quantum Mechanics*. Dordrecht: Kluwer.

Earman, J. 1992. *World Enough and Spacetime: Absolute vs. Relational Theories of Space and Time*. Cambridge, Mass.: MIT Press.

Earman, J. 1995. *Bangs, Crunches, Whimpers, and Shrieks: Singularities and Acausalities in Relativistic Spacetimes*. Cambridge, Mass.: MIT Press.

Earman, J. 2002. "Thoroughly Modern McTaggart: Or What McTaggart Would Have Said if He Had Learned the General Theory of Relativity." *Philosophers' Imprint*. Http://www.philosophersimprint.org.

Einstein, A. 1916. "Die Grundlage der allgemeinen Relativitätstheorie." *Annalen der Physik* 49: 769–822. English translation from W. Perrett and G. B. Jeffrey eds. *The Principle of Relativity*. New York: Dover, 1952.

Fewster, C. J. 1999. "Bisolutions to the Klein–Gordon Equation and Quantum Field Theory on 2-Dimensional Cylinder Spacetimes." *Classical and Quantum Gravity* 16: 769–788.

Fleming, G. N. 1996. "Just How Radical Is Hyperplane Dependence?" in R. Clifton ed. *Perspectives on Quantum Reality*. Dordrecht: Kluwer Academic.

Geroch, R. 1977. "Prediction in General Relativity." In J. Earman, C. Glymour, and J. Stachel eds. *Foundations of Space-Time Theories, Minnesota Studies in the Philosophy of Science* 8. Minneapolis, Minn.: University of Minnesota Press.

Ghirardi, G. C., T. Rimini, and T. Weber. 1986. "Unified Dynamics for Microscopic and Macroscopic Systems." *Physical Review D* 34: 470–491.

Hogarth, M. 1993. "Predicting the Future in Relativistic Spacetimes." *Studies in History and Philosophy of Science* 24: 721–739.

Horowitz, G. T., and D. Marolf. 1995. "Quantum Probes of Spacetime Singularities." *Physical Review D* 52: 5670–5675.

Horowitz, G. T., and R. Myers. 1995. "The Value of Singularities." *General Relativity and Gravitation* 27: 915–919.

Howard, D. 1999. "Point Coincidences and Pointer Coincidences: Einstein and the Invariant Content of Space-Time Theories." In H. Goenner, J. Renn, J. Ritter, and T. Sauer, eds. *The Expanding Worlds of General Relativity*. *Einstein Studies*, volume 7. Boston, Mass.: Birkhäuser.

Johnson, C. V., A. Peet, and J. Polchinski. 2000. "Gauge Theory and the Excision of Repulsion Singularities." *Physical Review D* 61: 086001.

Lanford, O. E. 1974. "Time Evolution of Large Classical Systems." In J. Moser, ed. *Dynamical Systems, Theory and Applications*. *Lecture Notes in Mathematics,* volume 38. Berlin: Springer-Verlag.

Laraudogoitia, J. 1996. "A Beautiful Supertask." *Mind* 105: 81–83.

Norton, J. D. 1987. "Einstein, the Hole Argument and the Reality of Space." In J. Forge, ed. *Measurement, Realism, and Objectivity*. Dordrecht: D. Reidel.

Pearle, P. 1989. "Combining Stochastic Dynamical State Vector Reduction with Spontaneous Localization." *Physical Review A* 39: 2277–2289.

Penrose, R. "Gravitational Collapse: The Role of General Relativity." *Revisita del Nuovo Cimento*. Serie 1, 1 Numero Speciale: 252–276.

Radin, C. 1977. "The Dynamical Instability of Non-Relativistic Many-Body Systems." *Communications in Mathematical Physics* 54: 69–79.

Rovelli, C. 1991. "What Is an Observable in Classical and Quantum Gravity?" *Classical and Quantum Gravity* 8: 297–316.

Saunders, S. 1999. "The `Beables' of Relativistic Pilot Wave Theory." In J. Butterfield and C. Pagonis, eds., *From Physics to Philosophy*. Cambridge: Cambridge University Press.

Suppes, P. 1993. "The Transcendental Character of Determinism." *Midwest Studies in Philosophy*, volume 18. Notre Dame, Ind.: University of Notre Dame Press.

Wald, R. M. 1984. *General Relativity*. Chicago: University of Chicago Press.

Wald, R. M. 1994. *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*. Chicago, Ill.: University of Chicago Press.

Wald, R. M. 1998. "Gravitational Collapse and Cosmic Censorship." In C. V. Vishveshwara, B. R. Iyer, and B. Bhawal, eds., *Black Holes, Gravitational Radiation, and the Universe*. Dordrecht: Kluwer Academic.

Wald, R. M. 1999. "Gravitation, Thermodynamics, and Quantum Theory." *Classical and Quantum Gravity* 16: A177–A190.

Xia, X. 1992. "The Existence of Noncollision Singularities in the N-Body Problem." *Annals of Mathematics* 135: 411–468.