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The Ontology of Spacetime
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Chapter 1

The Implications of General Covariance for the Ontology and Ideology of Spacetime

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Abstract

It generally agreed that the requirement of formal general covariance (i.e. the demand that laws be written in a form that is covariant under arbitrary coordinate transformation) is a condition of the well-formedness of a spacetime theory and not a restriction on its content. Physicists commonly take the substantive requirement of general covariance to mean that the laws exhibit diffeomorphism invariance *and* that this invariance is a gauge symmetry. This latter requirement does place restrictions on the content of a spacetime theory. The present paper explores the implications of these restrictions for interpreting the ideology and ontology of classical general relativity theory and loop quantum gravity.

1. Introduction

The story I have to tell here is not new. But it is worth retelling in various forms because it is not well known to philosophers. And some of the philosophers who know the story are in a state of denial. Perhaps the retelling will awaken them from their dogmatic slumber. If not, I have another message for them: physics is marching on despite your scruples. More generally, the story illustrates the trials and tribulations of scientific realism. Suppose that we resolve — as I think we should — to be realists in interpreting scientific theories. In carrying out our resolution we have to be aware of Roger Jones' (1991) question: what are we to be realists *about*? One of the morals that will emerge below is that for a generally covariant theory we cannot be naive realists and read the ontology/ideology of spacetime directly off the surface structure of the theory.

2. Two concepts of general covariance

I will distinguish two senses of general covariance, *formal* and *substantive*. A spacetime theory satisfies *formal general covariance* (FGC) just in case its laws are covariant under arbitrary spacetime coordinate transformations or, equivalently, its laws are true in every coordinate system if they are true in any. This is a condition on the well-formedness of a theory, not on its content[1]. There is nothing to celebrate about the fact that Einstein's general theory of relativity (GTR) satisfies FGC; or rather if there is, then celebration is also in order for many Newtonian and special relativistic theories since these theories can, without change of physical content, be formulated in a formally generally covariant manner. In hindsight, this should have been blindingly obvious: spacetime theories can be formulated in a completely coordinate-free manner, so coordinates cannot possibly matter in any substantive way[2].

To introduce some language I will be using throughout, FGC is an example (albeit a rather trivial example) of a gauge symmetry — that is, a symmetry that relates different descriptions of the same physical situation. To make this more concrete, assume that the models of a spacetime theory have the form (M, O_1, O_2, …, O_N), where M is a differentiable manifold (assumed for convenience to be C^∞) and the O_j are geometric object fields on M. In the case of GTR, the key geometric object fields are the spacetime metric g_{ab} and the stress-energy tensor T_{ab}. Each (local) coordinate system $\{x^i\}$ gives rise to a representation of these objects in terms of their coordinate components g_{ij} and T_{ij} in the given coordinate system. The result is a huge redundancy in description with many different coordinate representations, all corresponding to the same (M, g_{ab}, T_{ab}). The coordinate transformations that shuttle between the different representations are, thus, examples par excellence of gauge transformations (see the top portion of Fig. 1).

Now let d: $M \to M$ be a diffeomorphism (i.e. a one–one C^∞ map of M onto itself). Then a spacetime theory satisfies *substantive general covariance* (SGC) just in case (i) if (M, O_1, O_2, …, O_N) satisfies the laws of the theory, then so does

[1]At least on the assumption that the reality spacetime theories seek to capture can be completely described in terms of geometric object fields on a manifold; see below.

[2]For example, instead of characterizing, say, contravariant tensors as objects with coordinate components that transform in a specified way under coordinate transformations, they can be characterized in a coordinate-free manner as multilinear maps of tuples from tangent vectors to $\mathbb{R}$.

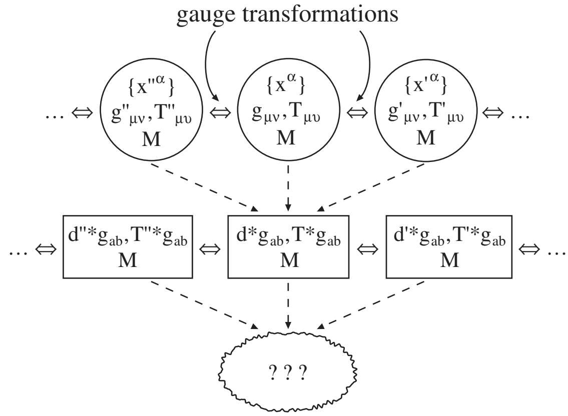

Fig. 1. Two levels of gauge redundancy in general relativity theory.

$(M, d^*O_1, d^*O_2, \ldots, d^*O_N)$ for any $d \in$ *diff* (M), where d^*O_j stands for the drag along of O_j by d, and (ii) this diffeomorphism invariance is a gauge symmetry of the theory, i.e. $(M, O_1, O_2, \ldots, O_N)$ and $(M, d^*O_1, d^*O_2, \ldots, d^*O_N)$ are descriptions of the same physical situation. SGC thus implies that there is a second level of descriptive redundancy (see the bottom portion of Fig. 1)[3].

The distinction between the two concepts of general covariance lacks bite unless it is accompanied by an account of gauge symmetries. Here I have several claims. First, there is an extant account of gauge symmetries that is widely accepted in the physics community. My assumption is that this account should be taken as the default account of gauge. This assumption is, of course, defeasible; but philosophers who want to override it are obligated to provide an alternative account. Second, the application of the recommended account to GTR implies that diffeomorphism invariance is a gauge symmetry of this theory and, hence, that GTR satisfies SGC. Third, this account implies that diffeomorphism invariance is not a gauge symmetry of typical pre-general relativistic spacetime theories. Thus, while Einstein had no reason to celebrate because his GTR fulfilled FGC, the fact that the theory fulfills SGC is something to celebrate — or at least to underscore. The hesitancy here has to do with the question marks in Fig. 1; namely, what is the nature of the reality that underlies the second level of redundancy of description? Or to put it in the terminology physicists use, what are the gauge-invariant quantities (a.k.a. "observables") of

[3]The two concepts of general covariance are sometimes confused because a coordinate transformation $x^{\upsilon} \to x'^{\upsilon}(x^{\mu})$ can be taken to indicate a mere relabeling of points of M or as indicating a (local) diffeomorphism that sends a point $p \in M$ in the common domain of the coordinate systems to another point $p' \in M$ where $x'^{\upsilon}(p') = x^{\upsilon}(p)$.

GTR in particular and of substantively generally covariant spacetime theories in general? It is far from clear what the best positive answer is. But on the negative side, one thing is clear from the start: accepting the interpretation of SGC I am recommending implies that *none* of the quantities used in standard textbook presentations of GTR — not even "scalar invariants" — are observables. Evidently then, SGC implies a rejection of the naive realism that would have us read off the ideology and ontology of GTR from standard presentations of the theory. But first things first. In the next section, I will give a brief sketch of the account of gauge symmetries on which these claims are based.

3. Gauge symmetries

In this section, I will attempt to provide a bare-bones, non-technical sketch of what I will call the standard analysis of gauge symmetries. Those interested in the details are referred to Earman (2003b, 2006). The standard analysis has a broad scope since it applies to any theory whose equations of motion are derivable from an action principle $\delta\mathfrak{A} = 0$, $\mathfrak{A} = \int \mathscr{L}(x, u, u^{(n)})\, \mathrm{d}x$, where x stands for the independent variables, u for the dependent variables, and the $u^{(n)}$ are derivatives of the dependent variables up to some finite order n with respect to the independent variables. The equations of motion, thus, take the form of (generalized) Euler–Lagrange (EL) equations. This is a substantive restriction, but it is satisfied by the vast majority of theories studied in modern physics[4]. Associated with an action principle is the notion of a *variational symmetry group* — a Lie group $\mathscr{G} \ni g : (x, u) \to (x', u')$ whose generators leave the action invariant up to a divergence term. Variational symmetries are necessarily symmetries of the equations of motion, i.e. they carry solutions of the equations of motion to solutions; the converse is not necessarily the case. If the action is such that there is an associated variational symmetry group $\mathscr{G}$, which is a finite dimensional Lie group with N parameters, then *Noether's first theorem* shows that, as a consequence of the EL equations, there are N conserved currents. Under appropriate conditions, the Noether currents can be integrated to give N quantities whose values are constant over time. Thus, for theories whose equations of motion are derivable from an action principle, the first Noether theorem provides a connection between the symmetries of the equations of motion and conservation laws of the familiar form[5].

Gauge symmetries are concerned with the case where the variational symmetry group $\mathscr{G}$ is an infinite dimensional Lie group whose parameters are

[4] There are, however, some exceptions. For instance, some — but not all — of the various versions of Cartan-style formulations of Newtonian gravitational theory are such that not all of the equations of motion can be derived from a single action principle; see Bain (2004).

[5] For a good introduction to the Noether theorems, see Brading and Brown (2003).

arbitrary functions of all the independent variables in the action. In this case, *Noether's second theorem* shows that the EL equations are not independent — a case of underdetermination. Generally, this underdetermination expresses itself by the appearance of arbitrary functions of the independent variables in the solutions of the EL equations. If time is one of the independent variables, this result means that the initial-value problem will not have a unique solution — an *apparent* violation of determinism. The reason for the emphasis on "apparent" is that the applicability of Noether's second theorem is taken to signal the presence of gauge freedom. The doctrine of determinism was never meant to imply that all magnitudes evolve deterministically — otherwise the doctrine would be trivially false — but only that genuine physical magnitudes evolve deterministically. And whatever other conditions a genuine physical magnitude satisfies, it must be a gauge-invariant quantity. (*Example*: that the potentials for the Maxwell electromagnetic field do not evolve deterministically is no insult to determinism because these quantities are gauge dependent. Determinism holds in this case because the Maxwell equations for the electric and magnetic fields, which are related one–many to the potentials, do admit a well-posed initial-value problem.)

I will emphasize how the development of these ideas is carried out using the Hamiltonian formalism because I eventually want to discuss the loop quantum gravity (LQG), which aims to provide a quantum theory of gravity by applying canonical quantization techniques to GTR. From the Lagrangian state space $\mathscr{S}(Q, \dot{Q})$ (here Q stands for the configuration variables and $\dot{Q}$ for their time rate of change) one moves to the Hamiltonian-phase space $\Gamma(Q, P)$ where the canonical momenta P are defined by the Legendre transformation $P = \partial\mathscr{L}/\partial\dot{Q}$. In cases where Noether's second theorem applies, the Hamiltonian system is not of the familiar kind treated in introductory mechanics texts, but is a constrained system because from the definitions of the canonical momenta follow identities of the form $\phi(P,Q) = 0$, called the *primary* constraints. Demanding that the primary constraints be preserved by the equations of motion may produce *secondary* constraints, etc. The total set of constraints picks out a hypersurface $\mathscr{C} \subset \Gamma$, called the constraint *surface*. The *first-class* constraints, which are the constraints that commute weakly with all the constraints (i.e. commute on $\mathscr{C}$ with all the constraints), are taken to generate the gauge transformations. The gauge-invariant dynamical variables $F : \Gamma(P, Q) \to \mathbb{R}$ are the ones that are constant along the gauge orbits. These quantities evolve deterministically, confirming that, when viewed through the lens of the recommended account of gauge, the apparent failure of determinism was due to mistaking gauge-dependent variables for genuine physical magnitudes[6].

[6]For details of the constrained Hamiltonian formalism, the reader may consult Henneaux and Teitelboim (1992).

Most of the familiar theories of Newtonian physics fall within the ambit of the first Noether theorem. For typical actions, the (inhomogeneous) Galilean group is a variational symmetry group, and the resulting conserved quantities are the familiar ones (energy, momentum, and the constancy of velocity of the center of mass). There are no non-trivial constraints and, thus, no gauge freedom. Realists are free to be naive realists and read off the ontological commitments of these theories from their surface structure without fear of generating indeterminism. Analogous results hold for special relativistic theories. To take a concrete example that will be elaborated below, consider a scalar Klein–Gordon field Φ on Minkowski spacetime. The equation of motion can be written so as to fulfill FGC by using the covariant derivative operator ∇_a determined by the Minkowski metric η_{ab}:

$$\eta_{ab}\nabla_a\nabla_b\Phi - m^2\Phi = 0 \tag{1}$$

where $m \geq 0$ is the mass of the field. This equation can be derived from an action principle with

$$\mathfrak{A}(\Phi, \eta) = \int \frac{1}{2}\left(\eta_{ab}\nabla_a\Phi\nabla_b\Phi + m^2\Phi^2\right)\sqrt{-\eta}d^4x \tag{2}$$

in which Φ is varied but η_{ab} is not because it is an "absolute object"[7]. The action admits the Poincaré group as a variational symmetry group. The first Noether theorem applies and yields a proper conservation law in which the stress-energy tensor for Φ is conserved as a consequence of (1). In the Hamiltonian formulation, there are no non-trivial constraints so that, once again, the proffered account of gauge renders the verdict that there is no non-trivial gauge freedom in the offing.

The application to GTR of the proffered account of gauge leads to a radically different result. The diffeomorphism group is a variational symmetry group for the standard Hilbert action for Einstein's gravitational field equations (EFEs). Since arbitrary functions of the independent variables — here the spacetime position variables — are involved, Noether's second theorem applies and, thus, EFEs are subject to underdetermination. This explains why in the rigorous statements of the (local) existence and uniqueness theorems for the initial-value problems for EFEs, the uniqueness result is qualified with an "up to diffeomorphism" clause. And the justification for this clause is that the diffeomorphism invariance is a gauge symmetry. In 1913, Einstein discovered this underdetermination by a different route, using his infamous "hole argument."

[7]Indeed, for theories whose equations of motion are derivable from an action principle, the distinction between "dynamical" vs. "absolute" objects can be drawn in terms of the objects that are vs. those that are not varied in the action. This way of drawing the distinction has bite because trying to shift objects from the "absolute" to the "dynamical" category typically results in different equations of motion and different sets of observables.

Because at that juncture he was not willing to treat the metric and matter fields as gauge-dependent quantities and because he was not willing to abandon determinism, he concluded that general covariance had to be rejected. And because he did not distinguish formal and substantive general covariance, he forsook FGC and tried to work with gravitational field equations that were not covariant under general coordinate transformations. He was just able to rescue himself from this morass of confusions in time to beat David Hilbert to the generally covariant field equations that now bear his name[8].

It is important to realize that examples where non-trivial gauge freedom is in the offing are not confined to 20th-century physics. Indeed, the apparatus I have been touting can be used to illuminate the debate over absolute vs. relational theories of motion that raged in the 17th and 18th centuries. Start with theories of particle motion formulated against the backdrop of neo-Newtonian spacetime characterized by absolute simultaneity, a Euclidean metric structure for the instantaneous three spaces, a time metric that gives the temporal interval between non-simultaneous events, and a flat affine connection that defines the inertial structure. This background spacetime structure is rich enough to support absolute quantities of motion — in particular, it makes good sense to ask for the value of the acceleration of a point particle or for the magnitude of the rotation of an extended body even if the particle or extended body is alone in the universe. Those who are relationists about motion will want to modify this background structure to get rid of the absolute quantities of motion. But any such modification seems at first blush to lead to a failure of determinism. For example, consider the semi-relationist who only wants to weaken the inertial structure of neo-Newtonian spacetime to the extent that there is still absolute rotation but not absolute acceleration in general. The appropriate modification produces what I have dubbed Maxwellian spacetime (see Earman, 1989). The symmetries of this spacetime are rich enough that they contain mappings with the property that they are the identity on or below some chosen plane of absolute simultaneity but non-identity above. But since a symmetry of the spacetime should also be a symmetry of the laws of motion, the said mappings produce from any system of particle world lines satisfying the laws of motion, another system also satisfying the laws of motion such that the two systems coincide for all past times but diverge in the future — a violation of determinism. The Newtonians will conclude that in order to secure the possibility of determinism it is necessary to swallow absolute acceleration and return to the safe haven of neo-Newtonian spacetime[9]; here, the spacetime symmetries are

[8]For an account of how Einstein found his field equations, see Norton (1984).

[9]But, as is now well known, it is not necessary to retreat all the way to full Newtonian spacetime, which singles out a particular inertial frame — "Absolute Space" — that underwrites absolute velocity.

given by the (inhomogeneous) Galilean transformations, and any such transformation that reduces to the identity for any finite stretch of time is the identity.

Those who are familiar with the absolute-relational controversy know that the relationist will not be cowed by this argument. She will conclude that in order to be a relationist about motion it is necessary to be thorough-going relationists and to reject the ''container'' view of spacetime implicit in the above argument: it is the redundancy of description in the Newtonian theory that counts two systems of particle world lines related by a spacetime symmetry as corresponding to different physical situations and thereby produces faux violations of determinism.

In evaluating this response, it is illuminating to work through concrete examples and to subject them to the analysis of the touted account of gauge. Consider, for instance, the semi-relationist who wants to construct a theory of motion using the structure provided by Maxwellian spacetime. She would have to produce equations of motion derivable from an action principle that admits the Maxwell symmetry group as a variational symmetry group. Since this symmetry group[10] contains arbitrary functions of time t (which is the only independent variable in the action), she will find that the second Noether theorem applies and that arbitrary functions of t show up in the solutions to the EL equations, apparently wrecking determinism. But she knows not to be swayed by first appearances. When she switches to the Hamiltonian formulation, she finds that she is dealing with a constrained Hamiltonian system. And when she solves for the first-class constraints, she finds that the quantities that are constant along the gauge orbits are quantities like relative particle position and relative particle momenta and that these gauge-invariant quantities evolve deterministically[11]. Of course, it remains to be seen whether the relationist or semi-relationist can produce equations of motion that match Newton's in terms of empirical adequacy, simplicity, and explanatory power. But that is an issue that is beyond the brief of the current paper.

Einstein's GTR is undeniably a theory that is as impressive today as was Newton's theory in his day in terms of empirical adequacy, simplicity, and explanatory power. But one major difference is that the touted account of gauge entails that Einstein's theory, but not Newton's theory, contains gauge degrees of freedom. Does the gauge-invariant content of GTR characterize a reality that answers to the relationist's dreams, or do the terms of the absolute-relational controversy no longer suffice to adequately describe what Einstein wrought? Before turning to these questions, I need to respond to a challenge to the distinction between FGC and SGC on which the above discussion is premised.

[10]And the group of spacetime symmetries for any classical spacetime that eliminates or substantially weakens the inertial structure of neo-Newtonian spacetime.

[11]For the details of a specific example, see my (Earman, 2003b).

The challenge is that just as it is possible, with sufficient cleverness, to rewrite non-formally generally covariant theories so as to be formally generally covariant, so it is possible, again with sufficient cleverness, to turn a theory that does not fulfill SGC into one that does. For sake of concreteness, consider the above example of the Klein–Gordon field. The equation of motion is typically presented in textbooks in terms of inertial coordinates. But only a minimal amount of cleverness was needed to produce formulation (1), which is valid in arbitrary coordinates and which reduces to the familiar textbook form in inertial coordinates. A bit more cleverness is needed to conceive the following maneuver. Replace the Minkowski metric η_{ab} in (1) by a general Lorentzian metric g_{ab} to get

$$g_{ab}\nabla_a\nabla_b\Phi - m^2\Phi = 0 \tag{3}$$

and add the equation

$$R_{abcd} = 0 \tag{4}$$

where R_{abcd} is the Riemann tensor computed from g_{ab} and where ∇_a is now the covariant derivative operator determined by g_{ab}. The solution sets for (1) and for (3) and (4) are the same[12]. The new pair of equations strike one as fulfilling SGC, but the official doctrine of gauge cannot be applied until an action principle is found that has (3) and (4) as its EL equations. This step, which requires genuine cleverness, was supplied by Sorkin (2002). The diffeomorphism group is the variational symmetry group of the rewritten Klein–Gordon theory and in this sense diffeomorphism invariance is a gauge symmetry of the theory[13]. But arguably, the rewritten Klein–Gordon theory is not merely a notational variant of the original theory, but a theory with a different physical content. In the first place, the rewritten theory contains additional physical variables satisfying an additional field equation over and above (3) and (4). Sorkin (2002) has conjectured that there is a hidden gauge symmetry, which effectively cancels out this equation. Even if this conjecture proves correct, there is a second difference; namely, whereas in the original theory the scalar field Φ is counted as a genuine physical magnitude, in the rewritten theory it is a gauge-dependent variable. Since the two theories differ on what they count as genuine physical magnitudes, they should be counted as different theories rather than just different presentations of the same theory.

[12]It is assumed that the spacetime manifold is $\mathbb{R}^4$.

[13]The constraint formalism for the reworked Klein–Gordon theory has not been worked out. This leaves a gap in the analysis, but there is no doubt that the Hamiltonian formulation involves non-trivial constraints. What remains to be seen is how the first-class constraints reflect the diffeomorphism invariance.

It seems very plausible that other attempts to trivialize SGC will fail for similar reasons, confirming that SGC really is a substantive requirement on the content of theories. But the investigation will have to be carried out elsewhere.

4. Implications of substantive general covariance: classical spacetime[14] theories

I take it for granted that we want to be realists in interpreting spacetime theories. I will also take it for granted that a satisfactory answer to the question "What would the world have to be like for the theory to be true?" must be couched in terms of what the theory takes to be "observables" or gauge-invariant quantities. In this way, general covariance as a gauge symmetry imposes a constraint on a realistic interpretation of GTR in particular and of a theory satisfying SGC in general.

In more detail, there are two ways to get at the observables of a spacetime theory satisfying SGC. On the *spacetime approach*, the observables of a theory with models of the form $(M, O_1, O_2, \ldots, O_N)$ consist of diffeomorphically invariant, real-valued functions constructed from the object fields O_i. In the case of the source-free solutions to EFE for GTR, the observables are the diffeomorphically invariant, real-valued functions constructed from the metric g_{ab} and its derivatives. On the *canonical approach*, the observables are (as said above) phase space functions $F : \Gamma \to \mathbb{R}$ that weakly commute with all of the first-class constraints or, equivalently, that are constant along the gauge orbits. Alternatively, the gauge orbits can be quotiented out to produce the reduced phase space $\tilde{\Gamma}$, and then the observables are functions $\tilde{F} : \tilde{\Gamma} \to \mathbb{R}$. These latter observables are in one–one correspondence with the former if two gauge invariants Fs that are equal on the constraint surface $\mathscr{C} \subset \Gamma$ are identified. In the Hamiltonian formulation of source-free GTR the phase space consists of pairs (h_{ab}, π^{ab}), where the configuration variable h_{ab} is a Riemann metric on a three-dimensional manifold Σ and the canonical momentum π^{ab} is a symmetric tensor field on Σ. When Σ is embedded as a spacelike hypersurface of spacetime, h_{ab} and π^{ab} become respectively the spatial metric and the exterior curvature of Σ. There are two first-class constraints, the momentum (or vector) constraint and the Hamiltonian constraint[15]. When the momentum constraint is smeared with an arbitrary shift vector, which is tangent to the embedded hypersurface, it generates the gauge change in a dynamical variable $F(h_{ab}, \pi^{ab})$ that corresponds to the gauge change generated by performing an arbitrary diffeomorphism on the hypersurface. And when the Hamiltonian constraint is smeared with an arbitrary lapse function that measures distance along a normal to the

[14] Here the contrast for "classical" is "quantum." So I speak of classical GTR.

[15] More precisely, one should speak of two families of constraints since there is a family member for each point of space.

hypersurface, it generates the gauge change in a dynamical variable that corresponds to evolving the initial data via the equations of motion[16].

There are some obvious consequences of the proposed account of general covariance for the interpretation of classical (= non-quantized) spacetime theories. The first two are somewhat repetitive, but are stated for sake of emphasis. (C1) Since typical pre-general relativistic theories satisfy FGC, but not SGC, the general covariance of these theories does not rule out naive realism that takes the theory at face value as characterizing a world in terms of a manifold on which live various geometric object fields. (C2) For GTR and other spacetime theories that satisfy SGC, there are two immediate negative implications: (i) the so-called metrical essentialism is ruled out from the start since it is incompatible with diffeomorphism invariance as a gauge symmetry. (ii) Naive realism is also ruled out. In GTR, for example, the metric and matter fields, g_{ab} and T_{ab} are not observables, and the correspondence between the models (M, g_{ab}, T_{ab}) and the physical situations they describe is many–one. (C3) It also seems that in GTR no form of manifold substantivalism is salvageable because there are no local or even quasi-local observables. Consider first the spacetime approach to observables. In source-free solutions to EFE, a quantity like the Ricci curvature, scalar R is not an observable. More generally, there do not seem to be any diffeomorphically invariant quantities that are constructible from the metric and its derivatives and that attach to spacetime points, or to local spacetime neighborhoods, or to time slices. A completely non-local quantity like $\int_M R\sqrt{-g}d^4x$ is, if the integral converges, an observable. In the canonical approach, Torre (1993) has shown that in spatially closed solutions to the vacuum Einstein equations, no local or even quasi-local quantity constructed by integrating over a compact Σ local function of the canonical variables (h_{ab}, π^{ab}) and their derivatives is an observable[17]. In sum, it seems that spacetime points or proper subsets of spacetime points are not needed to support the observables of GTR.

So much for the easy and obvious consequences of SGC. To make more progress on what to fill in for the question marks in Fig. 1 would require, as a necessary first step, making explicit the various ontologies/ideologies for GTR that are compatible with SGC. In order to avoid false appearances, it would seem advisable to go about this task by concentrating on complete sets of observables. Call a set $\mathfrak{S}$ of quantities a complete set of observables for a set $\mathfrak{M}$ of models of a theory T iff every element of $\mathfrak{S}$ is an observable with respect to $\mathfrak{M}$, and also every observable for $\mathfrak{M}$ is a functional of the elements of $\mathfrak{S}$; and call

[16]The details of how the first-class constraints of the Hamiltonian formulation of GTR express the diffeomorphism invariance of the theory is a delicate matter; the interested reader is referred to Isham and Kuchař (1986a, 1986b).

[17]For spatially open asymptotically flat spacetimes, some quasi-local canonical observables are known to exist, e.g. the ADM energy.

$\mathfrak{S}$ complete simpliciter for T iff it is complete with respect to the full set of models of T. No explicit characterization of a complete set of observables for GTR is known. What follows are two partial examples, one from the spacetime approach and the other from the canonical approach.

Example 1. One idea for producing observables for GTR can be traced back to Kretchmann (1915, 1917) and was worked out in some detail four decades later by Komar (1958). Consider a solution M, g_{ab} to the vacuum EFE. Construct, if possible, four independent scalar fields φ^{μ}, $\mu = 1,2,3,4$, from algebraic combinations of the components of the Riemann curvature tensor, such that the four-tuples $(\varphi^1(p), \varphi^2(p), \varphi^3(p), \varphi^4(p))$ and $(\varphi^1(p'), \varphi^2(p'), \varphi^3(p'), \varphi^4(p'))$ are different whenever $p \neq p'$ for any $p, p' \in M$. Thus, the values of these fields can be used to coordinatize the spacetime manifold. Note that these scalar fields are *not* observables. But they can be used to support such observables in the following way. If $g^{\alpha\beta}$ are the contravariant components of the metric tensor in a coordinate system $\{x^{\upsilon}\}$, the new components in the $\{\varphi^{\mu}\}$ system are given by $g^{\mu\nu}(\phi^{\lambda}) := (\partial\phi^{\mu}/\partial x^{\alpha})\,(\partial\phi^{\upsilon}/\partial x^{\beta})g^{\alpha\beta}$. These quantities do count as observables because they are diffeomorphic invariants. And when they are available, they form a complete set of observables. Unfortunately, they are not available across the board since in spacetimes with sufficiently high symmetries the four independent scalar fields required by the construction may not exist. One could try to dismiss such cases by proving that they form a set of "measure zero" in the full set of solutions to EFE. But such a dismissal would be shortsighted in view of the fact that historically the debate about the ideology and ontology of spacetime theories has often revolved around cases of spacetime symmetries. Of course, one might try to turn this round and argue that we have been misled by constructions that use spacetime symmetries. In the end, this attitude may turn out to be correct. But at the outset it would seem a better strategy not to beg the question and to strive for an account of observables that does not pre-suppose that spacetime symmetries are absent[18].

[18]For another example of how to construct observables in the spacetime approach, see Rovelli (2001). Suppose that there are four particles of small enough masses that their effect on the spacetime metric can be neglected. Assume that the world lines of these particles are timelike geodesics; that these geodesics intersect at a point o; and that at o their four velocities form the vertex of a tetrahedron. Let p be a spacetime point to the future of o. The past lightcone of p will intersect the particle geodesics in four points p_{α}, $\alpha = 1,2,3,4$. Assign to p the four numbers s^{α}, which are the spacetime distances, as measured along the geodesics, from o to the p_{α}. The s^{α} take the place of the Komar coordinates. Thus, the components of the metric $g^{\mu\nu}(s^{\alpha})$ in these coordinates are observables. These "GPS observables" obviously have a practical and operational significance lacking in the Komar observables — they can be implemented with current satellite technology! On the other hand, their applicability is limited to a small subset of the models of GTR.

Example 2. The line element for a cylindrically symmetric spacetime has the form

$$\mathrm{d}s^2 = \exp(\gamma - \psi)\left[\left(-N^2 + (N^1)^2\right)\mathrm{d}t^2 + 2N^1\,\mathrm{d}t\mathrm{d}r + \mathrm{d}r^2\right] + R^2\exp(-\psi)\,\mathrm{d}\phi^2 + \exp(\psi)\,\mathrm{d}z^2 \tag{5}$$

where N, N^1, $R \geq 0$, γ, and ψ are functions of the radial coordinate r and time t only. An explicit characterization of a complete set of observables in the canonical approach for cylindrically symmetric vacuum solutions to EFE has been given by Torre (1991). Elements of this set take the form ($Q(r)$, $P(r)$), $r \in [0, +\infty)$, with

$$Q(r) = \int_0^\infty \mathrm{d}\omega \frac{1}{2} J_0(\omega r)\left\{\int_0^\infty \mathrm{d}x \exp\left(-i\omega \int_\infty^x \mathrm{d}y\, \Pi_\gamma(y)\right)(\omega R(x)\psi(x)\right. \\ x\left[-i\Pi_\gamma(x)J_1(\omega R(x)) + R'(x)J_0(\omega R(x))\right] + iJ_0(\omega R(x))\,\Pi_\psi(x)) \\ \left. + \text{complex conjugate}\right\} \tag{6}$$

where Π_γ and Π_ψ are the momenta conjugate to γ and ψ, respectively and j_n is the n^{th}-order Bessel function. The expression for $P(r)$ is similar. Together $Q(r)$ and $P(r)$ form a complete set since they are invariant under reparametrization of r. And they are observables because they commute weakly with the momentum and Hamiltonian constraints.

The physical meaning of such observables can be seen from the fact that $Q(r)$ and $P(r)$ are respectively the values that ψ and Π_ψ take when $\Pi_\gamma = 0$ and $R(r) = r$, respectively. This idea can in principle be generalized to cover all solutions of EFE. In the canonical approach, a complete set of observables is obtained by expressing in terms of observables, the values of the true degrees of freedom at a given time. But obtaining explicit expressions for such a complete set is tantamount to solving the EFE, something that is beyond the capabilities of mere mortals except in very special cases. While this route to obtaining observables is useless for practical applications, it is revealing for purposes of getting a grip on the deep-level ontology and ideology of GTR.

What conclusions can be drawn from such examples? In both examples, the observables that aspire to completeness have a coincidence character: in Example 2, the observables are the values that ψ and Π_ϕ take when $\Pi_\gamma(r) = 0$ and $R(r) = r$, respectively; in Example 1, the observables are the values metric components $g^{\mu v}$ take when the fields ϕ^μ take on values such-and-so. Historically it is interesting to note that Einstein (1916) hit on a limited version of the notion of coincidence observables in extricating himself from the hole he had dug himself with his "hole argument." His "point coincidences" were quite literally that — the intersection of two light rays or the like. This coincidence character of observables should not come as a surprise. What is going on in the canonical approach can be described as follows. Some subset of dynamical variables — which in themselves are not observables — are being used to fix a gauge. Geometrically, a hypersurface $S \subset \Gamma$ in phase space transverse to the gauge orbits is picked out, and the values of the chosen variables are used to coordinatize S. Then writing other dynamical variables — which again are not in themselves observables — as functions of these coordinates does serve to define observables. But once you see the trick, you see that it can be done in many different ways, yielding many different sets of observables. Completeness does not serve to single out a preferred set. Nor apparently does any other non-pragmatic requirement.

While the lens of SGC does not focus on a specific ontology/ideology for GTR, it does reveal an ontology/ideology that does not fit comfortably with either of the traditional absolute/substantivalist vs. relational alternatives. It is tempting to say that the gauge-invariant content of GTR is closer to the relational side because the coincidence nature of observables has a relational flavor. But this misses the really radical change that general covariance as a gauge symmetry has wrought. Both sides in the absolute/substantivalist vs. relational debate accept the traditional subject–predicate parsing of spacetime ontology and ideology. For the relationist, the subjects are material bodies and/or events in their histories, and the spatiotemporal predicates are relational properties of these subjects. For the absolute/substantivalist, the subjects are points or regions of space or spacetime, and the spatiotemporal predicates are relational and non-relational properties of these subjects. Twentieth-century physics has been unkind to both sides. First, classical field theory elevated fields to coequal status with particles, and subsequently quantum field theory (QFT) (arguably) demoted particles to a second class if not epiphenomenal status. This ascendancy of fields seems at first to be a god-send for the substantivalist since it seems to lend itself to the view that spacetime is the only basic substance *qua* object of predication. But diffeomorphism invariance as a gauge symmetry seems to wipe away spacetime points as objects of predication.

Coming to grips with the ramifications of SGC indicates that we need to rethink the traditional subject–predicate ontology/ideology. I want to tentatively suggest that the gauge-invariant content of GTR is best thought of in terms of a new ontological category that I will call a *coincidence occurrence*. I use "occurrence" rather than "event" since the latter is traditionally conceived in subject–predicate terms, whereas coincidence occurrences lack subjects and, thus, also predicates insofar as predicates inhere in subjects; rather, a coincidence occurrence consists in the corealization of values of pairs of (non-gauge invariant) dynamical quantities. The textbook models of GTR are to be thought of as providing many–one representations of coincidence occurrences in terms of the co-occurrence of the relevant values of the pairs of quantities at a spacetime point. If further pressed to give a representation-free characterization of coincidence occurrence, I have nothing to offer. But I doubt that the defender of the traditional subject–predicate ontology can do much better in explaining what it is for a subject to take on or lose a predicate. If pressed far enough, any ontological view eventually reaches the stage where the basic concepts cannot be further explained except through gestures and analogies. Ultimately, the competing ontological views have to be judged on how well they facilitate an understanding of the best theories science has to offer, not the folk theories on which philosophy is largely based. And my feeling is that spacetime theories satisfying SGC are telling us that traditional subject–predicate ontologies, whether relational or absolute, have ceased to facilitate understanding.

Of course, it is one thing to keep an open mind, it is quite another thing to be so open-minded that your brains fall out. And it might be urged that the sort of open-mindedness I am encouraging is of the latter sort. One indication of this (it might be urged) is that the sort of disappearance-of-subjects view I have been running would seem to undercut B-series change since such change consists, for instance, of a subject s being P-*at*-t_1 and being $\neg P$-*at*-t_2 for $t_1 \neq t_2$ (endurance version) or a subject stage s-*at*-t_1 being P and subject stage s-*at*-t_2 being $\neg P$ (perdurance version). But rather than serving as an indication of the brains-falling-out open-mindedness, this consequence can be seen as a quick and dirty way of arriving at the "problem of time" in GTR. The "problem" takes its most dramatic form in the canonical approach where all the observables are "constants of the motion": because of the Hamiltonian constraint, the dynamics in the canonical approach is pure gauge, and so any observable, which, by definition, is constant along the gauge orbits, does not change its value as the system evolves[19].

Several comments are called for. First, I think that this "problem" is not a problem to be avoided, but a result that has to be accommodated in any thorough-going understanding of the foundations of GTR (see Earman, 2002). Second, this result is not a consequence of SGC *per se*. A counterexample is provided by unimodular gravity, which satisfies SGC but does not yield the result in question (see Earman, 2003a, 2006). Formally unimodular gravity resembles Einstein's GTR when a cosmological constant term is added to EFE. The difference is that in standard GTR the cosmological constant Λ is not only a spacetime constant — having the same value at each point of spacetime in any solution to EFE — but is also a universal constant having the same value in every solution, while in unimodular gravity the value of Λ can vary from solution to solution. This might seem to be a small difference, but unimodular gravity introduces additional spacetime structure over and above the metric g_{ab}. When unimodular gravity is run through the constraint algorithm of the canonical approach, it is found that some of the observables are not constants of the motion. I conjecture that in the absence of additional spacetime structure over and above the metric,

[19] As noted above, if one restricts GTR to models that are asymptotically flat, then there are some non-trivial quasi-local canonical observables, such as ADM mass, which can be used to define a non-zero Hamiltonian and to give conventional change.

SGC does entail that all canonical observables are constants of the motion. Third, the "no change" result for GTR is not an artifact of the canonical approach to observables, for a similar result holds on the spacetime approach to observables. But those who take the "no change" result to be an absurdity may say: so much the worse for general covariance interpreted as a gauge symmetry!

This leads me back to my first comment, which calls for elaboration. There is no need for hysteria in the face of the "no change" result at issue. Consider the following dilemma designed to saddle the advocate of taking seriously the "frozen" dynamics of GTR with an absurdity. Ask him: is the universe expanding? If he answers no, dismiss him as a kook who goes against the best available evidence, which indicates that our universe is expanding and at an increasing rate at that. If he answers yes, then dismiss him as holding contradictory views since he has denied that there is change. This is a little too quick. Consider solutions to EFE with compact spacelike slices. For such solutions, spatial volume-at-a-time is not a canonical observable. Nevertheless, the expansion of the universe is "observable" in a broader sense: there are enough genuine canonical observables to distinguish between (gauge equivalence classes of) those familiar spacetime models — manifold plus metric — where the volume of space is expanding and those where the volume of space is unchanging (*pace* Smolin, 2000; see also Earman, 2002). What is going on here mirrors some familiar moves in the classical phase of the absolute vs. relational debate. The relationist says that, at base, there is no space *per se* but only material bodies and their spatial relations. The absolutist may be upset: "You are taking away my space! I want space as a container in which bodies reside!" The relationist can reply: "If you want a container space you can have it — as a representation of relational reality. And as such, a container space is not a chimera because it can be part of an accurate mapping of relational reality. But you go astray if you take the representation literally, which is to say that you take the container space representations to correspond one–one rather than many–one to physical reality." And so it is with B-series change in the sense described above. If you want it, you can find it in the familiar (non-stationary) spacetime models that represent the gauge-invariant content of GTR. And this change is not a chimera because it can be part of an accurate mapping of the gauge-invariant reality. But again you go astray if you take these models literally.

It remains to explain how conscious observers experience the world as a B-series of subject–predicate events. But I view this task as being on a par with explaining how conscious observers experience "temporal passage." In the latter case, enlightened opinion has it that the task is to be carried out without forcing physics to recognize an A-series or a shifting "now." Equally enlightened opinion will, I think, come to see that the former task is to be carried out without backsliding from SGC by forcing physics to recognize a B-series of subject–predicate events. Nor need the former task be more difficult than the latter; indeed, if one starts with a temporally ordered series of coincidence occurrences and one assumes that conscious observers accurately perceive this order, then the former task reduces to that of explaining how conscious observers perceive the coincidence occurrences as subject–predicate events.

5. The implications of substantive general covariance: quantum spacetime theories

Even if it is conceded that a goal of 21st-century physics should be to develop a quantum theory of gravity, there is no knock-down argument to the effect that such a theory must proceed by way of quantizing the gravitational field (cf. Callender & Huggett, 2001 and Wüthrich, 2006). But since a sizable number of theoretical physicists are devoting their careers to developing such a theory, it seems a worthwhile enterprise for philosophers of science to try to discern the implications of quantum gravity for the ontology and ideology of spacetime. Of course, the philosophical work of providing an interpretation — realistic or otherwise — of a quantum theory of gravity can only begin in earnest after such a theory has been constructed. And while the candidate quantum theories of gravity are still very much works-in-progress, LQG has reached a stage of maturity that makes worthwhile some initial philosophical spadework. This is fortunate for present purposes since SGC plays a key rule in LQG.

LQG aims at a quantum theory of gravity by following the canonical quantization program for GTR. Thus, in this program the classical quantities that get transmuted into quantum observables — in the sense of self-adjoint operators on the Hilbert space of quantum gravity — are the observables of the canonical formulation of GTR, i.e. the phase space quantities that are gauge invariant in the sense that they commute weakly with the first-class constraints. LQG follows Dirac's constraint-quantization scheme. The idea is to turn the classical constraints into operators on a Hilbert space $\mathscr{H}$ and then to enforce gauge invariance by requiring that the physical sector $\mathscr{H}_{\text{phy}} \subset \mathscr{H}$ of the Hilbert space is the subspace corresponding to the kernel of the constraint operators. LQG is able to make progress on this program by replacing the canonical variables discussed in the previous section by a different set of variables invented by Amitaba Sen and deployed by Abay Ashtekar. At first, this replacement seems to make the problem worse since instead of the two families of constraints discussed above, there are now three — called the Gauss constraint, the vector (or diffeomorphism) constraint, and the scalar (or Hamiltonian) constraint. But in fact, the new variables facilitate the handling of the constraints. The first two constraints have been solved using the so-called spin network states $|s\rangle$, which describe discretized three geometries. Work on the scalar or Hamiltonian constraint is too technical to report here, but the implications for some of the issues discussed above can be summarized in a relatively non-technical manner.

Formally what one wants is a projection operator $\hat{P} : \mathscr{H}_{\text{Gauss, vector}} \rightarrow \mathscr{H}_{\text{phy}}$ that projects the subspace $\mathscr{H}_{\text{Gauss, vector}}$ lying in the kernel of the Gauss and vector constraints onto the kernel of the Hamiltonian constraint. Using this projector one defines the transition amplitudes $W(s,s') := \langle s|\hat{P}|s'\rangle$ between spin network states[20]. These quantities are gauge invariants and, thus, genuine observables of LQG; and they also form a complete set of observables (see Perez & Rovelli, 2001). In theories for which general covariance is not a gauge symmetry, analogous transition amplitudes can be thought of as matrix elements of the time evolution operator. But in LQG, which is a quantum implementation of the idea that the diffeomorphism invariance of GTR is a gauge symmetry, these transition amplitudes do not give time evolution in any conventional sense. Rather, they solve the Hamiltonian constraint and define the inner product on $\mathscr{H}_{\text{phy}}$. This is the quantum expression of the disappearance of time in the canonical formulation of classical GTR.

Assuming that the states $\hat{P}|s\rangle$ form a basis for $\mathscr{H}_{\text{phy}}$, one can ask whether this basis is unitarily equivalent to a natural basis that lies in the common kernel of the constraint operators that derive from the more familiar canonical formulation of GTR (a.k.a. geometrodynamics) discussed above in Section 4. Callender and Huggett (2001) have opined that if the answer is no, then "spacetime is not fundamental, but a result of a more basic reality" (p. 21). The first issue here is whether or not the physical Hilbert spaces of the two formulations of canonical quantum gravity are separable. Some versions of $\mathscr{H}_{\text{Gauss, vector}}$ for LQG are non-separable, raising the worry that $\mathscr{H}_{\text{phy}}$ might not be separable. But it has been shown that some technical tweaking can restore separability (see Fairburn & Rovelli, 2004). The basis elements that have typically been contemplated for the kinematic Hilbert space of geometrodynamics are neither normalizable nor countable, but this does not settle the issue of whether the physical Hilbert space of geometrodynamics has a countable orthonormal basis. A negative answer would indicate that this route to quantum gravity is defective since non-separability is generally regarded as a pathology in QFT. A positive answer would entail a positive answer to the Callender–Huggett question since all infinite dimensional Hilbert spaces are the same (isomorphic) and all orthonormal bases of any such space are unitarily equivalent. But, I would maintain, the answer to their question has little to do with whether spacetime is retained as a fundamental entity. (i) If LQG leads to what comes to be regarded as the correct quantum theory of gravity, then *classical* general relativistic spacetime can no longer be taken as a fundamental entity because none of the states of the physical Hilbert space of LQG describes a classical general relativistic spacetime at the subPlanck scale, and some of the states fail to describe a classical general relativistic spacetime at the macroscopic scale. Of course, a condition of adequacy on a quantum theory of gravity requires a demonstration that there are appropriate conditions under which classical general relativistic spacetime

[20]How to add flesh to the bare formalism by providing a means of calculating values for these transition amplitudes is another matter. The so-called spin-foam models can be seen as means for accomplishing this task.

emerges in some classical limit. Giving such a demonstration for LQG is one of the most important challenges that the proponents of this theory now face. The lack of progress to date could be taken as an indication that LQG strikes a Devil's bargain: by making it easier to handle the constraints, it makes it more difficult to see how classical general relativistic spacetime emerges[21]. Here again philosophers might take a wait-and-see attitude. But I think that it is a useful exercise to try to provide a nomenclature of the various senses in which classical general relativistic spacetime can be an emergent entity (see Wüthrich, 2006). One thing is clear from the outset, however: if LQG is the correct theory of quantum gravity, then classical general relativistic spacetime is not emergent in a sense that is congenial to relationism; in particular, classical spacetime does not emerge from the matter fields and their interactions since LQG is quite happy to quantize curved empty general relativistic spacetimes. (ii) On the other hand, although *classical* general relativistic spacetime has been demoted from a fundamental to an emergent entity, spacetime *per se* has not been banished as a fundamental entity. After all, what LQG offers is a quantization of classical general relativistic spacetime, and it seems not unfair to say that what it describes is *quantum* spacetime. This entity retains a fundamental status in LQG since there is no attempt to reduce it to something more fundamental.

6. Conclusion

Some of the issues that philosophers debate about the ontology/ideology of spacetime are not merely philosophical — they make a difference to ongoing scientific research programs. The nature and status of the requirement of general covariance is an example par excellence of such an issue. Indeed, I want to suggest that there is a kind of empirical test of the hypothesis that one of the lessons classical GTR teaches is that general covariance should be a gauge symmetry of spacetime theories. This hypothesis receives confirmation if LQG, which incorporates this hypothesis as a central tenet, prospers in the way that good scientific theories do. And it receives disconfirmation if LQG degenerates as a research program for reasons can be traced to the hypothesis. SGC also plays an important role in differentiating LQG from the string theory approach to quantum gravity. SGC strongly suggests that a quantum theory of gravity ought to be "background independent," i.e. should not rely on a split of the spacetime metric into a part that provides a fixed background metric and a part that encodes the dynamical degrees of freedom of the gravitational field[22]. But

[21]For a treatment of the semi-classical limit in loop quantum cosmology, see Bojowald (2001).

[22]I say "strongly suggests" rather than "implies" because I have not seen a tight argument to the effect that SGC entails background independence. But it seems plausible that on the analysis of SGC I propose here, a spacetime theory cannot satisfy SGC if it uses a fixed or absolute background metric.

so far, the string theory approach to quantum gravity has not been formulated in a background-independent fashion.

A decade ago John Norton (1993) published a masterly review article entitled ''General covariance and the foundations of general relativity: Eight decades of dispute.'' The dispute is now over nine decades old, and it will undoubtedly continue for many more decades to come. The main reason for the longevity of the dispute is that pursuing the nature and status of general covariance leads directly to some of the most fundamental issues in the foundations of spacetime theories, issues that do not easily yield to neat solutions. I also want to suggest that some of the implications of SGC are counterintuitive, so much so that those who have glimpsed them have turned away in search of an avoidance strategy. Philosophers are nothing if not clever, and sufficient cleverness will undoubtedly produce a variety of avoidance strategies. But it seems to me that the road to wisdom goes not by way of avoidance, but by way of facing the implications and in trying to understand how they change the terms of the debate about the ontology and ideology of spacetime.

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