Chapter 3 – Postulates

1. State of QM system completely specified by wavefunction $\Psi(x,t)$

$$P(x_0,t_0) = \Psi(x_0,t_0)^* \Psi(x_0,t_0)dx = \left|\Psi(x_0,t_0)\right|^2 dx$$

↑ probability of finding the particle within dx of x_0 at time t_0

$$\int_{-\infty}^{\infty} \left|\Psi(x,t)\right|^2 dx = 1$$

← probability of finding the particle somewhere

$\Rightarrow$ Ψ is single valued

Ψ and $\frac{d\Psi}{dx}$ are continuous

Ψ cannot be ∞ over a finite interval

2. Each observable is associated with a QM operator

position: $\hat{x} = x$

momentum: $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$

KE: $\hat{E}_{kin} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} = \frac{\hat{p}^2}{2m}$

PE: $\hat{E}_{pot} = V(x)$

total E: $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$

angular momentum: $\hat{l}_x = \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$

. . . .

QM operators are Hermitian
$\Rightarrow$ eigenvalues are real

$$\psi^*[A\psi]dx = \psi[A\psi]^*\mathrm{dx}$$

Suppose

$$A = \frac{d}{dx}, \quad \psi = e^{ix}$$

$$\int \psi^* A\psi dx = \int e^{-ix}(i)e^{ix}dx = \int i dx$$

$$\int \psi[A\psi]^* dx = \int e^{ix}(ie^{ix})^* dx = -\int i dx$$

$\frac{d}{dx}$ not Hermitian

3. In a single measurement of an observable associated with Â, only an eigenvalue of Â can be measured.

4. Expectation value:

$$< a >= \frac{\int_{-\infty}^{\infty} \Psi^* \hat{A} \Psi dx}{\int_{-\infty}^{\infty} \Psi^* \Psi dx}$$

← for normalization

Average of the observable a, if many measurements are done

If Ψ is an eigenfunction of Â, all measurements give the same result

If Ψ is not an eigenfunction of Â, it can be represented in terms of a sum over eigenfunctions

$$\Psi = \sum b_n \phi_n(x)$$

↑ eigenfunctions of Â

$$< a >= \sum |b_m|^2 a_m \ ,$$

assuming Ψ is normalized

Suppose $\psi(x) = \frac{1}{2}\phi_1(x) + \frac{\sqrt{3}}{2}\phi_2(x), \quad \phi_1, \phi_2$ being eigenfunctions of Â

$$\hat{A}\phi_1 = a_1\phi_1, \quad \hat{A}\phi_2 = a_2\phi_2$$

How frequently do we measure a_1? a_2?

$$\langle \psi | \hat{A} | \psi \rangle / \langle \psi | \psi \rangle$$
$$= \frac{1}{4}a_1 + \frac{3}{4}a_2$$

Each individual measurement gives either a_1 or a_2

Whichever one we get, if we do a subsequent measurement we will get the same value

5. The time evolution of a QM system is given by

$$i\hbar \frac{\partial \Psi}{\partial t}(x,t) = \hat{H} \ \Psi(x,t)$$

For stationary states, we can write

$$\Psi = \psi(x)e^{-iEt/\hbar}$$