Name ______________________

Chemistry 1410
Exam 1

(20 pts) 1. (a) e^{kx}, k real, is an acceptable eigenfunction of the momentum operator | $\hat{P}_x e^{kx}$ is not real — T **F**

(b) The energy of a quantum mechanical particle can never be zero free particle, $k=0 \Rightarrow E=0$ — T **F**

(c) Systems tend to act more classically as $T \to 0$. False this is the case as $T \to \infty$ — T **F**

(d) Consider two eigenfunctions ϕ_1 and ϕ_2 of an operator $\hat{A}$. $c_1\phi_1 + c_2\phi_2$ cannot be an eigenfunction of $\hat{A}$. If ϕ_1 and ϕ_2 are degenerate (have the same eigenvalue) then $c_1\phi_1 + c_2\phi_2$ is an eigenfunction of $\hat{A}$ — T **F**

(20 pts) 2. (a) Consider the ground state of the 1D particle-in-the-box problem where the box runs from x = 0 to x = a. If you do an individual measurement of the momentum, what value(s) can you obtain?

$$\psi = \frac{\sqrt{2}}{\sqrt{a}}\left[e^{i\pi x/a} + e^{-i\pi x/a}\right]\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{a}}\left[e^{i\pi x/a} + e^{-i\pi x/a}\right]$$

momentum eigenfunctions =

$e^{i\pi x/a}$ and $e^{-i\pi x/a}$

eigenvalues: $\frac{\hbar\pi}{a}, -\frac{\hbar\pi}{a}$

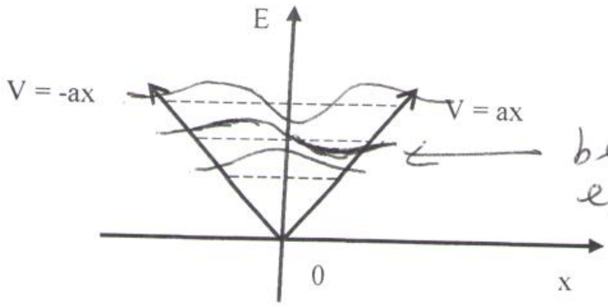
(b) What is the probability of the particle being at x = a/4?

$$P_1\left(x=\frac{a}{4}\right) = \frac{2}{a}\left[\sin\left(\frac{\pi a}{4a}\right)\right]^2 = \frac{2}{a}\left(\sin\frac{\pi}{4}\right)^2 = \frac{2}{a}\,\frac{1}{2} = \frac{1}{a}$$

(c) Consider the first excited state of the system. What is the probability of the particle being at a/2?

$P_2\left(x=\frac{a}{2}\right)$ for the first excited state $= 0$ as this is the position of the node in the wavefunction

(20 pts) 3. Consider the following potential. The dashed lines schematically represent the energies of the first three energy levels.



be sure to show exponential fall off in the tunelling region.

(a) For each of these levels, sketch on the figure the corresponding wavefunction.

(b) Do you expect the energy levels to increase more or less steeply in energy with increasing quantum number n than those of the particle-in-the box problem? Justify your answer.

As one proceeds to higher energy, the width of the potential grows, in contrast to the particle-in-the-box problem. Thus we expect a less steep increase in energy with increasing quantum number

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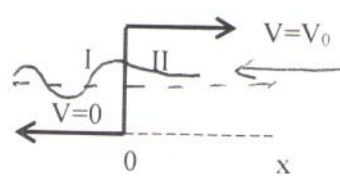
$m_e = 0.00059$ amu; $\mu_H = 0.0005896$ amu, $\mu_D = 0.0005898$

$E = -\frac{m_e e^4}{8 \varepsilon_0^2 h^2}$ • So need to multiply E by $\frac{\mu_H}{m_e}$ and by $\frac{\mu_D}{m_e}$,

(20 pts) 4. In the expression for the energy levels of the H atom, the mass should actually be the reduced mass rather than the mass of e^- alone. Recall that the reduced mass μ for two objects of mass m_1 and m_2 is $\mu = \frac{m_1 m_2}{m_1 + m_2}$. Realizing this, predict the change in the ionization potential (in eV) in going from H to D (deuterium) atom.

(20 pts) 5. Consider the potential shown to the right.



falls off exponentially in this region

(a) Consider a particle incident from the left with $E < V_0$. Sketch the wavefunction (superimposed on the above figure).

$\psi_I = Ae^{\cdot} + Be^{\cdots}$
$\psi_{II} = C e^{-\kappa x}$, $\kappa = \sqrt{2m(V-E)}/\hbar$

(b) Write the wavefunction in regions I and II.

at $x=0$ $\left.\begin{array}{l} \psi_I(0) = \psi_{II}(0) \\ \psi_I'(0) = \psi_{II}'(0) \end{array}\right]$ from this and normalization one can determine the coefficients A, B, C

(c) Describe how you would go about solving the Schrödinger Equation for this problem. You do not have to work the problem to completion, but only to state how you would go about solving it.