CHAPTER **10**

Pruning Some Branches from "Branching Spacetimes"

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Abstract

Discussions of branching time and branching spacetime have become common in the philosophical literature. If properly understood, these conceptions can be harmless. But they are sometimes used in the service of debatable and even downright pernicious doctrines. The purpose of this chapter is to identify the pernicious branching and prune it back.

1. INTRODUCTION

Talk of "branching time" and "branching spacetime" is wide spread in the philosophical literature. Such expressions, if properly understood, can be innocuous. But they are sometimes used in the service of debatable and even downright pernicious doctrines. The purpose of this paper is to identify the pernicious branching and prune it back.

Section 2 distinguishes three types of spacetime branching: individual branching, ensemble branching, and Belnap branching. Individual branching, as the name indicates, involves a branching structure in individual spacetime models. It is argued that such branching is neither necessary nor sufficient for indeterminism, which is explicated in terms of the branching in the ensemble of spacetime models satisfying the laws of physics. Belnap branching refers to the sort of branching used by the Belnap school of branching spacetimes. An attempt is made to situate this sort of branching with respect to ensemble branching and individual branching. Section 3 is a sustained critique of various ways of trying to implement individual branching for relativistic spacetimes. Conclusions are given in Section 4.

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2. THREE TYPES OF SPACETIME BRANCHING

The issue of futuristic (respectively, historical) determinism is sometimes posed in terms of the question of whether the laws of physics allow branching in the future (respectively, the past).[1] The relevant sense of "branching" is what I will call *ensemble branching*. The relevant ensemble is the collection of spacetime models (worlds, histories, ...) satisfying the laws.[2] Future branching means that the ensemble contains models that are isomorphic at a time (or isomorphic over some finite stretch of time, or isomorphic for all times less than or equal to a given time) but nonisomorphic for future times;[3] past branching is understood analogously. The isomorphism at issue may be construed either as literal identity or as a counterpart relation.[4] This distinction will be relevant below, but it can be ignored for the nonce. The presence of branching in the ensemble of spacetime models satisfying given physical laws is a *prima facie* indication that these laws are not deterministic. The qualification is needed because although the branching creates a presumption of the failure of determinism, the presumption can be rebutted by identifying gauge freedom; two examples help to illustrate the point.

The first example comes from the context of the special theory of relativity (STR). Consider the source-free Maxwell equations for electromagnetism written in terms of the electromagnetic (four-) potentials.[5] The instantaneous values of the potentials and their first time derivatives do not suffice to fix a unique solution. This failure of non-uniqueness is not taken as an insult to Laplacian determinism because (according to the standard story) the electromagnetic potentials are mere auxiliary devices. The genuine physical magnitudes are electric and magnetic field strengths, and formulated in terms of these quantities the Maxwell equations do admit a well-posed initial value problem, securing Laplacian determinism for worlds described purely in terms of (source free) electromagnetic fields. The values of the electromagnetic potential correspond many-one to the values of the electric and magnetic field strength, and the transformations among the many that correspond to the one are gauge transformations. In this example all of the spacetime models use a single fixed spacetime background, Minkowski spacetime. The second example moves from the context of STR to the context of the general relativity theory (GTR) where the spacetime structure can vary from model to model.[6] The initial value problem for the source-free Einstein gravitational field equations admits a solution that is guaranteed to be unique (locally

[1] For a recent overview of the fortunes of determinism in modern physics, see Earman (2007a).

[2] Since for present purposes it makes little or no difference whether one works with models, or worlds, or histories, I will feel free to shift back and forth amongst them.

[3] The first type of branching is relevant to the issue of Laplacian while the others are relevant to weakened cousins of Laplacian determinism.

[4] For those familiar with the language of differential geometry, think of a spacetime model as having the form $\langle \mathcal{M}, O_1, O_2, \ldots, O_N \rangle$ where $\mathcal{M}$ is a differentiable manifold and the O_i are geometric object fields on $\mathcal{M}$ that characterize either the structure of spacetime or the matter-fields that inhabit spacetime. The models for Newtonian, special relativistic, and general relativistic theories can all be put in this form. Two such models $\langle \mathcal{M}, O_1, O_2, \ldots, O_N \rangle$ and $\langle \mathcal{M}', O'_1, O'_2, \ldots, O'_N \rangle$ are isomorphic in the relevant sense iff there is a diffeomorphism $d: \mathcal{M} \to \mathcal{M}'$ such that $d^*O_i = O'_i$ for all i, where d^*O_i denotes the "drag along" of O_i by d.

[5] The spacetime models have the form $\langle \mathbb{R}^4, \eta_{ab}, \phi_1, \phi_2, \phi_3, \phi_4 \rangle$ where η_{ab} is the Minkowski metric and the ϕ_i are scalar fields on $\mathbb{R}^4$.

[6] The spacetime models have the form $\langle \mathcal{M}, g_{ab} \rangle$ where g_{ab} is a Lorentz signature metric on $\mathcal{M}$. See Section 3.2 below.

in time) only up to a diffeomorphism.[7] The "up to" qualification is not taken to signal a failure of Laplacian determinism because (according to one persuasive interpretational stance) the diffeomorphism invariance of the theory is to be interpreted as a gauge symmetry. From here on I will assume that gauge freedom has been removed so that indeterminism can be safely inferred from ensemble branching.[8]

Note that ensemble branching can obtain even though no spacetime model in the ensemble has a structure that, by any reasonable standard, can be deemed to be branching. But it is precisely the branching in individual spacetime models that seems to be presupposed by some versions of the many worlds interpretation of quantum mechanics (QM). Individual spacetime branching is explicitly advocated by Storrs McCall (1994, 1995, 2000) and Roy Douglas (1995). And it is considered (but rejected) by Roger Penrose (1979). This nonensemble *individual branching* will be the main target of my pruning operation.

But before I set my sights on this target, I want to introduce a third sense of branching—what I will call *Belnap branching*—that lies somewhere between ensemble branching and individual branching, although its exact location is not easy to pin down. The Belnap school's endorsement of branching begins with an anti-Lewisian stance.[9] In *On the Plurality of Worlds* David Lewis informs the reader that he rejects what he dubs 'branching' of possible worlds but accepts 'divergence'. The difference between the two is characterized as follows.

> In branching, worlds are like Siamese twins. There is one initial spatiotemporal segment; it is continued by two different futures—different both numerically and qualitatively—and so there are two overlapping worlds. One world consists of the initial segment plus one of its futures; the other world consists of the identical initial segment plus the other future.
> In divergence on the other hand, there is no overlap. Two worlds have duplicate initial segments, not one that they share in common. I, and the world I am part of, have only one future. There are other worlds that diverge from us. ... Not I, but only some very good counterparts of me, inhabit these other worlds. (Lewis, 1986, p. 206)

Contra Lewis, the Belnapians insist that the openness of the future requires 'branching' and not mere 'divergence'. This insistence might be interpreted as an endorsement of the branching of individual spacetimes, but a more conservative reading is possible. On this reading, Lewis' 'divergence' and 'branching' are both subsumed under ensemble branching. In endorsing the former and rejecting the latter Lewis is asserting that in ensemble branching the relation of isomorphism should not be interpreted as identity but as a counterpart relation. The Belnap school is read as insisting that indeterminism requires that the relation of

[7] See Section 3.4 below for more details.

[8] Note that on this interpretational stance neither the spacetime metric nor any of the scalar invariants formed from the metric count as observables or gauge invariant quantities. The issue of how to characterize the gauge-independent content of GTR is controversial; see Earman (2006).

[9] The Belnap school includes, of course, Nuel Belnap, his collaborators (Mitchell Green, Michael Perloff, and Ming Xu), and those who accept his framework for treating branching time and branching spacetime (e.g., Thomas Müller and Thomas Placek). Representative references from this school include Belnap (1992, 2002, 2003), Belnap et al. (2001), Kowalski and Placek (1999), Müller (2004), Placek (2000) and Placek and Müller (2005).

isomorphism must be interpreted as identity and, thus, as opting for 'branching' over 'divergence'. Leaving aside the merits of this dispute, the important point for present purposes is that this dispute does not require taking a stance on branching of individual spacetimes.

In support of this conservative reading, there are passages in which the Belnapians seem to be rejecting individual branching in favor of ensemble branching. Thus, in Belnap et al.'s *Facing the Future* we read:

> [A]lthough we use the phrase 'branching time' because of its fixed place in the literature, we never, ever mean to suggest that time itself—which is presumably best thought of as linear—ever, ever, 'branches.' The less misleading phrase ... is 'branching histories,' with an essential plural to convey that it is the entire assemblage of histories that has a branching structure. (Belnap et al., 2001, p. 29, fn 1)

And what holds for branching times presumably holds *inter alia* for branching spacetimes. Note, however, that in the very same passage Belnap et al. refer the reader to McCall (1994) for "similar ideas" on branching spacetimes. This reference is explicit in endorsing individual branching, and it also returns the complement by referring to Belnap (1992) as a source of inspiration (see McCall, 1994, p. 4, fn 4). The reader is left wondering whether Belnap branching spacetimes involve only ensemble branching or something more.

Another characteristic feature of the Belnap school—the denial of the "thin red line"—strongly hints of something beyond ensemble branching. The doctrine of the thin red line is the doctrine that, as of an indeterministic moment ("branch point"), exactly one future reaching branch is the actual future. John MacFarlane has opined that "positing a thin red line amounts to giving up on objective indeterminism" (2003, p. 325). This is a non-sequitur if indeterminism is explicated as ensemble branching, which I claim, is the sense needed to illuminate issues of determinism and indeterminism in physics. On this explication futuristic indeterminism means no more and no less than that there is more than one possible future history compatible with the combination of the laws of physics and the history up-to-now.[10] It certainly does not follow that (as of now) there is no fact of the matter as to which of the possible futures is the actual future. To get that result some additional piece of metaphysics would have to be added. It might be supplied by Storrs McCall's model of indeterminism in terms of individual spacetime branching and the attrition of branches with the advance of "now" (see Section 3.1 below). An alternative piece of metaphysics that would bridge the non-sequitur without involving a commitment to individual branching would be supplied by C.D. Broad's (1923) growing block universe picture of spacetime according to which the block is built up as successive layers of "now" come into existence. On this picture there is no line, thin or thick, red or any other color, that marks the course of the future since there literally is no future. But note well that this is so regardless of whether or not the growth of Broad's growing block is indeterministic; this severing of the

[10] I do not understand what MacFarlane's qualifier "objective" is supposed to add, other than to emphasize that the indeterminism is not purely epistemic. But this is already captured by ensemble branching when the ensemble is taken to be the models of the physical laws.

link between indeterminism and the "openness" of the future is contrary to the spirit of the Belnap school.

Perhaps the no-thin-red-line doctrine is not supposed to be given an ontological reading but only a semantic reading on which some future tensed statements are neither true nor false. Consider the semantic rule (*R*): as of an indeterministic moment, a statement asserting the future occurrence of an event of type *E* (e.g. 'sea-battle') is true (respectively, false, indeterminate) iff a token *e* of *E* is present in every (respectively, no, some but not every) future reaching branch. Branching in individual spacetimes is not needed to get the consequence that, as uttered now, "There will be a sea battle" has an indeterminate truth value. What is needed is (i) futuristic indeterminism with respect to sea battles, i.e. ensemble branching of physically possible future histories, with some branches containing sea battles and some containing nonesuch, and (ii) the decision to take 'branch' in (*R*) to range over possible future histories compatible with the combination of the laws of physics and the history up-to-now.

But although the semantic reading of the no-thin-red-line doctrine does not necessarily imply a commitment to individual branching, it does not ring true without some additional ontological underpinning. Suppose that Broad's growing block universe is rejected—as I think it should be since it is difficult if not impossible to reconcile it with the relativistic conception of spacetime—and suppose further that branching in individual spacetimes is eschewed. Then the denial of truth values to future contingents has a hollow ring to it. For on the stated suppositions there are no actual future reaching branches sprouting from the trunk of the actual world up to now; there is only a single actual trunk extending into the future—in this sense not only is there a thin red line but also a broad red band painting the unbranching trunk of the actual world. And a natural rule for assigning truth values to future tensed statements is (*R*) with 'branch' taken to range over actual branches. This makes "There will be a sea battle" true in case the (hypothesized) unique actual future contains a sea-battle token and false otherwise. Of course, it is consistent to adopt the form of (*R*) that takes 'branch' to range over possible future histories compatible with the combination of the laws of physics and the history up-to-now. But if indeterminism *per se*, without the aid of an evolving block universe or individual branching, supports the adoption of this form of (*R*), then it equally supports the analogous version of (*R*) for past tensed statements, yielding a version of presentism on which contingent statements about the past as well as the future lack truth values. But the Belnap school has no truck with presentism; indeed, it posits that while there is branching in the future, there is no branching in the past. This posit does not follow from indeterminism if indeterminism is explicated as ensemble branching and the indeterminism is the indeterminism of time reversal invariant laws. For laws that are time reversal invariant, futuristic and historical determinism stand or fall together; in branching jargon, either there is ensemble branching in the ensemble of physically possible histories in both the past and the future or in neither. Time reversal invariance is a property of all of the known fundamental laws of physics, save for some that govern the weak interactions of elementary particles, processes that are presumably irrelevant to explaining any of the temporal asymmetry we observe at the macrolevel.

In sum, both the ontological and the semantic readings of the no-thin-red-line doctrine seem to presuppose something beyond indeterminism as explicated by ensemble branching.[11] This is implicitly acknowledged in *Facing the Future* where it is announced that "indeterminism" is to be understood in a sense that validates the no-thin-red-line doctrine, a future-branching tree structure for time and spacetime, and the other postulates imposed on this structure (see Belnap et al., 2001, pp. 133–141). While there is nothing objectionable *per se* to such a procedure, it leaves unclear what the connection is between the sense of indeterminism needed to satisfy these demands and the sense of indeterminism that (I claim) is relevant to understanding physics.[12]

Since I have been unable to get a fix on what Belnap branching involves, all I can say for the present is this: insofar as Belnap branching eschews individual branching, then for present purposes I have no quarrel with it; but insofar as it is committed to individual branching, it will be subject to the pruning operation to be undertaken in the next section.

3. PROBLEMS WITH INDIVIDUAL BRANCHING

3.1 Faulty motivation

McCall (1994, 1995, 2000) takes individual branching as a way of modeling indeterminism. I claim that there is no necessary connection, in either direction, between determinism and individual branching. As noted above, ensemble branching can obtain even when each and every model (world, history) in the ensemble describes an unbranching spacetime. Thus, if indeterminism is adequately explicated by ensemble branching, indeterminism does not entail individual branching. A detailed examination of determinism and indeterminism in modern physics (see Earman, 2006, 2007a, 2007b) supports the antecedent.

In the other direction, branching in individual spacetimes by itself need not entail indeterminism. If there is no branching in the ensemble of models satisfying given laws, then these laws are deterministic. But it is *a priori* possible that the ensemble contains models whose individual spacetimes branch; that is, although

[11] MacFarlane's claim that "positing a thin red line amounts to giving up on objective indeterminism" is a not-so-subtle attempt to put his opponents on the defense by attributing to them a "posit" of a metaphysical entity—the thin red line. On the contrary, it is those deniers of the thin red line—like MacFarlane and the Belnap school—who hold that future tensed but not past tensed statements may lack a determinate truth value that need to rely on some posit beyond indeterminism in the ensemble branching sense to motivate their semantic rules. MacFarlane's own account of future contingents is much more intricate than anything considered here; in particular, he proposes to relativize truth assignments to "contexts of assessments."

[12] The postulates used by the Belnap program seem to place *a priori* constraints on physics. For example, members of the Belnap school speak of "choice points," which are to be thought of as the loci of indeterministic influences that radiate outwards along or within the future null cones of these loci. On this way of conceptualizing indeterminism, the members of the ensemble of physically possible models will typically agree at all points except those that lie on or inside the future null cones of the choice points. I know of no theory in modern physics which will produce this kind of indeterminism. Relativistic field theories, whether classical or quantum, typically entail lawlike connections among relatively spacelike events.

the laws plus initial conditions pick out a unique temporal evolution, that evolution might involve a spacetime that literally branches. (In fact, however, one means of implementing individual branching in general relativistic spacetimes does lead to indeterminism; but the problem is that it leads to too much indeterminism (see Section 3.4 below).) Indeterminism can emerge from branching in individual spacetimes by adding a chance or random falling off or withering away of branches as "now" creeps up the trunk of the branching spacetime tree.[13] But in addition to the use of the dubious metaphysical notion of a shifting "now," this approach makes the analysis of indeterminism hostage to the even less well understood concepts of chance and randomness.

Finally, it will be seen in Section 3.4 below that the most plausible way of implementing branching for relativistic spacetimes is incapable of modeling some forms of indeterminism.

3.2 No-go results on topology change in classical GTR[14]

Mathematical details aside, the intuitive idea of a future branching spacetime seems clear enough: spacetime is like a tree in that it starts with a main trunk which sends off future reaching branches, each of which in turn may send off its own branches, etc.

As intuitively appealing as the idea of literally branching spacetime might seem to be on first impression, there are serious difficulties in implementing it in terms of relativistic spacetime structure. Start by accepting the standard definition of a relativistic spacetime as a pair $\mathcal{M}, g_{ab}$, where $\mathcal{M}$ is a Hausdorff differentiable manifold and g_{ab} is a Lorentz signature metric defined at every point $p \in \mathcal{M}$.[15] One might contemplate a literal branching of a relativistic spacetime as pictured in Figure 10.1, which shows an upside down "trousers universe" for which the "trunk" bifurcates into two "legs". However, such a contemplation involves a change in the spatial topology and, thus, it runs up against no-go results for topology change. This subsection will review some of these results. The following subsection will discuss various escape options.

RESULT 1 (Tipler, 1977). Let $\mathcal{M}, g_{ab}$ be a compact n-dimensional (Hausdorff) spacetime with boundary consisting of the disjoint union of two compact $(n-1)$-dimensional spacelike manifolds Σ_1 and Σ_2, and let T_{ab} be a stress energy tensor describing the matter fields that are the sources of gravity. Suppose that the spacetime is time orientable. Suppose further that Einstein's gravitational field equations are satisfied by g_{ab}, T_{ab} at every point of $\mathcal{M}$ and that T_{ab} satisfies the weak

13 "[I]indeterminism occurs again in the random or chance selection of the branch which becomes actual" (McCall, 1995, p. 156).

14 'GTR' is being used here in a sense that includes STR as a special case. In particular, all of the considerations below apply equally to flat spacetimes.

15 A topological space is said to be *Hausdorff* just in case for any pair of distinct points there are disjoint open neighborhoods. A *differentiable manifold* is a topological space equipped with a differentiable structure that makes it possible to talk of degrees of smoothness or differentiability that go beyond simple continuity. The exact form of differentiability is not relevant here; for sake of definiteness the spacetime manifold may be assumed to be C^∞. The relevant mathematics for relativistic spacetimes is explained in Wald (1994).

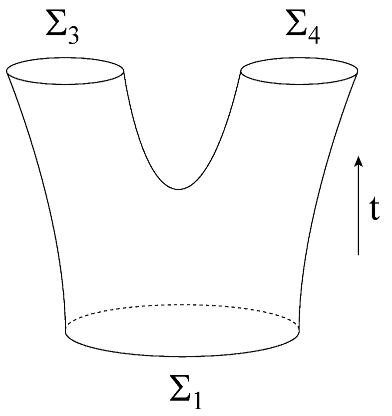

FIGURE 10.1 Trousers spacetime.

energy condition and the generic condition. Then Σ_1 and Σ_2 have the same topology.

A spacetime $\mathcal{M}, g_{ab}$ is said to be *time orientable* iff it admits a non-vanishing, continuous, timelike vector field. Two such fields can be deemed equivalent if at every point $p \in \mathcal{M}$ the vectors they define fall into the same lobe of the null cone at p. The choice of one of two equivalence classes constitutes a *time orientation* for $\mathcal{M}, g_{ab}$. The *weak energy condition* says intuitively that there are no negative energy densities; technically it requires that $T^{ab}V_aV_b \geqslant 0$ for every timelike vector V^a. The *generic condition* says intuitively that every timelike or null geodesic feels a tidal force at some point; technically it requires that every timelike or null geodesic contains at least one point p at which $V^aV^bV_{[c}R_{d]ab[e}V_{f]} = 0$ where V^a is tangent vector at p to the geodesic, R_{abcd} is the Riemann curvature tensor, and the square brackets denote antisymmetrization. Tipler’s result can be paraphrased by saying that in a generic time orientable model of GTR with physically reasonable source fields, topology change cannot take place in a spatially finite universe. The power of this result is that it does not appeal to any causality conditions over and above time orientability. For present purposes its weakness lies in the fact that it appeals not only to conditions on the geometry of spacetime but also to physical assumptions, i.e. Einstein’s field equations and energy conditions. The other no-go results to be discussed below have the opposite trade off.

In a time oriented spacetime a spacelike hypersurface $\Sigma \subset \mathcal{M}, g_{ab}$ is said to be a *Cauchy surface* iff Σ is intersected exactly once by every future directed timelike curve without past or future endpoint.

RESULT 2 (Geroch, 1967a). Let $\mathcal{M}, g_{ab}$ be a connected time oriented (Hausdorff) spacetime without boundary. Suppose that it admits a Cauchy surface Σ. Then $\mathcal{M}$ is topologically $\Sigma \times \mathbb{R}$.[16]

16 Further, $\mathcal{M}$ can be foliated by Cauchy surfaces with the topology of Σ; see Wald (1994, Theorem 8.3.13).

Since the trousers universe is not topologically $\Sigma \times \mathbb{R}$ for any Σ, it follows from Result 2 that this universe does not admit a Cauchy surface. Advocates of individual branching who are out to model indeterminism may be unmoved by this result. They can cheerfully give up the existence of Cauchy surfaces, for the absence of such surfaces spells doom for (global) Laplacian determinism.[17] The next result should give them more pause.

RESULT 3 (Geroch, 1967a). Let $\mathcal{M}, g_{ab}$ be a compact n-dimensional (Hausdorff) spacetime with boundary consisting of the disjoint union of two compact $(n-1)$-dimensional spacelike manifolds Σ_1 and Σ_2. Suppose that the spacetime is time oriented and that it contains no closed timelike curves (CTCs).[18] Then Σ_1 and Σ_2 have the same topology.

Apply Result 3 to the trousers universe by taking Σ_1 to be a waist slice, Σ_2 to be the disjoint union of two leg slices Σ_3 and Σ_4 (see Figure 10.1), and $\mathcal{M}$ to be the portion of the trousers universe between and including Σ_1 and Σ_2. Since Σ_1 and Σ_2 have different topologies (in particular, Σ_1 is connected but Σ_2 is not), conclude that this spacetime is either: not time orientable; or is time orientable and contains CTCs; or else $\mathcal{M}$ is non-compact. The last possibility would signal that the spacetime is singular in the sense that the metric is ill-defined somewhere on the portion of the spacetime sandwiched between Σ_1 and Σ_2. Geroch (1967b) and Hawking (1992) generalize Result 3 to the spatially open universes by showing that topology change in bounded regions of a time orientable spacetime entails the existence of CTCs.

A time oriented spacetime $\mathcal{M}, g_{ab}$ is said to be *causally stable* iff (intuitively speaking) it contains no CTCs and, further, there exists a finite widening of the null cones of g_{ab} for which there are no CTCs. The formal definition can be found in Hawking and Ellis (1973, p. 198). But for present purposes the crucial fact is that stable causality is equivalent to the existence of a *global time function*, i.e. a smooth map $t: \mathcal{M} \to \mathbb{R}$ such that $t(p) < t(q)$ for any $p, q \in \mathcal{M}$ such that there is a future directed timelike curve from p to q.

RESULT 4 (Hawking and Ellis, 1973). Let $\mathcal{M}, g_{ab}$ be a stably causal spacetime. If the level surfaces $t = const$ of a global time function are all compact, then they are all topologically the same and $\mathcal{M}$ is topologically $\Sigma \times \mathbb{R}$ for some spacelike hypersurface Σ.

Some additional no-go results are worth mentioning. Gibbons and Hawking (1992a, 1992b) show that existence of a spin structure for spacetime prevents certain kinds of topology change. Alty (1995) shows that for a large class of spacetimes, topology change implies spacetime singularities, whether or not CTCs are

[17] Intuitively, if Σ fails to be a Cauchy surface there will be possible causal processes that fail to register on Σ and, thus, it is unreasonable to expect that even the most precise specification of the state on Σ will suffice to fix, via the laws, the state everywhere in the spacetime.

[18] A CTC is a closed curve whose parametrization can be chosen such that the tangent to the curve is everywhere timelike and future-pointing according to the time orientation.

present. Hawking and Sachs (1974) promote the requirement of *causal continuity*: "There is some reason, but no fully convincing argument, for regarding causal continuity as a basic macrophysical property." Intuitively, this requirement says that the causal past (respectively, future) of any point $p \in \mathcal{M}$—i.e. the portion of spacetime that can be reached from p by a past directed (respectively, future directed causal curve)—should depend continuously on p. Causal continuity is stronger than the requirement of stable causality but weaker than the existence of a Cauchy surface.[19] It is incompatible with the trousers universe of Figure 10.1 and, presumably, with most branching spacetime structures.

3.3 Attempted escapes from the no-go results

Needless to say, all of the no-go results on topology change discussed above rely on substantive assumptions and, thus, can be escaped by rejecting one or more of these assumptions.

Escape 1. Drop the requirement of time orientability. Then none of the no-go results of Section 3.2 is applicable. On the positive side, it is known that dropping time orientability opens the way to topology changes that were not possible with time orientability. As a preliminary stating one precise result, some additional concepts are required.

Two $(n-1)$-dimensional manifolds Σ_1 and Σ_2 are said to be *topologically cobordant* just in case there is an n-dimensional manifold $\mathcal{M}$ with boundary consisting of the disjoint union of the two $(n-1)$-manifolds in question. In the case Σ_1 and Σ_2 are compact it is natural to require that the interpolating $\mathcal{M}$ be compact as well. The necessary and sufficient condition for topological cobordance of compact Σ_1 and Σ_2 is that their Stiefel–Whitney numbers be equal (see Milnor, 1965). It follows that when $n = 4$, any two 3-dim compact Σ_1 and Σ_2 are topologically cobordant. The topologically cobordant manifolds Σ_1 and Σ_2 are said to be *Lorentz cobordant* just in case the interpolating $\mathcal{M}$ admits a time oriented Lorentz signature metric for which Σ_1 and Σ_2 are spacelike. The existence of such a metric is equivalent to the existence of a non-vanishing vector field t^a that is nowhere tangent to Σ_1 or Σ_2. It is natural to require that t^a points into $\mathcal{M}$ on (say) Σ_1 and out of $\mathcal{M}$ on Σ_2. The necessary and sufficient condition for Lorentz cobordance in this sense is that $\chi(\mathcal{M}) = 0$ for even n and $\chi(\Sigma_1) = \chi(\Sigma_1)$ for odd n, where $\chi(\Sigma)$ denotes the Euler characteristic of Σ (see Sorkin, 1986a).

An example of how dropping of time orientability opens up possibilities of topology change is given in Sorkin (1986b). Let Σ_1 and Σ_2 be topologically cobordant $(n-1)$-dimensional compact manifolds. If n is odd then there is a connected manifold $\mathcal{N}$ which has a boundary consisting of the disjoint union of Σ_1 and Σ_2 and which admits a (possibly non-time orientable) Lorentz metric such that Σ_1 and Σ_2 are spacelike.

However, giving up time orientability is not an escape that the advocates of individual branching want to use since they assume that branching is compatible with a consistent time directionality. In addition, Borde (1994) shows that even if

[19] In more detail, the causality conditions discussed above are related as follows: $\exists$ a Cauchy surface (a.k.a. global hyperbolicity) $\Rightarrow$ causal continuity $\Rightarrow$ $\exists$ a global time function (stable causality) $\Rightarrow$ no closed CTCs.

time orientability is dropped, topology change implies CTCs[20] if the spacetime is *causally compact*. A (possibly) non-time orientable spacetime $\mathcal{M}$ with boundary consisting of the disjoint union of two spacelike hypersurfaces Σ_1 and Σ_2 is said to be causally compact just in case the topological closure of $I(p)$ is compact for each $p \in \mathcal{M}$, where $I(p)$ consists of all points of $\mathcal{M}$ that can be reached from p by a timelike curve. Intuitively, this condition says that points of $\mathcal{M}$ cannot be causally connected to points at infinity or to "holes" that have been cut in the manifold.

Escape 2. Drop causality requirements, e.g. the requirement that there are no CTCs. Again, this is not an escape that the advocates of individual branching want to use since they assume that branching is compatible with a globally consistent time order, which requires the existence of a global time function.[21] Moreover, this escape does not avoid Result 1 or the result of Alty (1995) neither of which uses causality requirements beyond time orientability.

Escape 3. Go to open universes where the space slices are non-compact. Then Results 2 and 3 do not apply. There are two problems with this escape. First, what if the actual universe is spatially closed? Note that cosmologists use the Friedmann–Robertson–Walker (FRW) models to describe the large scale structure of our universe, and the latest astronomical data is (barely) consistent with a $k = +1$ FRW universe, which has compact space slices. Second, going to spatially open universes does not escape the generalizations of Result 3 proved by Geroch (1967b) and Hawking (1992) if the branching is localized, i.e. is confined to a compact region of space.

Escape 4. Escape by permitting spacetime singularities. This escape can be described in two ways. First, continue to operate with the definition of a relativistic spacetime as a pair $\mathcal{M}, g_{ab}$ where $\mathcal{M}$ is a Hausdorff differentiable manifold, but allow that g_{ab} may not be defined at every point $p \in \mathcal{M}$. Then Results 1 and 3 do not apply even if $\mathcal{M}$ is a compact Hausdorff manifold with boundary, for both results implicitly assume that the metric is everywhere defined on $\mathcal{M}$. On the positive side, it is known that if g_{ab} is allowed to be undefined at (at most) a finite number of interior points of a manifold with boundary, then any two compact three manifolds are Lorentz cobordant; and, moreover, the causal structure is well defined even at the points where the metric is not defined (see Sorkin, 1990 and Borde et al., 1999).[22] Alternatively, one could stick to the original definition of a relativistic spacetime by deleting those interior points of the manifold $\mathcal{M}$ with boundary where the metric is ill-defined, with the result that the originally compact $\mathcal{M}$ is

[20] If the spacetime is not time orientable, then a CTC must be understood as a closed curve whose tangent is everywhere timelike, dropping the condition that the tangent is everywhere future pointing.

[21] However, the advocates of branching spacetimes might want to cite Sorkin's (1986a) interpretation of Result 1 above as showing that causality violations connected with topology changes will be confined to Planck scale regions of spacetime and, therefore are "harmless."

[22] The construction makes use of a *Morse function*. Let $\mathcal{M}$ be a compact n-manifold with boundary consisting of the disjoint union of the $(n-1)$-manifolds Σ_1 and Σ_2. A Morse function is a C^∞ map $f : \mathcal{M} \to [0,1]$ such that $f^{-1}(0) = \Sigma_1$ and $f^{-1}(1) = \Sigma_2$. The critical points of f are those for which $\partial_a f = 0$. These are assumed to be finite in number, and are non-degenerate in the sense that the matrix $\partial_a f \partial_b f$ is non-singular. Since $\mathcal{M}$ is compact it admits a Riemann metric h_{ab}. The associated More metric is then defined by $g_{ab} := (h^{cd} \partial_c f \partial_d f) h_{ab} - \varsigma \partial_a f \partial_b f$, where $\varsigma > 1$ is a real number. This Morse metric vanishes only at the (finite number) of critical points of the Morse function and is Lorentzian everywhere it does not vanish. The version of the construction used in Borde et al. (1999) assigns a causal future and a causal past even to the points where the metric vanishes.

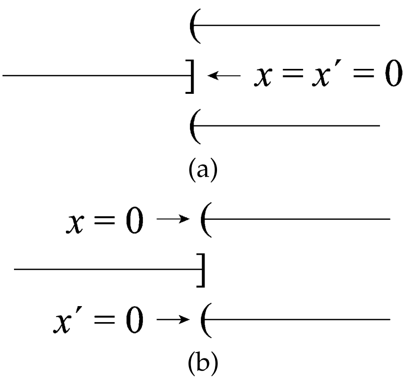

FIGURE 10.2 One-dimensional branching space. (a) The space is non-Euclidean at $x = x' = 0$. (b) The points $x = 0$ and $x' = 0$ are not Hausdorff separated.

no longer compact. Consider the first way of describing the escape. The following dilemma arises. Either there is an isometric embedding ι of $\mathcal{M}, g_{ab}$ into a spacetime $\mathcal{M}', g'_{ab}$ such that any point $p \in \mathcal{M}$ where the metric g_{ab} is not defined has an image point $p' = \iota(p)$ where the metric g'_{ab} is defined, or else not. If the first horn applies, then the no-go results have been evaded by artificially erasing the metric at various points of $\mathcal{M}$ (e.g. the crotch points in the trousers spacetime). Such an artifice ought to be disallowed in constructing models of physically reasonable spacetimes; for physical laws as we know them presuppose a spacetime metric and, thus, the points at which the metric has been erased are literally lawless locations. If the second horn applies, then the points of $\mathcal{M}$ at which the metric is not defined represent genuine singularities in the metric field. The singularity theorems of Hawking and Penrose indicate that spacetime singularities are a generic feature of the solutions of Einstein's field equations;[23] in this sense, classical GTR forces us to seize the second horn in some circumstances. But the advocates of individual branching would be engaging in wishful thinking if they relied on the mechanisms of singularity formation in classical GTR (e.g. gravitational collapse) to create singularities just where they think branching should happen. Furthermore, it is conjectured that quantum gravity effects will smooth out the spacetime singularities of classical GTR.[24]

Escape 5. Allow for spacetimes that are topological spaces but are not locally Euclidean. Such spaces certainly include branching structures, as illustrated for the case of a one-dimensional topological space in Figure 10.2(a) which is constructed as follows. Take two copies of the real line $\mathbb{R}$ coordinatized by x and x', and identify the points of the two copies such that $x = x'$ and $x, x' \leqslant 0$. The resulting tree structure is non-Euclidean at $x = x' = 0$.[25] But such topological spaces cannot be made into spacetimes in the sense of GTR. For topological spaces that

23 See Wald (1994, Ch. 9) for a review of these results. Most of the theorems use geodesic incompleteness as a criterion of the existence of singularities; geodesic incompleteness may or may not be connected with singularities in a more intuitive sense, e.g. the "blow up" of curvature scalars.

24 See Earman (2007b) for a review of some of the relevant considerations.

25 This construction corresponds to McCall's (1994, Appendix 1) "lower cut" option.

are not locally Euclidean cannot be assigned a differentiable structure, and such a structure is essential in formulating the very notion of a Lorentzian metric and in formulating the Einstein field equations.[26]In short, Escape 5 is not viable if one wants to do anything resembling classical GTR.

Escape 6. Keep the standard definition of a relativistic spacetime as a differentiable manifold but abandon the requirement that the manifold be Hausdorff. Now spacetime branching can occur even in spatially closed universes without abandoning temporal orientability, without having to swallow acausalities such as CTCs, and without violating Einstein's field equations, energy conditions, or the generic condition. To get a feel for how dropping the Hausdorff condition allows branching while still allowing the manifold structure needed to formulate GTR, modify the above example by identifying the points of the two copies of $\mathbb{R}$ such that $x = x'$ and $x, x' < 0$ (see Figure 10.2(b)). The result is a locally Euclidean space in which the points $x = 0$ and $x' = 0$ are not Hausdorff separated.[27] Explicit details on the construction of non-Hausdorff spacetimes can be found in Douglas (1995), McCabe (2005), and Visser (1996, 250–255). The non-Hausdorff branching can be global; e.g. take a $k = +1$ FRW model, and for some branch time $t = t_b$ attach non-Hausdorffly a future "leg" to produce a spacetime resembling the trousers universe of Figure 10.1.[28] Or the non-Hausdorff branching might be local; e.g. it could take place along the future light cones of selected spacetime points (see Penrose, 1979 and McCabe, 2005).

Escapes 1–5 have sufficiently high prices that they do seem to merit further consideration, leaving Escape 6 as the only serious contender. But as we will now see, the non-Hausdorff option has many unattractive features.

3.4 Shortcomings of the non-Hausdorff option

The assumption of Hausdorffness is explicitly invoked only sporadically in textbooks on general relativity. But it is implicitly assumed in so many standard results in GTR that dropping it would require a major rewriting of textbooks. Here are two examples of widely used results that depend on Hausdorffness. (i) A compact set of a topological space is closed—if the space is Hausdorff. (ii) If a sequence of points of a topological space converges, the limit point is unique—if the space is Hausdorff. The situation is best summed up by a dictum of Robert Wald in response to my query of what relativistic physics would be like without the assumption that the spacetime manifold is Hausdorff: "Asking what relativistic

[26] The metric g_{ab} is a bilinear map from pairs of tangent vectors of the manifold $\mathcal{M}$ to the real numbers. Einstein's field equations are formulated in terms of second-order derivatives of the metric, the definition of which relies on the differentiable structure of $\mathcal{M}$.

[27] This construction corresponds to McCall's (1994, Appendix 1) "upper cut" option.

[28] Here t is the global time function for the FRW spacetime metric as displayed in the line element

$$ds^2 = a(t)\left[\frac{dr^2}{1 - kr^2} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\varphi^2\right] - dt^2,$$

where $a(t)$ is the scale factor (sometimes called the radius of the universe) and $k = 0, -1$, or $+1$, corresponding respectively to space sections of zero curvature, constant negative curvature, and constant positive curvature.

physics would be like without Hausdorffness is like asking what the earth would be like without its atmosphere."[29]

Specific illustrations of difficulties for relativistic physics that would crop up in non-Hausdorff spacetimes are easy to produce. For instance, non-Hausdorff spacetimes can admit bifurcating geodesics; that is, there can be smooth mappings γ_1 and γ_2 from, say, $[0,1] \subset \mathbb{R}$ into $\mathcal{M}$ such that the image curves $\gamma_1[0,1]$ and $\gamma_1[0,1]$ are geodesics that agree for $[0,b)$, $0 < b < 1$, but have different endpoints $\gamma_1(1)$ and $\gamma_2(1)$.[30] This does not contradict a fundamental theorem for both Riemannian and Lorentzian manifolds which asserts the *local* existence and uniqueness of geodesics: consider any point $p \in \mathcal{M}$ and any (non-zero) vector $V_p \in T_p(\mathcal{M})$; then for some interval $-\delta < t < +\delta$, $\delta > 0$, there is a unique geodesic $\gamma(t)$ such that $\gamma(0) = p$ and $(d\gamma/dt)_{t=0} = V_p$. But in the presence of non-Hausdorff branching this local theorem cannot be used to conclude that each geodesic is contained in a unique maximal geodesic. According to GTR, the worldline of a massive test particle not acted upon by non-gravitational forces is a timelike geodesic. But how would such a particle know which branch of a bifurcating geodesic to follow? This problem has led general relativists to shun non-Hausdorff spacetimes that involve non-Hausdorff branching (see Hajicek 1970, 1971; and Miller, 1973).[31]

Another example concerns the failure of either local or global conservation laws. Local differential geometry can be developed per usual on non-Hausdorff spacetimes. In particular, one can impose the local conservation law $\nabla_a T^{ab} = 0$ for the stress energy-tensor T^{ab}.[32] The implicit assumption is that T^{ab} is continuous and, indeed, differentiable. But this assumption can be violated when a branch is non-Hausdorffly glued on unless T^{ab} is smoothly extended along the branch. Thus, to the above rhetorical question, "Which branch of a bifurcating geodesic should a free-falling test particle follow?", the answer should be "All!" if local conservation is to hold.

So suppose then that T^{ab} is smoothly extended along all branches. Then global conservation laws will be violated. To see this, consider how the local conservation law $\nabla_a T^{ab} = 0$ for the stress energy-tensor T^{ab} is normally integrated to give a global conservation law. Suppose that the spacetime $\mathcal{M}, g_{ab}$ is stationary, i.e. admits a non-vanishing timelike vector field V^a such that $\nabla_{(a} V_{b)} = 0$.[33] Define the momentum flow (associated with V^a) as $P^a := -T^{ab} V_b$; then $\nabla_a P^a = 0$. Suppose finally that T^{ab} satisfies the dominant energy condition;[34] then P^a is a future directed non-spacelike vector. Then using the spacetime version of Gauss' theorem it can be shown that total energy is conserved:

29 Private communication to the author.

30 Non-Hausdorffness is a necessary but not sufficient condition for the existence of bifurcating geodesics; see Hajicek (1971) where a necessary and sufficient condition for a non-Hausdorff spacetime to have no bifurcating geodesics is given. But such bifurcation is a product of the structure advocates of individual branching want and need to escape the no-go results on topology change given above.

31 Of course, those who want to use branching in individual spacetimes to express indeterminism may like bifurcating geodesics. They should read on.

32 ∇^a is the covariant derivative determined by the metric g_{ab}.

33 This is the invariant way of saying that the metric is not time dependent.

34 This condition, which is needed to prove that there is no superluminal transmission of energy-momentum, requires that for any timelike vector V^b, $-T^a_b V^b$ is a future directed timelike or null vector, is thought to be satisfied by all of the matter fields encountered in classical GTR; see Wald (1994, p. 219).

LEMMA. Let $\mathcal{M}, g_{ab}$ be a compact n-dimensional (Hausdorff) spacetime with boundary consisting of the disjoint union of two compact $(n-1)$-dimensional spacelike manifolds Σ_1 and Σ_2. Under the above assumptions about P^a, the energy $E(\Sigma_1) := \int_{\Sigma_1} P^a \, dS_a$ at "time" Σ_1 equals the energy $E(\Sigma_2) := \int_{\Sigma_2} P^a \, dS_a$ at "time" Σ_2.

This lemma on the conservation of energy can be generalized to the case of non-compact space slices when the support of P^a is confined to a finite world tube. But this lemma and its generalizations can fail for non-Hausdorff branching spacetimes if T^{ab} is smoothly extended along the branch—the total energy can increase with time. If "observers" are associated with non-branching segments of (possibly) branching worldlines, then no observer will ever be directly aware of a violation of conservation of energy. But an observer who believes in individual branching will have reason to believe that it occurs.

The non-Hausdorff modeling of spacetime branching also suffers from the inability to accommodate various ideas. (i) The idea that the laws of physics plus what happens up to and including a given moment of time do not uniquely fix the future. The non-Hausdorff branching can (at best) capture the idea that what happens up to but not including a given moment of time does not uniquely fix the future. To include the given moment would seem to require non-locally Euclidean spacetimes. (ii) Indeterminism could involve a radical "openness" of the future in the sense that, compatible with the laws of physics and the past history, there are possible global alternative at *each and every* future moment of time. But, consistent with spacetime being a manifold, this cannot be modeled in terms of non-Hausdorff branching at every moment. Note that ensemble branching has no trouble in expressing the ideas in (i) and (ii).

As a final drawback of non-Hausdorffness I will mention the lack of control that results from allowing non-Hausdorff manifolds. As noted in Section 3.1, branching spacetimes do not in themselves entail indeterminism in the proper sense of non-uniqueness of dynamical evolution. But allowing non-Hausdorff branching does undermine the determinism of classical GTR. If Hausdorffness is assumed, then GTR provides for a (locally in time) deterministic evolution: the basic theorem on the initial value formulation of Einstein's field equations shows that corresponding to appropriate initial data on a three manifold (thought of as specifying the state at a given instant), there exits a unique (up to diffeomorphism) maximal solution for which the initial value hypersurface is a Cauchy surface (see Theorem 10.2.2 of Wald (1994)).[35] The uniqueness result fails if non-Hausdorff branching is allowed. The failure may be applauded by advocates of individual branching. But general relativists will be appalled by the lack of control on the addition of non-Hausdorff branches. Suppose for sake of illustration that the large scale structure of our universe is captured by a FRW big bang model. Start with the standard Hausdorff version of this model, and at some time t_b attach non-Hausdorffly n branches. What determines the choice of t_b and n? Nothing in the laws of classical GTR or any natural extension thereof provides an answer.

35 This theorem says nothing about how "big" the unique solution is or whether it is maximal simpliciter. These issues are connected with the problem of cosmic censorship.

The point here is not peculiar to non-Hausdorff branching spacetimes but applies equally to other constructions that are explicitly or implicitly shunned by physicists. For example, suppose that the triple $\mathcal{M}, g_{ab}, T_{ab}$ satisfies Einstein's gravitational field equations and that T_{ab} satisfies standard energy conditions (say, the weak and dominant energy conditions). Choose any closed set of points $\mathcal{C} \subset \mathcal{M}$. Then surgically removing this set of points results in a triple $\mathcal{M} - \mathcal{C}, g_{ab}|_{\mathcal{M}-\mathcal{C}}, T^{ab}|_{\mathcal{M}-\mathcal{C}}$ that also satisfies Einstein's field equations and energy conditions. Conspiracy theorists who want to explain strange disappearances will revel in this construction. Don't bother to look in Florida or California for the children whose pictures appear on milk cartons. In fact, don't bother to look for them anywhere in spacetime; for their world lines have fallen into "holes" carved into the spacetime (i.e. take $\mathcal{C}$ to be the disjoint union of closed balls of $\mathcal{M}$). Creationists will also be delighted. To model how the universe could have begun at 10,000 B.C., take $\mathcal{C}$ to consist of all of those points on or to the past of a spacelike slice that corresponds to 10,000 years ago. The result is a universe that pops into existence with the fossils and all of the other physical artifacts that the deluded proponents of evolution take as evidence for their theory. Since classical GTR has no way to control the pathologies such models entail, they must be excluded in a sensible version of the theory. And, indeed, it is explicitly or implicitly assumed by general relativists that physically reasonable models of spacetime should be maximal, i.e., cannot be isometrically embedded as a proper subset of a larger spacetime.[36] Non-Hausdorff branching spacetimes are, I submit, in exactly the same league with "holey" spacetimes.

If non-Hausdorff spacetime branching is to be taken seriously, what is needed is a physical theory that prescribes the dynamics of branching. What the advocates of individual branching offer is not a theory but the nostrum that branching takes place when an indeterministic process produces an outcome plus *ad hoc* rules, e.g. there is branching to the future but not towards the past. This is no more satisfactory than saying in QM that a reduction of the state vector takes places when a measurement occurs. Unlike other theoretical terms employed by the theory, "measurement" is not a term that can play a role in explanations; it is rather something that requires analysis and explanation. Similarly, "outcome of an indeterministic process" is not a term of any theory of physics but something that needs analysis. Note that I am not criticizing state vector reduction *per se*. There are honest theories of state vector reduction that offer a dynamics that explains how and when state vector reduction takes place (see, for example, Pearle 1976, 1989). I know of no extant honest theories of how and when non-Hausdorff branching takes place.

Where might one look for a dynamics of non-Hausdorff branching? Classical GTR is a theory that provides a dynamics for spacetime; but it eschews intrinsic randomness. The situation is just the opposite for QM and quantum field theory: these theories deal with intrinsic randomness, but they operate on a fixed spacetime background. The Holy Grail of current physics research is a quantum theory of gravity—a theory that combines the insights of GTR and quantum physics.

36 Whether or not this condition is sufficient to exclude "holey" spacetimes is an interesting issue, but it is not one that needs to be discussed here.

In one sense, quantum gravity will probably render moot the issue of branching spacetime since at a fundamental level the spacetime of classical GTR will dissolve into quantum foam. But one can ask: In those circumstances when a classical general relativistic spacetime emerges in some semi-classical limit of quantum gravity, will the emergent spacetime be a non-Hausdorff branching spacetime? My conjecture is that the answer is negative. Some support for this conjecture comes from the finding that quantum fields display singular behavior when propagating on a $(1+1)$-dimensional trousers spacetime (see Anderson and DeWitt, 1986; Manogue et al., 1988). This has led to the opinion that the kind of topology change involved in a trousers spacetime, and more generally in any spacetime that violates causal continuity, will be suppressed in the sum over histories form of quantum gravity (see Anderson and DeWitt, 1986; Dowker, 2003).[37]

In summing up the discussion of Escape 6, I agree with Roger Penrose, who after a brief consideration of non-Hausdorff branching spacetimes, concluded: "I must … return firmly to sanity by repeating to myself three times: 'spacetime is a *Hausdorff* differentiable manifold; spacetime is a *Hausdorff* …'!" (1979, p. 595).

4. CONCLUSION

Some of the ways of implementing branching in individual spacetimes have what strike me as fatal defects. None of the objections to non-Hausdorff branching can be deemed to be fatal, but the cumulative weight of these objections seems to me justify the attitude that non-Hausdorff branching is to be viewed as a desperate last resort rather than a device to be used as the basis for a program aimed at interpreting scientific or ordinary language concepts. Overall, the recommendation to the proponents of the many worlds interpretation of QM or the Belnap school of branching spacetimes is to proceed without recourse to individual branching.

Regardless of the merits of my stance, the study of branching spacetimes reveals itself as a fruitful way to bring together issues from philosophy (e.g. understanding the "openness" of the future), general philosophy of science (e.g. the analysis of determinism), and the foundations of physics (e.g. the role of branching spacetimes in classical GTR and quantum gravity).

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[37] The goal of the sum over histories approach is to calculate transition amplitudes between two n-geometries representing the states of an $(n+1)$-dimensional spacetime at two different times. This is to be accomplished by summing over a weighted average of the actions associated with all of the possible histories connecting the two states.

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